Another Interpretation of Bathtub Curve

Hiroe Tsubaki (tsubaki@ism.ac.jp)
Director, Risk Analysis Research Centre,
The Institute of Statistical Mathematics

The Erl King", by Albert Sterner, ca. 1910
http://www.answers.com/topic/der-erlk-nig

The Oldest Twins; Kin-san Gin-san

http://www.geocities.co.jp/SilkRoad-Ocean/2002/nati0000.htm

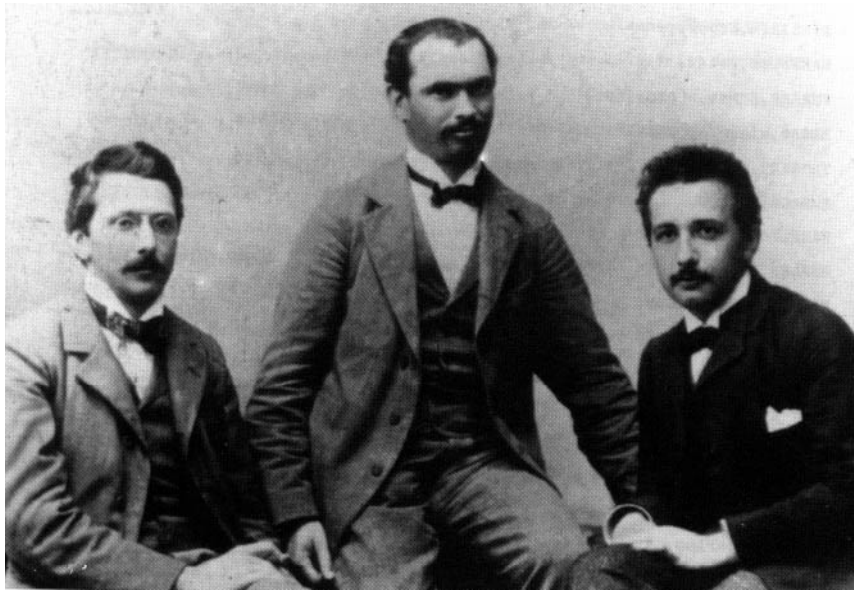
Contents

- The Grammar of Science
  - Classification before modeling
- Three Case Studies
  - Residual Analysis of Regression of non-anonymized data
  - Analysis of Outliers
  - Another Interpretation of Bathtub Curve
    - Mixture of Normal, Weak and Strong Populations
- Concluding Remarks

Two Typical Misinterpretations to Science & Statistics

- Business is beyond the scope of Science
  - Why Business Sciences?
    - Business is material for science!
  - Statistics is a kind of applied mathematics
    - Why Statistical Methodology?
      - Statistics is the grammar of science

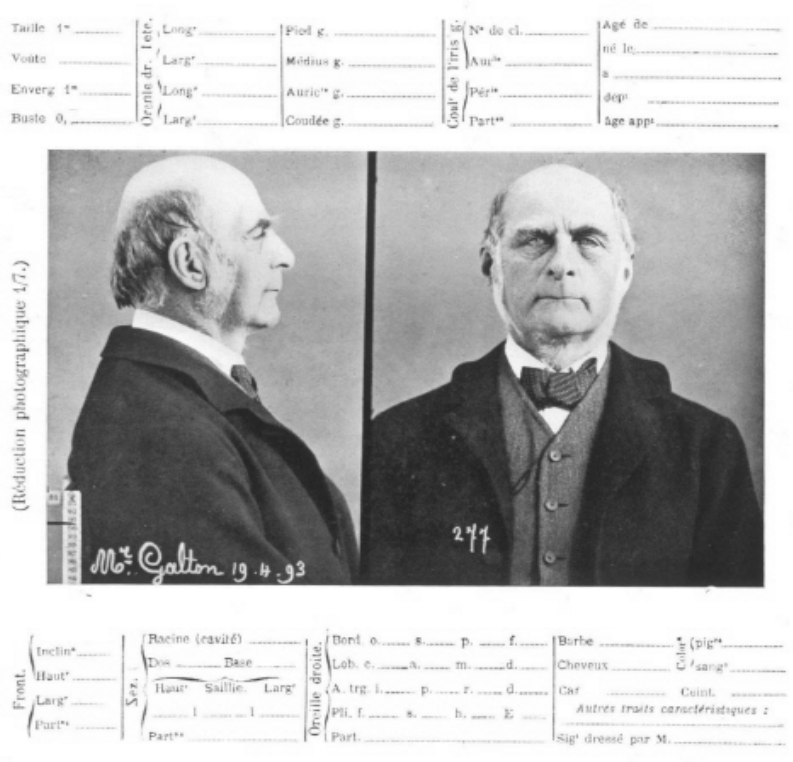
Definition of Science



Akademie Olympia

- **The attempt to make the chaotic diversity of *our sense experience* correspond to a logically uniform system of thought**
  - A. Einstein, 1940

Statistical Science



- To discover methods of condensing information concerning large groups of allied facts into brief and compendious expressions suitable for discussion
  - Francis Galton (1883)
    - *Inquiries into Human Faculty and its Development*

http://www.mugu.com/galton/

Business is also material for science

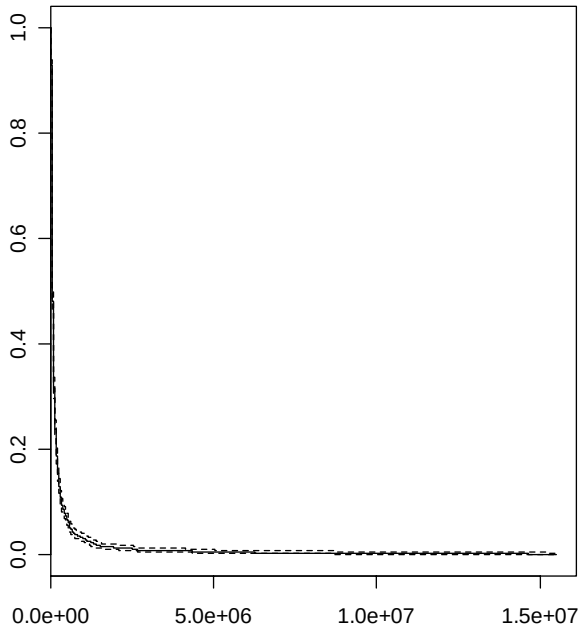
- Karl Pearson, 1892
  - ***The unity of all science consists in its method, not in its material.***
  - The field of science is unlimited; its material is endless, every group of natural phenomena, every phase of social life, every stage of past or present development is material for science.
  - The man who **classifies fact** of any kind whatever, who sees their **mutual relations** and describes their **consequences**, is applying the scientific method and is a man of science

