The Institute of Statistical Mathematics 26-28 July, 2012
Extreme Value Theory & It's Application

Two types of extrapolations for examining sea extremes

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Viaggio: 2012 0615-0708 t kitano


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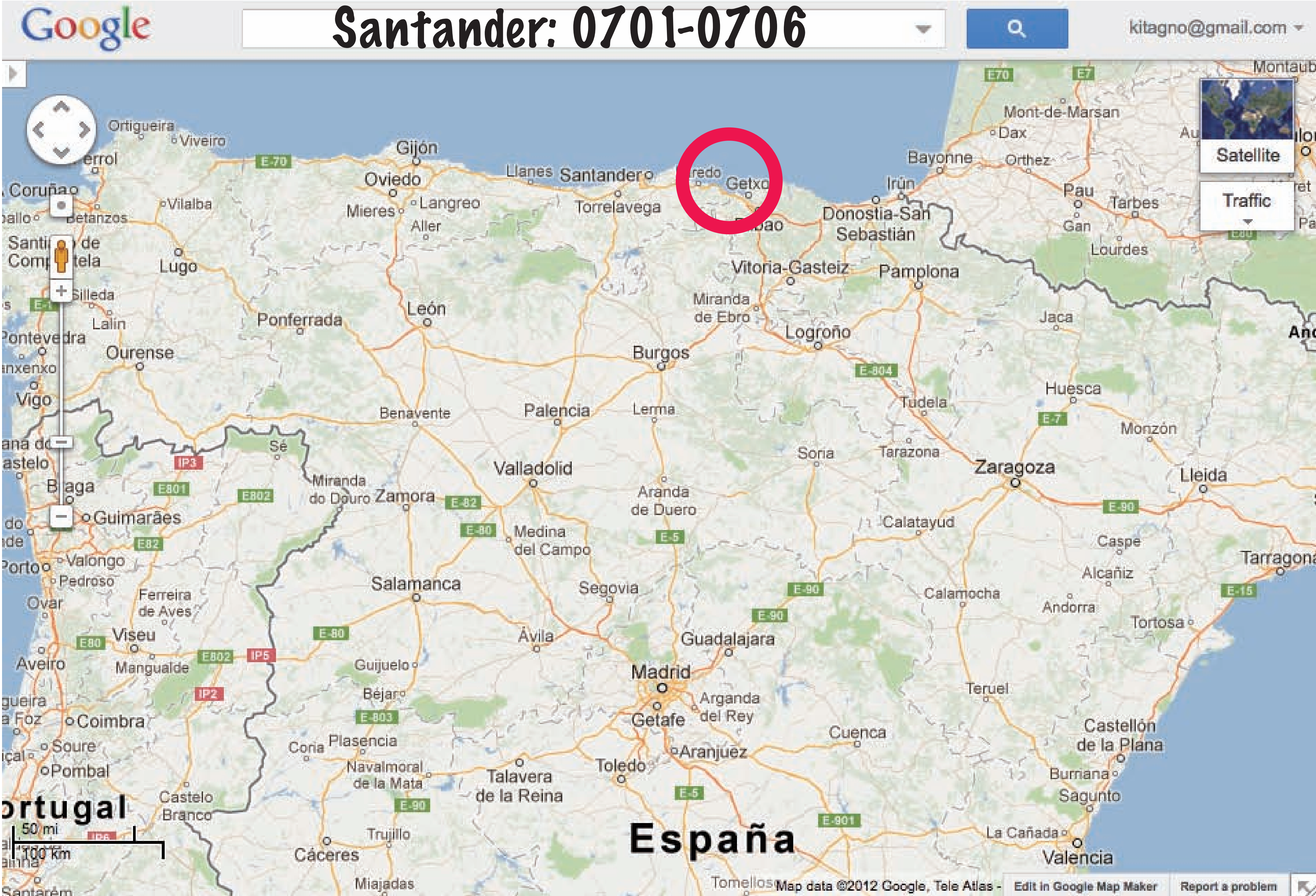
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International Conference on Coastal Engineering
ICCE2012
Santander Spain July 1-6 2012