www-groups.dcs.st-and.ac.uk/ ~history/Mathematicians/Pearson.html

Karl Pearson (1892) *The Grammar of Science*

- *A man gives a law to Nature*
  - Statistical Science as "a new way" to Scientific thinking
    - **Systematic ways** to derive a scientific law (= model)
    - Not Scientific Objects but **Scientific Process**
      - Model Planning: Statistical Methods for Planning
        - » ***Careful and accurate classification of facts***
        - » Observation of their correlation and sequence
      - Do (Fitting Model) : Constructing Scientific Laws
        - » Discovery of scientific laws by aid of creative imagination
      - C: Checking the Laws
        - » Self-criticism and the final touchstone of equal validity for all normally constituted minds
    - Development of Statistical Methodology as the Supporting tools for the Grammar
      - Statistical and Probabilistic **interpretation of causes and effects**
      - Statistical description of a scientific law

Case Studies



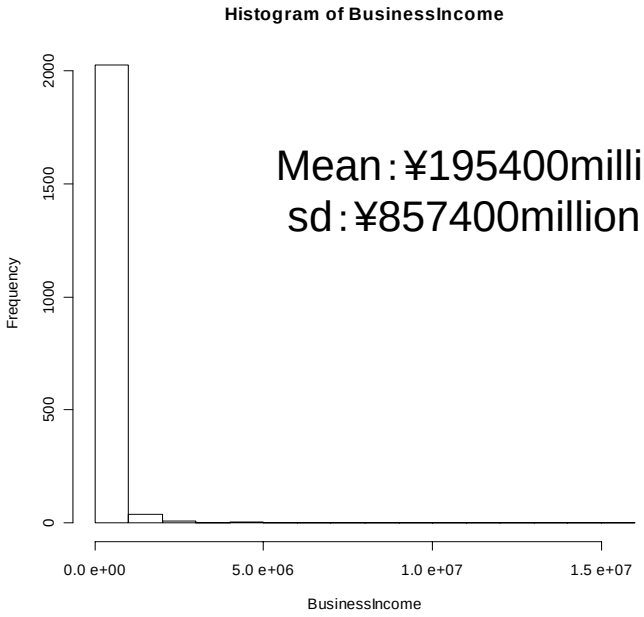
Cob-Douglas Production Function and Prediction of Profitability for Japanese Listed Enterprises

Sales Incomes and Profitability are hypothetically regarded as Survival Time

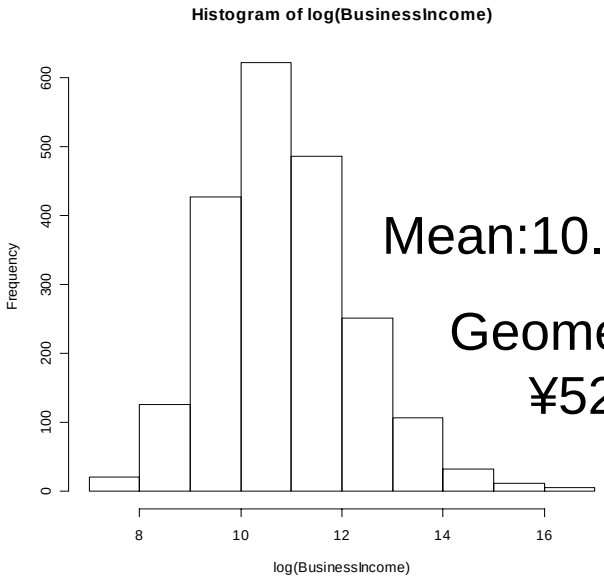
Laws in Japanese Financial data？

- # of all the Japanese listed enterprises in Japan in 1996 : 2091
- Output Variable
  - Business Income
- Input Variables
  - Total Asset, Working Force, Net Debt etc.
- Others
  - Company Name, Industrial Code