Long Wave & RunUp Workshop

June 29-30, 2012, Santander, Spain. **Click here for more info.**

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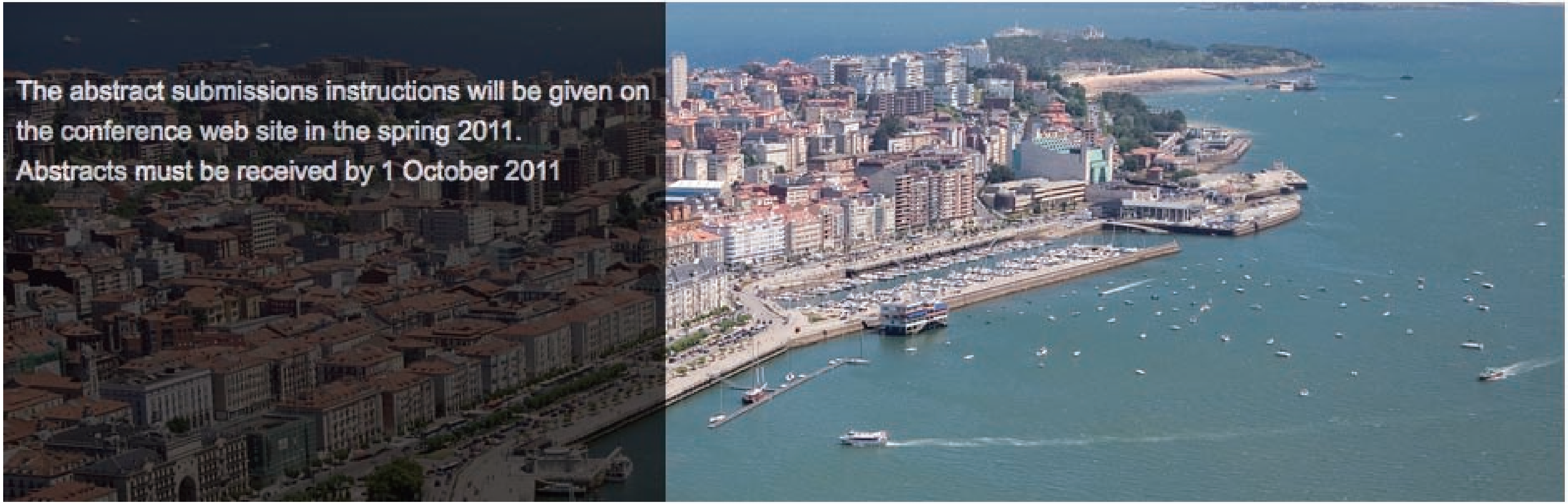

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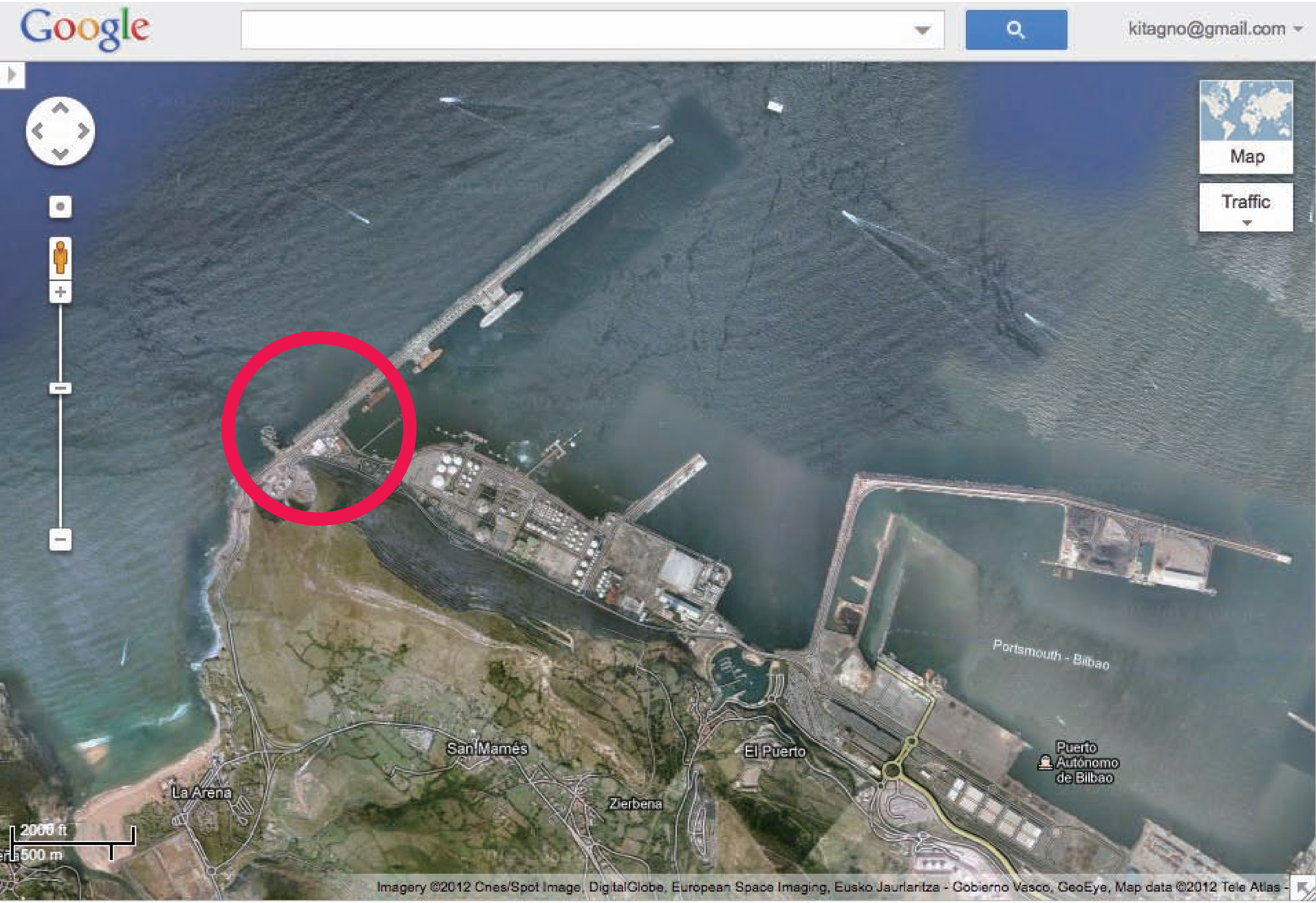