Description of Dispersion
Sales Income

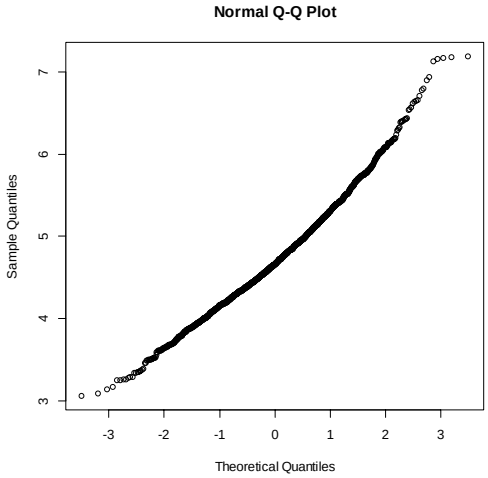


Mean：¥195400million
sd：¥857400million



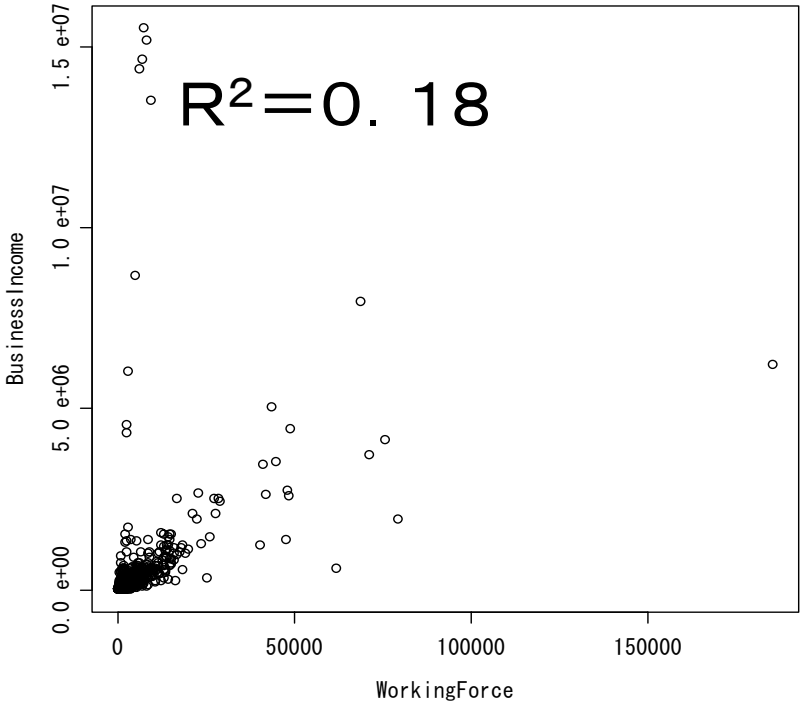
Mean:10.9 sd:1.40

Geometric Mean:
¥52560million

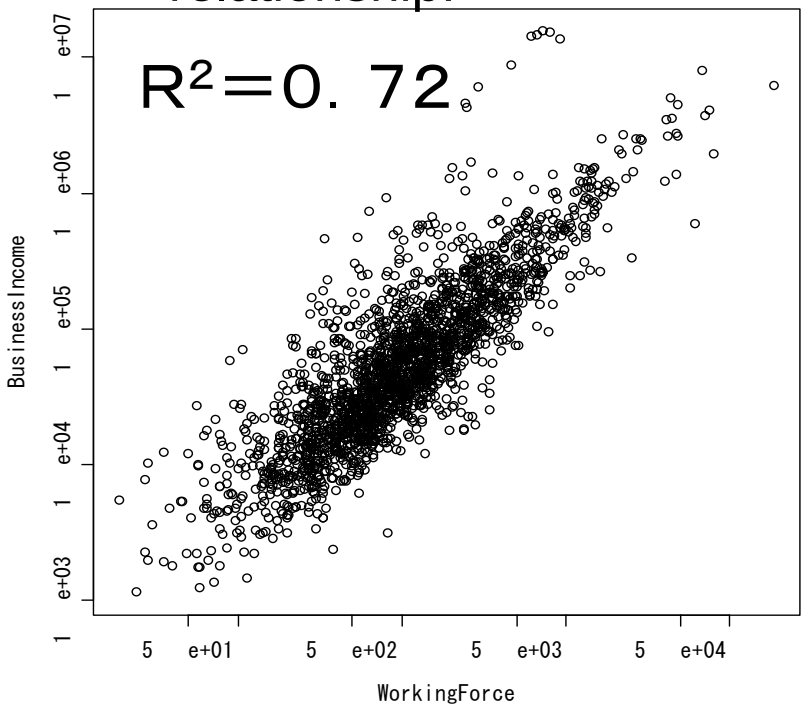


STEP 1 Planning
Description of Association

Linear relationship?



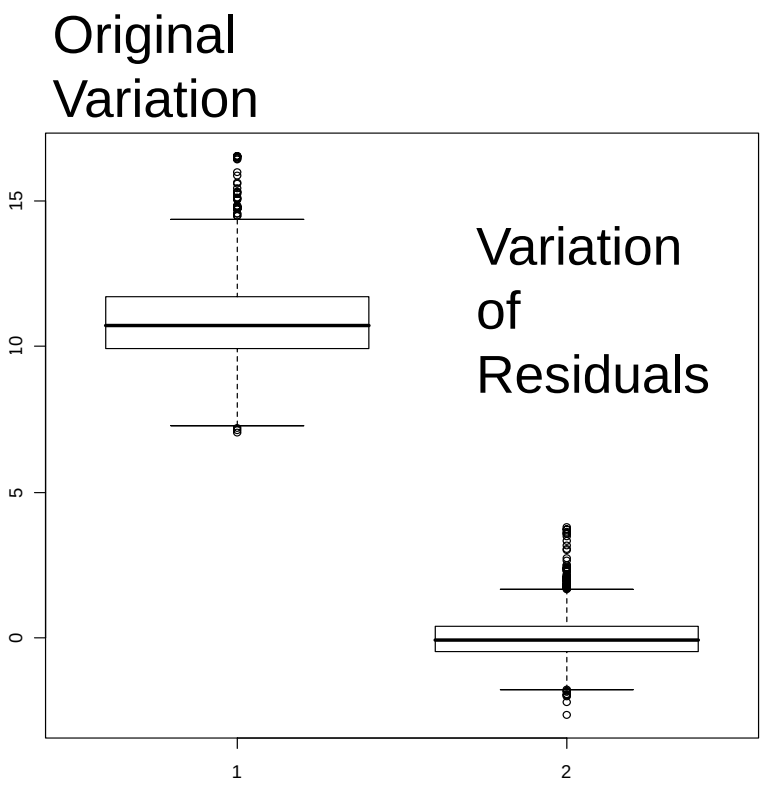
Log Linear relationship!

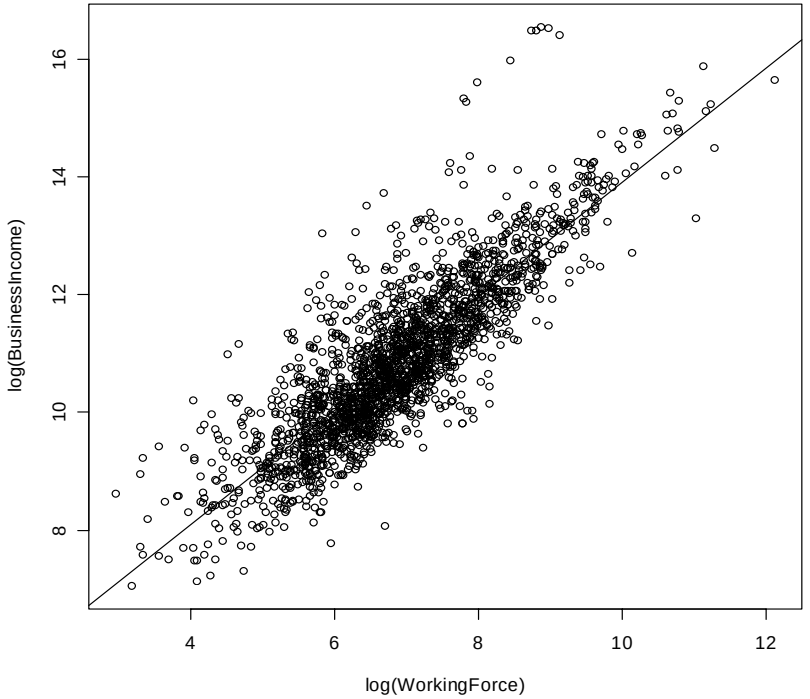


Step 2: Model Fitting

- Fitting the model or the hypothetical law to the related facts (data) to get the empirical law
  - Regression Analysis
    - Log (Sales Income) =4.24+0.97 log (Working Force)+residuals standard deviation of residual=0.74
      - sd of log Business Income = 1.40

Fitting Model





Step 3: Checking the Fitted Model

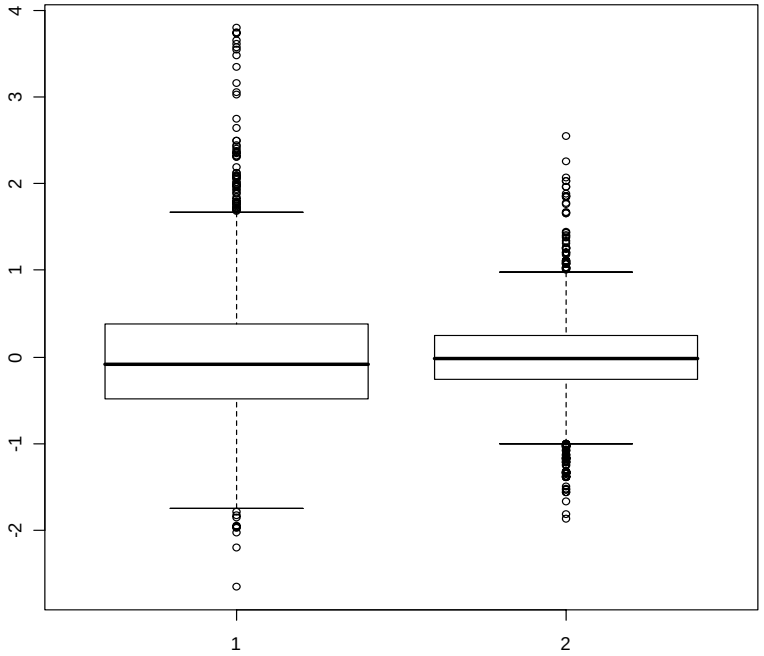
- Checking performance of the obtained law to clarify the needs of classification of the facts
  - Diagnostics
    - Total Performance Measures of the model
      - R^2, Residual SD
    - Exploring Needs for Further Classification
      - Residual Analysis

Evolution of Model

- Log(SI)=1.15
  +0.28 log(WF)
  +0.46log(Total Assets)
  +0.27log(Net Debt)
  +residuals
  - R^2=0.89
    residual SD=0.47
    - Residuals SD of the simple model =0.74
    - Total performance of the prediction model is significantly improved.

Residuals of the simple model

Residuals of the new model



Residual Analysis Clarifies Needs of Classification:
Companies such that the residuals>1.5

・	1500	ITOCHU	1.854051
・	1501	Marubeni	1.853562
・	1502	TOMEN	1.835570
・	1503	Nichimen	1.670540
・	1518	KANEMATSU	1.761537
・	1528	CHUO GYORUI	2.258017
・	1529	MITSUI & CO.	1.660330
・	1536	TOHTO SUISAN	2.064938
・	1537	TSUKIJI UOICHIBA	1.967221
・	1539	OSAKA UOICHIBA	1.878274
・	1542	DAITO GYORUI	2.032328
・	1548	SUMITOMO	1.960161
・	1557	Nissho Iwai	1.662237
・	1564	TOKYO SANGYO	2.552716
・	1625	CHUBU SUISAN	1.769532
・	2090	SHINKO GYORUI	2.031702

Companies such that the residuals< -1.3

・	9	Chugai Mining	-1.521919
・	480	KYOWA HAKKO KOGYO	-1.392347
・	548	Green Cross	-1.819432
・	568	INTERNATIONAL REAGENTS	-1.557378
・	955	ISEKI & CO.	-1.372723
・	1142	SANYO ELECTRIC	-1.338558
・	1762	HOKKAIDO SHINKO	-1.536938
・	1781	TOBU RAILWAY	-1.322300
・	1786	Keihin Electric Express Railway	-1.354466
・	1787	Odakyu Electric Railway	-1.333317
・	1789	Keisei Electric Railway	-1.389206
・	1798	Kinki Nippon Railway	-1.387198
・	1800	HANSHIN ELECTRIC RAILWAY	-1.381091
・	1801	Nankai Electric Railway	-1.490005
・	1803	Kobe Electric Railway	-1.667814
・	1804	Nagoya Railroad	-1.348491
・	1807	Sanyo Electric Railway	-1.558611
・	1876	Nihonbashi Warehouse	-1.327809
・	1945	WESCO	-1.861918
・	1954	Koshien Tochi Kigyo	-1.340120
・	1980	KYOTO HOTEL	-1.328580

Needs for Classification

- After Classification
  - Commerce (#181):
    - ~0.27+0.08logWF+0.77logTA+0.21logND
      - Residual SD: 0.51 R^2:0.89
  - Transportation(#51):
    - ~1.86+0.64logWF+0.59logTA-0.24logND
      - Residual SD: 0.35 R^2:0.93
  - Others:
    - ~1.26+0.38logWF+0.40logTA+0.24logND
      - Residual SD: 0.38 R^2:0.92
- **But if Data were anonymized**?.