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OFFSHORE WAVE CLIMATE

An extreme statistics for offshore storm waves of different directions of propagation was estimated by Prof. Y. Goda, mainly based on 13 years (1976-1988) of scalar Waverider buoy records located just outside the bay in 30 m and 50 m water depths, visual wave data for the Bay of Biscay for the period 1950-1985 provided by National Climatic Data Center of the US Navy, Ashville, and hindcast of larger storms in the period 1955-1981 provided by the Danish Hydraulic Institute.

Only larger storms with offshore wave directions within the sectors NW, NNW, N can have significant impact on the breakwaters. Table 1 gives the central estimate of return period of max significant wave heights $\hat{H}_s$ within single storms and the estimated standard deviations σ covering the statistical uncertainty due to limited data and an empirically determined uncertainty due to unknown true distribution.

Table 1. Estimated long term "offshore" wave climate at bay entrance in 30 m water depth.

Return period (year)	central estimates all directions $\hat{H}_s$ (m)	σ (m)	10% exceedence probability estimates NW H_s	NNW H_s	N H_s
1	6.4	0.5	6.7	6.0	5.0
10	8.3	0.6	8.6	7.7	6.4
50	9.5	0.9	10.1	9.0	7.5
100	10.0	1.0	10.7	9.6	7.9
200	10.5	1.2	11.4	10.7	8.4
500	11.1	1.4	12.3	11.0	9.0

Burcharth et al. (1995): Design of the Ciervana breakwater, Bilbao, Proc. of Coastal Structure and Breakwater.

The Commercial Harbor of Bilbao

The city of Bilbao, located at the northeast coast of Spain, was founded in 1300 as an Administrative Center for the control of harbor activities along the Nervion River and the Bay of Bilbao. In 1511, the Consulate of Bilbao, an old version of the Chamber of Commerce was created. In 1872, the Administration of the harbor was transfered to the Federal Government.

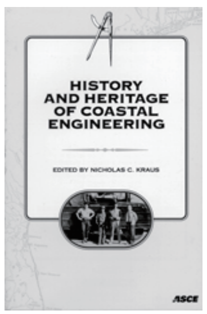
At that time, the entrance bar limited the development of the harbor. To solve the entrance problem the construction of the jetty of Portugalete was started in 1877, (Fig. 20). In 1901, the Harbor Authority finished the construction of the east breakwater, and, in 1902, King Alfonso XII placed the first stone of the breakwater *Dique de Santurce* in 20 m water (Fig. 21). During the construction, a storm destroyed part of the breakwater, and it was decided to start the construction again leewards of the destroyed structure, under its protection which worked as a submerged breakwater.

In the early 1970s, a new breakwater 2,500 m long was designed, *Dique de Pta. Lucero*, in 33 m water depth. Similar to the *Dique de Santurce*, during the construction, several storms delayed the completion of the works for several years. Actually, it may be said that the quantity of quarry used for the construction of the core was enough to build it twice. In December 1976, a storm with $Hs > 8.5$ m damaged several sections of the breakwater. The wave buoy failed after recording a wave height of 16 m. The breakwater was rebuilt with a new main layer of 150 Tn concrete blocks (Fig. 22).

494 COASTAL ENGINEERING HISTORY/HERITAGE

Nowadays, a new 3,150 m long breakwater is being built in the leeside of the *Dique de Pta. Lucero* (Fig. 23). The cross section of the breakwater is the traditional section used in Spain, following Iribarren's methodology: a main layer with a screen wall. The armor units are concrete 100-Tn. blocks (Fig. 24). The head of the breakwater is built with a caisson of approximately 29 m length. Again, during the construction, it has suffered some damages.