Analysis of Residuals by Rank Logit Modeling

- Qualitative Choice by Ascending Order
  - Proportional Hazard Model: Weak population
    - $\text{Log}(f/(1-F))=\log \lambda(t) + \boldsymbol{\beta}^T\mathbf{X}$
      - **Erl-king Selects Children.**
- ***Qualitative Choice by Descending Order***
  - **Proportional Reverse Hazard Model: Strong Population**
    - $\text{Log}(f/F)=\log \rho(t) + \boldsymbol{\beta}^T\mathbf{X}$
      - **God Celebrates Kinsan and Ginsan : Amadeus**

Rank Order Logit Regression of Residuals (Ascending Order)

	coef	exp(coef)	se(coef)	z	p
log(CurrentAsset)	0.048	1.05	0.048	1.006	0.310
log(LongTermAsset)	**-0.097**	**0.90**	**0.035**	**-2.765**	**0.005**
log(LongTermDebt + 0.5)	-0.000	0.99	0.006	-0.117	0.910
log(CurrentDebt)	-0.096	0.90	0.049	-1.941	0.052
log(PersonnelExpense) **However**	**0.142**	**1.15**	**0.038**	**3.732**	**0.000**
log(AdvertiseExpenses+0.5)	0.004	1.00	0.006	0.666	0.510
log(Exp&ResearchExp+0.5)	**0.024**	**1.02**	**0.006**	**3.680**	**0.000**

- Wald test = 40.7 on 7 df, p=9.31e-07
- Red: Accelerating Negative Residuals
- If the residuals greater than i-th quartile could be regarded as censored data, the Wald test statistics become
  - i=1 Wald Statistics = 10.5 (P Value = 0.161)
  - i=2 Wald Statistics = 10.9 (P Value = 0.143)
  - i=3 Wald Statistics = 28.0 (P Value = 0.0001)
  - **i=4 Wald Statistics = 40.7 (P Value = 9.31e-07)**
    - Should all the data be commonly regarded as complete data? **No!**

Rank Order Logit Regression (Proportional Reverse Hazard Modeling) of Residuals (Descending Order)

	coef	exp(coef)	se(coef)	z	p
log(CurrentAsset)	-0.027	0.97	0.048	-0.565	5.7e-01
log(LongTermAsset)	**-0.030**	**0.97**	**0.032**	**-0.967**	**3.3e-01**
log(LongTermDebt + 0.5)	-0.001	0.99	0.006	-0.278	7.8e-01
log(CurrentDebt)	**-0.093**	**0.91**	**0.047**	**-1.948**	**5.1e-02**
log(PersonnelExpense)	**0.129**	**1.13**	**0.032**	**3.939**	**8.2e-05**
log(AdvertiseExpense+0.5)	**0.007**	**1.00**	**0.007**	**1.014**	**3.1e-01**
log(ExperimentalAndResearchExpense+0.5)	**0.033**	**1.03**	**0.006**	**5.257**	**1.5e-07**

- Wald test = 56.1 on 7 df, p=8.93e-10
- If we regarded the residuals less than i-th quartile as censoring, the Wald test statistics become
  - ***i=1 Wald Statistics = 64.8 (P Value = 1.65e-11)***
  - i=2 Wald Statistics = 21.5 (P Value = 0.0031)
  - i=3 Wald Statistics = 18.9 (P Value = 0.0087)
  - **i=4 Wald Statistics = 56.1 (P Value = 8.93e-10)**
- **At least 25% data might be affected by specific qualitative choice mechanism!**

Rank Order Logit Regression of Descending Order Residuals with 75% Censoring

	coef	exp(coef)	se(coef)	z	p
log(CurrentAsset) but	-0.222	0.80	0.092	-2.412	1.6e-02
log(LongTermAsset)	0.211	1.23	0.061	3.432	6.0e-04
log(LongTermDebt + 0.5)	0.005	1.00	0.013	0.428	6.7e-01
log(CurrentDebt)	0.238	1.26	0.097	2.438	1.5e-02
log(PersonnelExpense)but	-0.229	0.79	0.060	-3.802	1.4e-04
log(AdvertiseExp+0.5)but	0.014	1.01	0.013	1.117	2.6e-01
log(Exp&ResExp+0.5)	-0.056	0.94	0.013	-4.241	2.2e-05

Blue: Accelerating positive residuals

- Wald test = 64.8 on 7 df, p=1.65e-11

- Enterprises, the residuals of which are greater than the 1st quartile, are specifically affected by R&D and PE negatively.

Selection or Classification by Erking & (Ama)Deus

The Erl King", by Albert Sterner, ca. 1910
http://www.answers.com/topic/der-erlk-nig

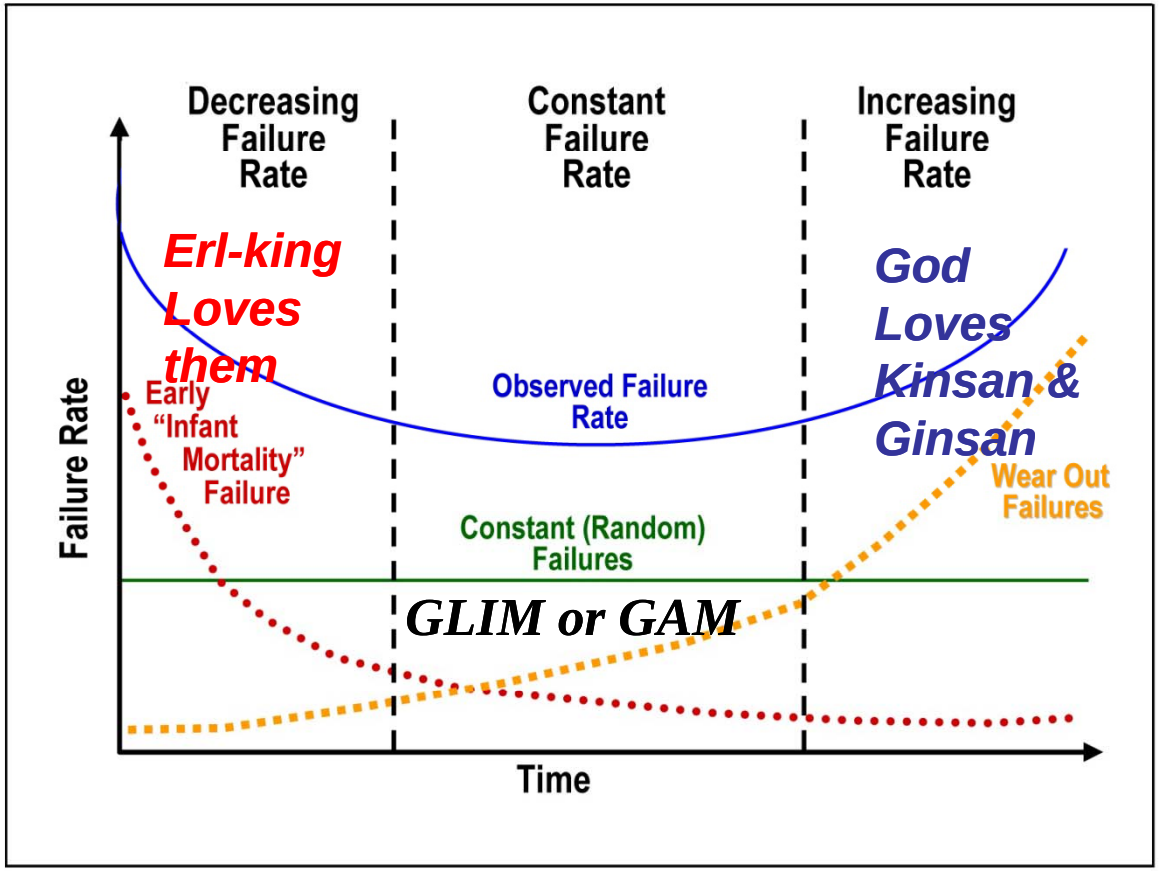
The Oldest Twins; Kin-san Gin-san

http://www.geocities.co.jp/SilkRoad-Ocean/2002/nati0000.htm

Typical Interpretation of Bath-tub Hazard functions

Is it generally true?

Interpreting by Qualitative Choices!



Two types of LSI:

Finite life time

Infinite life time as Kinsan and Ginsan

http://en.wikipedia.org/wiki/Image:Bathtub_curve.jpg

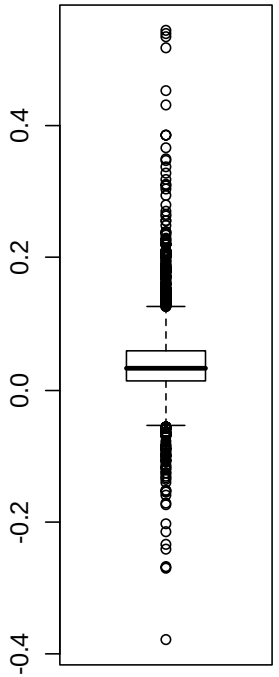
2nd Example
Predicting Profit Ratio

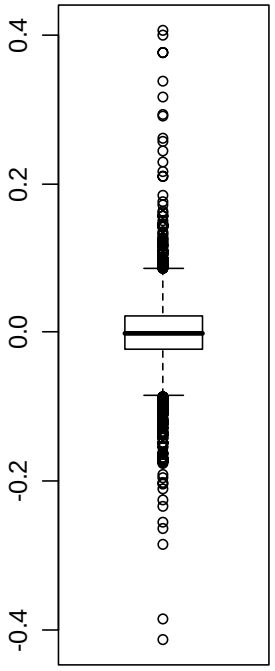
- Dependent variable : profitability
  - -log(1-Gross Profit/Sales Income) ~ Gross Profit Rate
- Independent variables
  - Total Asset, Sales Income, Fixed Liability, Floating Liability,
  - Working Force, Average Salary, Research & Development Expense
- OLS
  - RMSE=0.0543, Adjusted R^2=0.263
- Classification: not residuals but original profitability
  - Mixtures of Weak Population, Normal Population and Strong Population
    - Proportional hazard and reverse proportional hazard with appropriate censoring

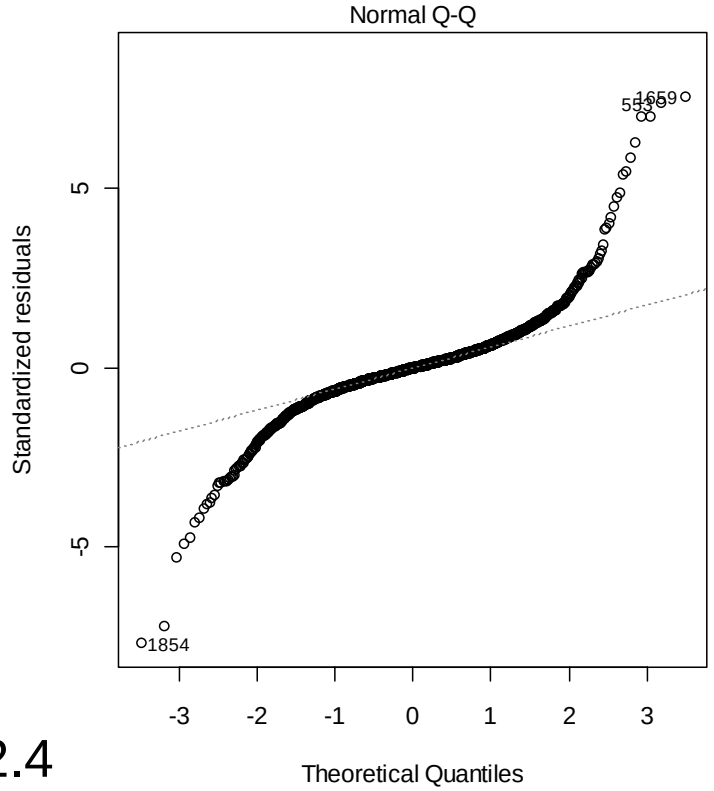
Simple linear prediction of profitability

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	-0.0628244	0.0193732	-3.243	0.00120	**
log(Asset)	0.1011546	0.0044365	22.801	< 2e-16	***
log(Sales Income)	-0.0163968	0.0028198	-5.815	7.00e-09	***
log(Floating Liability)	-0.0465029	0.0027301	-17.033	< 2e-16	***
log(Fixed liability)	-0.0054217	0.0010289	-5.270	1.51e-07	***
log(Capital)	-0.0214088	0.0022269	-9.614	< 2e-16	***
Average Age	-0.0018365	0.0003425	-5.362	9.15e-08	***
log(R & D Expense)	0.0011787	0.0003532	3.338	0.00086	***
log(#Employee)	-0.0095458	0.0019798	-4.822	1.53e-06	***

Box-plot of Profitability(Left)
Box-plot of the residuals(Center)
Normal Probability Plot（Right）







Kurtosis=12.4

Weak Population ~ 9.8%

Results of Wald Statistics for $\beta = \mathbf{0}$, where data more than q-quantiles are treated as censoring data

q	0.01	0.05	0.10	0.25	0.50	0.75	1.00
Wald χ^2	84.3	212.8	242.4	234.6	414.5	596.3	750.3