Bilbao is a very good example of the difficulties coastal engineers are facing to provide adequate protection against the wind waves generated in the Bay of Biscay.



History and Heritage of Coastal Engineering, ASCE, 1996

Edited by Nicholas C. Kraus

International Conference on Coastal Engineering
ICCE2012
Santander Spain July 1-6 2012

OUTLIER SENSITIVITY ON THE SEA EXTREMES BY THE TEMPORAL AND CLIMATE INDEX COVARIATIONS

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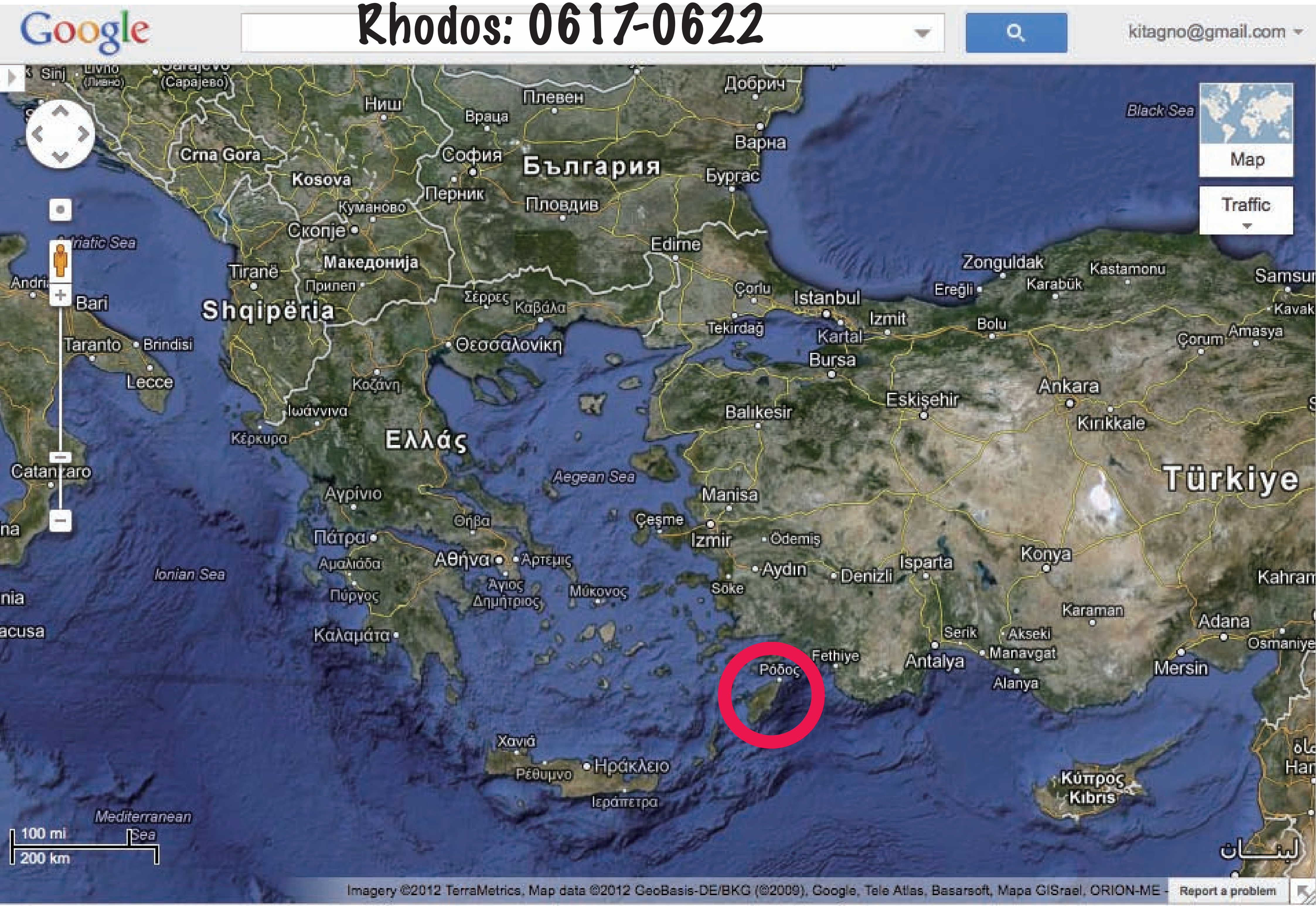
Nagoya Inst. of Tech.
& Rinya TAKAHASHI Kobe Univ.

Rhodos: 0617-0622



ISOPE-2012
Rhodes

ISOPE-2012 Rhodes Conference

The 22nd International Ocean and Polar Engineering Conference

Rodos Palace Hotel, Rhodes (Rodos), Greece, June 17–22, 2012

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Important Dates

Final Manuscript
March 24, 2012

News and Updates

Registration Form

Technical Program

Session List

Hotel Reservation
Rodos Palace Hotel:
Tower/Executive Rooms
Sold out
Try Garden Room!