Comparison of rank logit (proportional hazard) analysis of weak population and total population

Independent Variable	q_M=0.098 Coefficients	q_M=0.098 Z-value	q=1.00 Coefficients	q=1.00 Z-value
log(Asset)	-1.2564**	-4.87	-2.1561**	-22.22
log(Income)	**-1.3719****	-8.55	**0.8052****	13.73
log(Float L.)	0.9187**	5.90	0.8774**	14.97
log(Fixed L.)	0.1876**	2.82	0.1346**	6.31
log(Capital)	0.7642**	6.46	0.2433**	5.58
Ave. Age	0.0339	1.74	0.0355**	5.88
log(R&D Exp.)	-0.0249	-1.06	-0.0249**	-3.94
log(# Employee)	0.3393**	2.74	0.0987**	2.56

Strong Population ~ 31 % ⇒ Normal Population ~59.2 %

Wald Statistics for β =0, where data less than 1-q quartile are treated as censoring data

q	0.01	0.05	0.10	0.25	0.50	0.75	1.00
Wald χ^2	89.8	302.1	482.8	657.5	668.5	520.8	325.8

Comparison of reverse rank logit(reverse proportional hazard) analysis of strong population and total population

Independent variable	q_M=0.31 Coefficients	q_M=0.31 Z-vlue	q=0.37 Coefficients	q=0.37 Z-value	q=1.00 Coefficients	q=1.00 Z-value
Log(Asset)	2.4964	19.96**	2.3525	20.17**	0.97117**	12.428
log(Income)	**-0.8770**	-9.79**	**-0.7984**	-9.70**	-0.07361	-1.531
log(Fl. L)	-0.9413	-13.27**	-0.9096	-13.69**	-0.50562**	-11.426
log(Fixed L)	-0.0989	-3.41**	-0.1067	-3.97**	-0.05111**	-2.653
log(Capital)	-0.3256	-4.72**	-0.3251	-5.09**	-0.23350**	-6.212
Ave. Age	-0.0571	-5.10**	-0.0524	-5.05**	-0.03064**	-4.841
log(R&D)	0.0330	2.91**	0.0381	3.68**	0.02622**	3.927
log(#Emp)	**-0.1763**	-2.73**	-0.1166	-1.95	-0.00366	-0.105

Analysis of the Normal Population
OLSE after excluding the weak and strong populations from the analysis

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.0335826	0.0064448	5.211	2.20e-07	***
log(Asset)	0.0119804	0.0018034	6.643	4.60e-11	***
log(Sales Income)	-0.0075150	0.0008858	-8.484	< 2e-16	***
log(Fl. L.)	-0.0043485	0.0010411	-4.177	3.16e-05	***
log(Fixed L.)	-0.0005030	0.0003660	-1.374	0.169558	
log(Capital)	-0.0014063	0.0007498	-1.876	0.060955	.
Ave. Age	-0.0003476	0.0001123	-3.094	0.002016	**
log(R&D)	0.0002754	0.0001150	2.394	0.016811	*
log(#Emp.)	0.0021215	0.0006136	3.458	0.000564	***

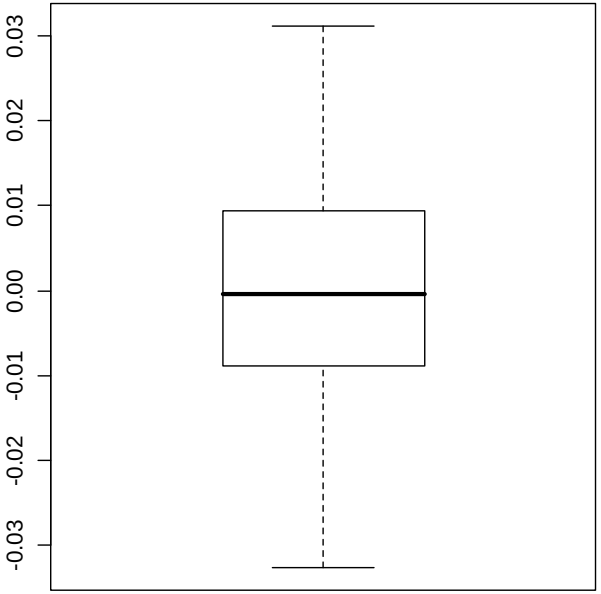
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

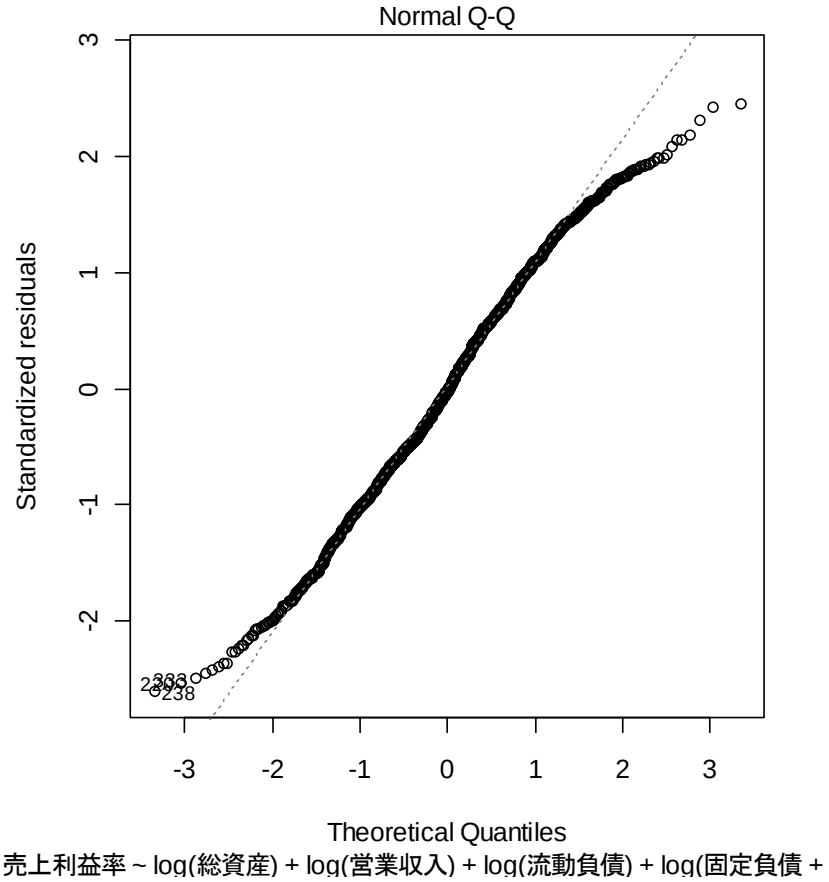
Residual standard error: 0.01284 on 1229 degrees of freedom