The 22nd International Offshore (Ocean) and Polar Engineering Conference will be held in Rhodes, Greece fron 17 to 22, 2012. The conference will cover the following technical areas and special symposia:

Offshore Technology & Ocean Engineering
Frontier Energy Resources Technology
Renewable Energy & Environment
Geotechnical Engineering
Offshore Mechanics
Hydrodynamics & CFD
Sloshing Dynamics & Design
Tsunami and Safety
Coastal Engineering
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Subsea, Pipelines, Risers, Positioning

High-Performance Materials Symposium
Nanotechnologies For Clean Energy
Strain-Based Design
Arctic Materials
Corrosion Control
Polar Science & Tech
Advanced Ship Technology
Underwater Systems & Oceanology
CFD & Computational Mechanics
Metocean
ISO, Codes and Standards

ARCTIC-2012: The 3rd Arctic Science and Technology Symposium
ARCTIC M-2012: The 2nd Arctic Materials Symposium

ISOPE-2012 Rhodes

Degree of Experience & Durability

- Indices for Two Types of Extrapolating Sea Extremes

Toshikazu Kitano, Wataru Kioka
Nagoya Institute of Technology
Rinya Takahashi
Kobe University

* In this study, we discuss on the restrictions on the statistical analysis for the design wave heights and the design sea levels.

* Extreme value analysis is a technique of extrapolating the observed data set for the target return period. However, we have NOT recognized the different types of extrapolations, NOR been aware of the limitations.

Return period:

We employ 50, 100 years (breakwaters in Japan), 200 yrs (principal rivers in Japan) and 1250 yrs (standards for the Netherland dikes).

Question: Does this return period lie on the time axis, extending from the present to the future?

In other words, will the validity of the estimating 50 years return level be kept over the 50 years for future?

Return period

is just the reciprocal number of the exceedance probability, or the occurence rate,

$$\frac{1 \text{ year}}{50 \text{ years}} \text{ or } \frac{1 \text{ time}}{50 \text{ years}}$$

It is NOT a part of real time.
We have been unconcerned about this fact, or
we don't have any tool for recognizing the return period and the elapsed time toward the future,
even if we are aware of distinguishing this feature.

Compare these figures!
Same data (on the y axis),
but different positions (arranged on the x axis)

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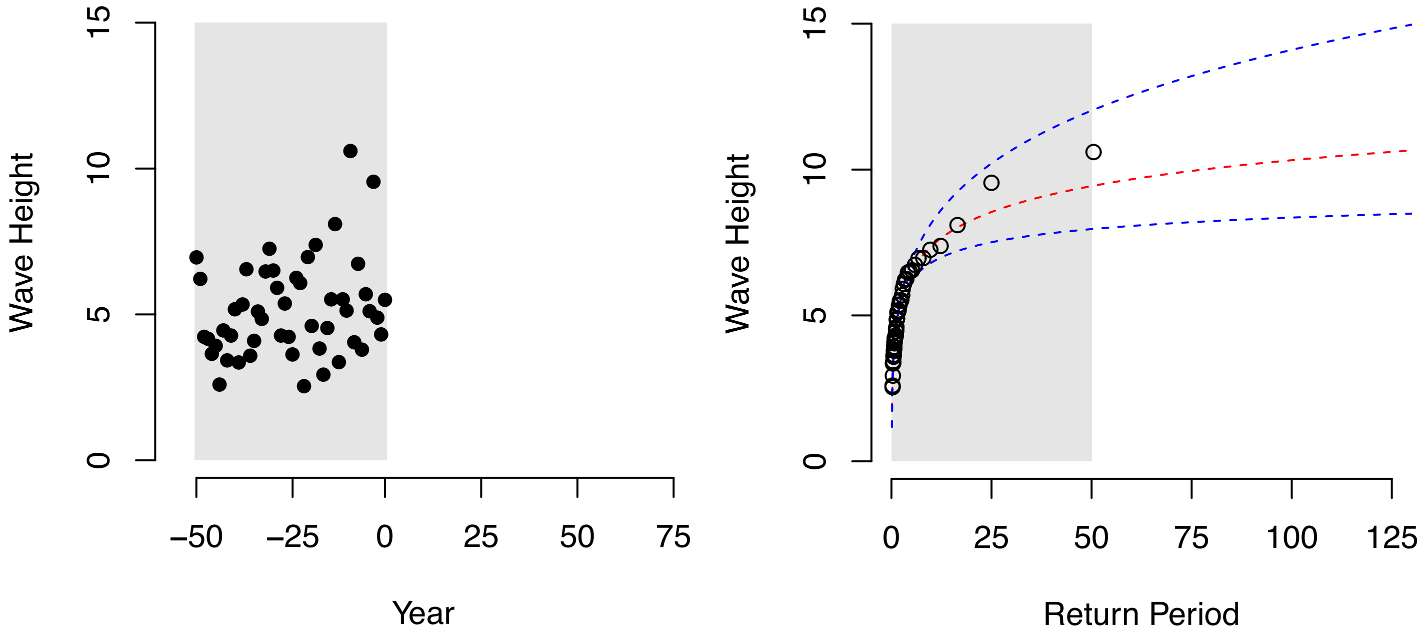