Multiple R-squared: 0.1205, Adjusted R-squared: 0.1147

F-statistic: 21.04 on 8 and 1229 DF, p-value: < 2.2e-16

Distribution of residuals of the normal population analysis
Kurtosis= -0.63

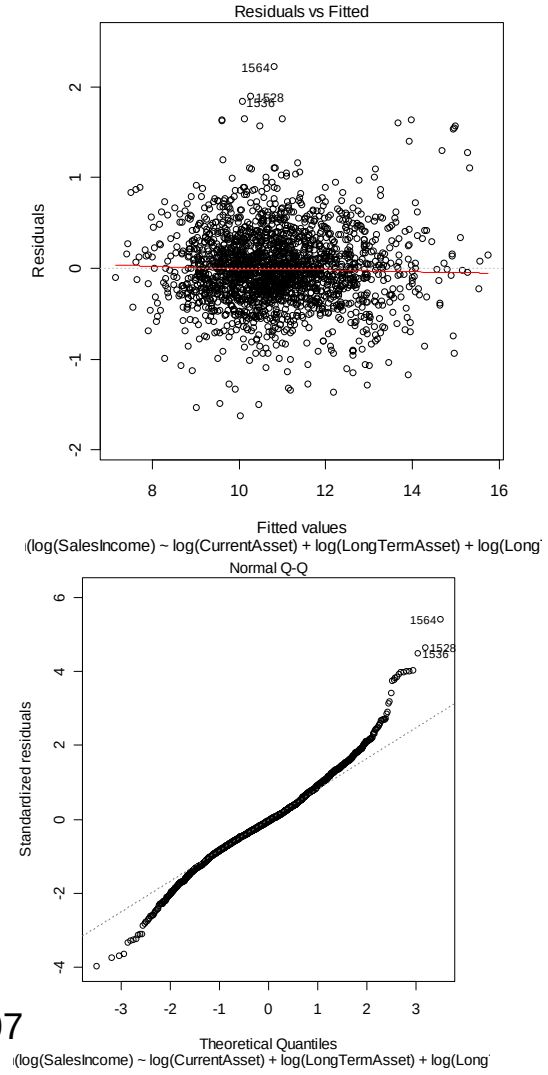




Little more elaborated models, Prediction of log(Sales Income)

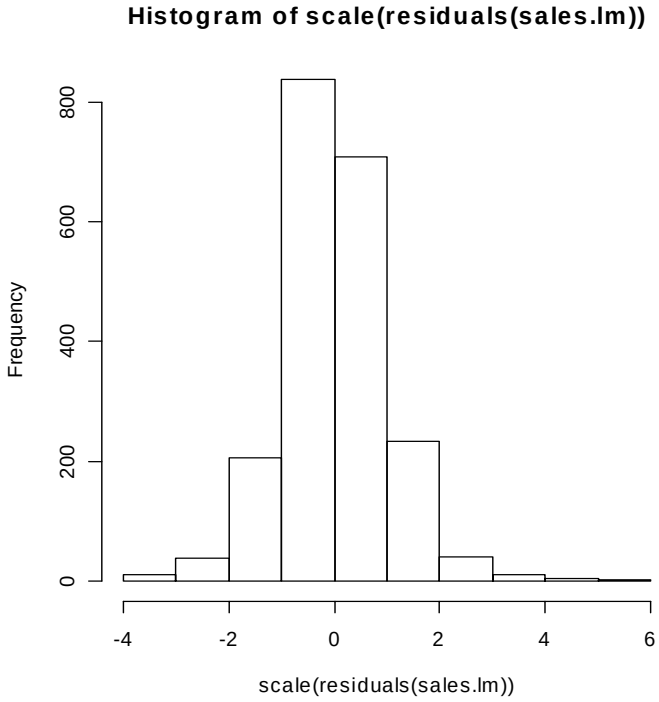
```
Residuals:
    Min      1Q  Median      3Q     Max
-1.6209 -0.2348 -0.0205  0.2273  2.2194
Coefficients: red: risk factor, blue: anti-risk factor
                              Estimate  SE   t value Pr(>|t|)
(Intercept)                    0.798   0.074  10.7  < 2e-16 ***
log(CurrentAsset)              0.245   0.019  12.3  < 2e-16 ***
log(LongTermAsset)             0.085   0.013   6.3 2.11e-10 ***
log(LongTermDebt + 0.5)       -0.020   0.002  -7.6 3.25e-14 ***
log(CurrentDebt)               0.414   0.020  20.6  < 2e-16 ***
log(PersonnelExpense)          0.303   0.014  21.4  < 2e-16 ***
log(AdvertiseExpense+0.5)      0.010   0.003   3.9 8.41e-05 ***
log(Exp&ResearchExpense+0.5) -0.022   0.003  -8.6  < 2e-16 ***???
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4095 on 2083 degrees of freedom
Multiple R-Squared: 0.9144,     Adjusted R-squared: 0.9141
```

Classical Regression Diagnostics Again



Slightly **Fat Tailed Residual Distribution**

No Remarkable Correlation between Linear Predicts of Sales Income and the corresponding Residuals from the view points of Quantitative Modeling



Concluding Remarks

- Quantitative Modeling (as GLIM or GAM) + Qualitative Choice Modeling of ***the both direction*** will be essentially useful in general statistical risk analysis and its diagnostics
  - Quantitative residuals are not orthogonal to predictors in terms of qualitative choice models

- Future Work
  - More Cases
  - Formal Mixture Inferences of Quantitative Modeling and Qualitative Choice Modeling using Weibull and Gumbel Distributions
  - Treatment of Censoring Data

I hope my twins shall be selected as Kinsan and Ginsan