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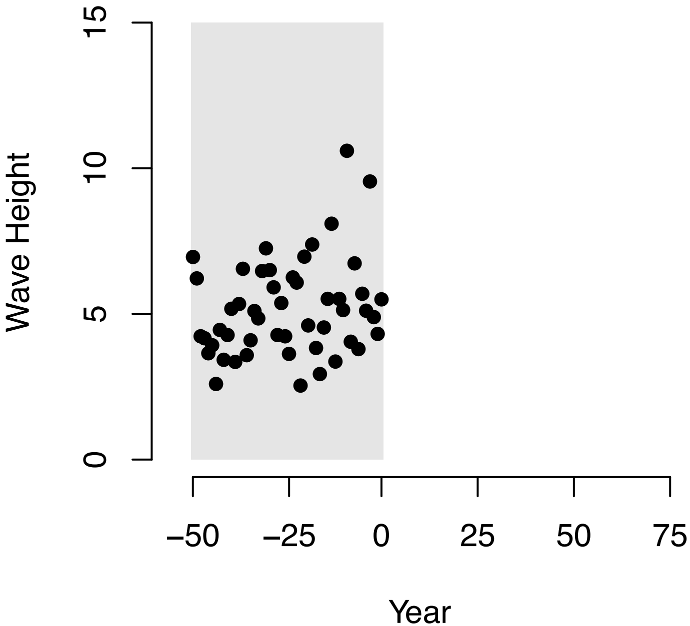
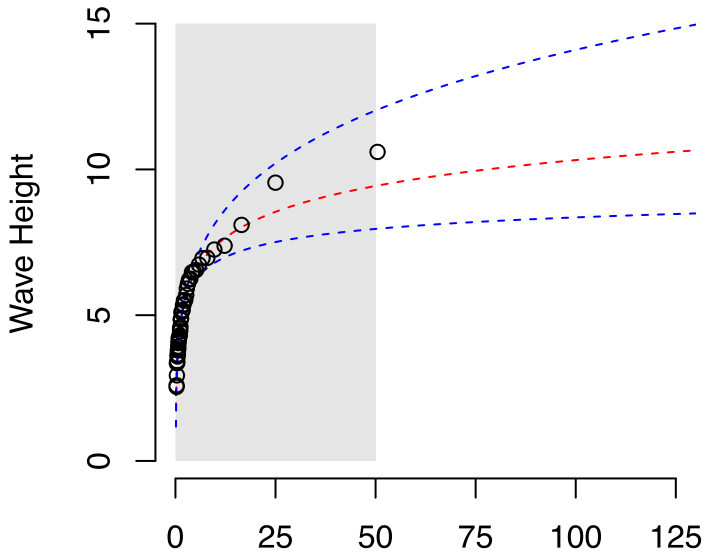
Compare these figures!
Same data (on the y axis),
but different positions (arranged on the x axis)



Real time



just 1/occurence rate, or
1/exceedance probability

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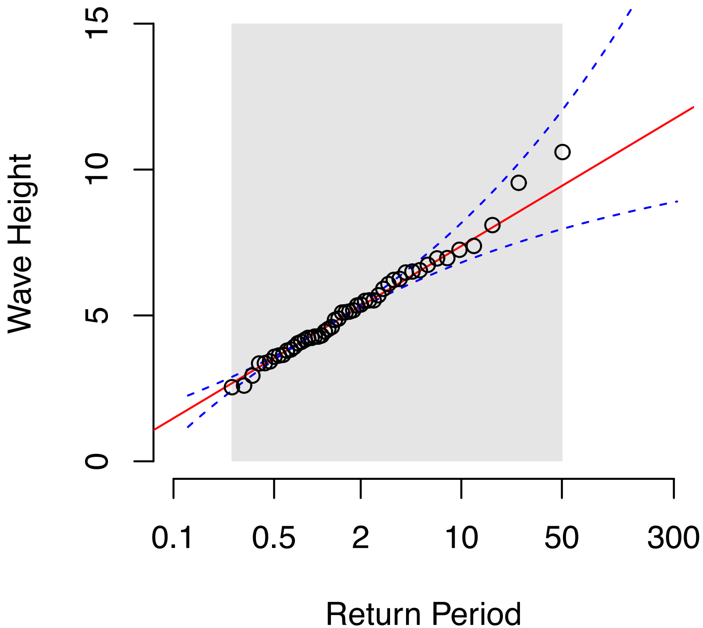
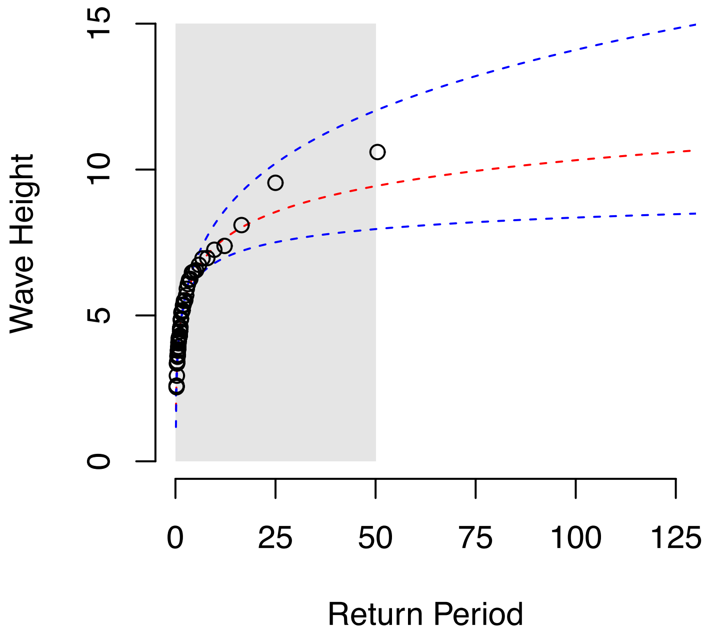
Occ. rate can be rearranged again in different scales. (log <- normal)

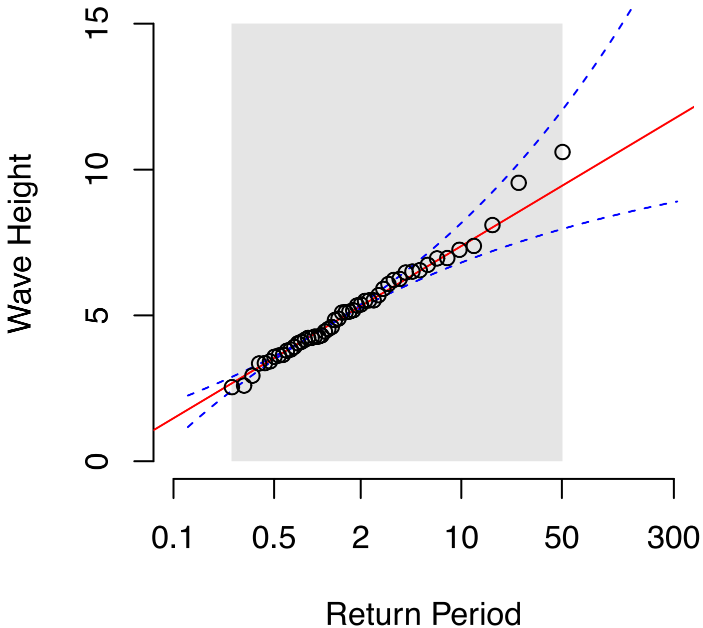


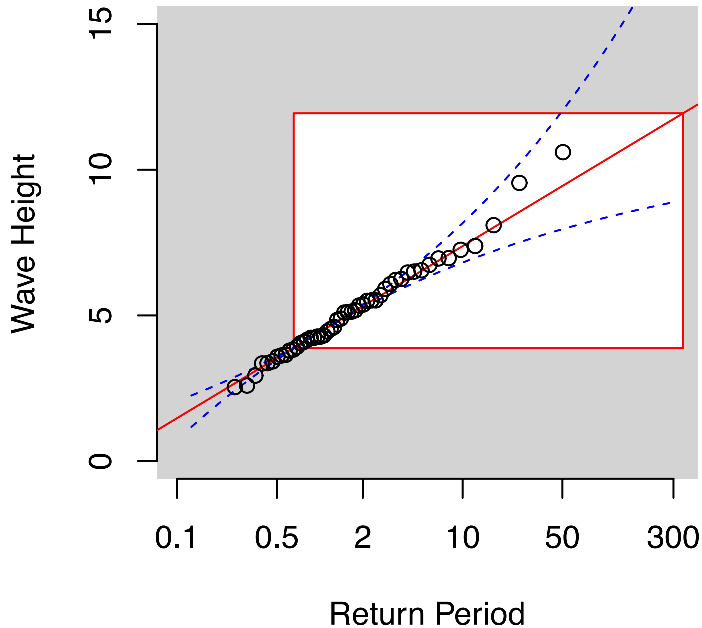
Log scale



Normal scale

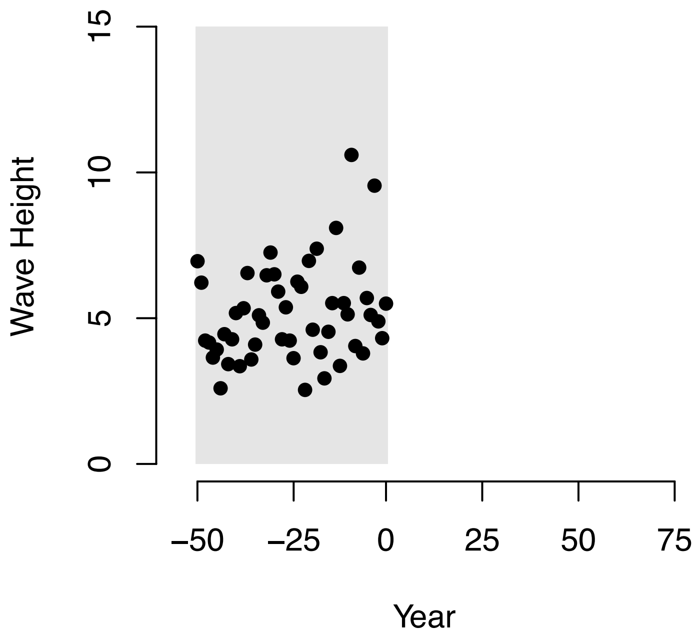
We can NOT extrapolate the fitted line without the limitation.



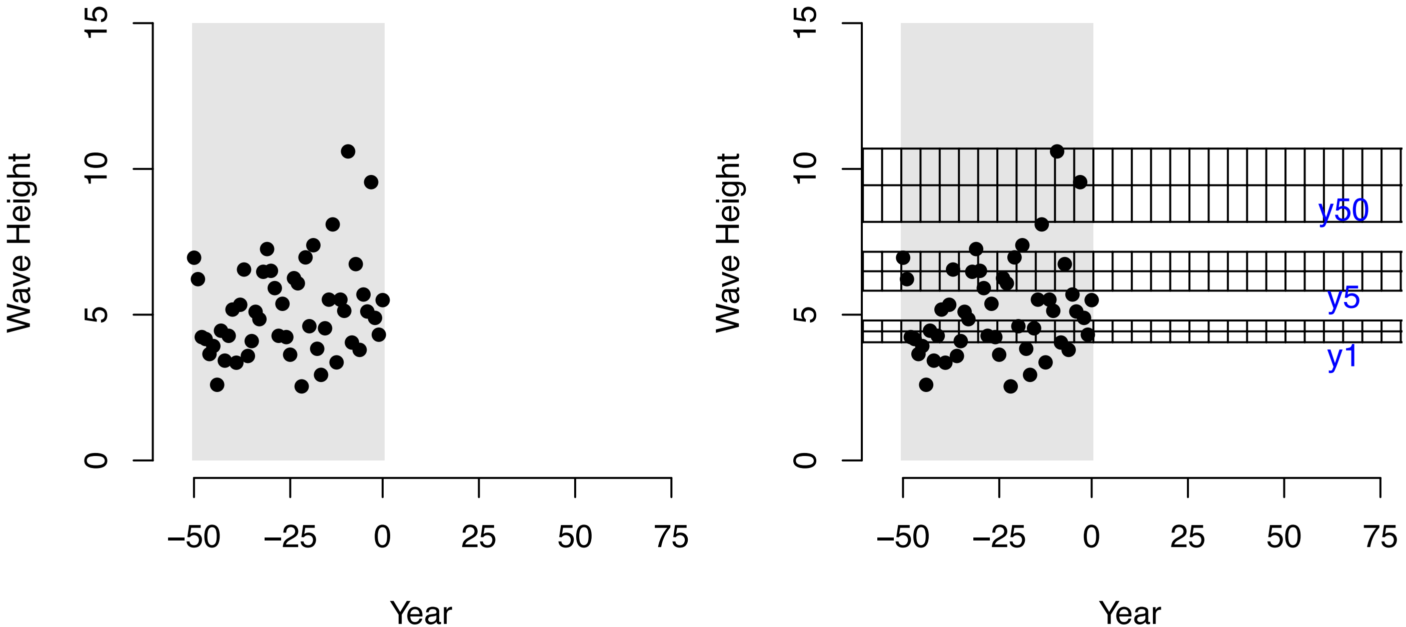


a frame for limits

We can show a time history for extremes,



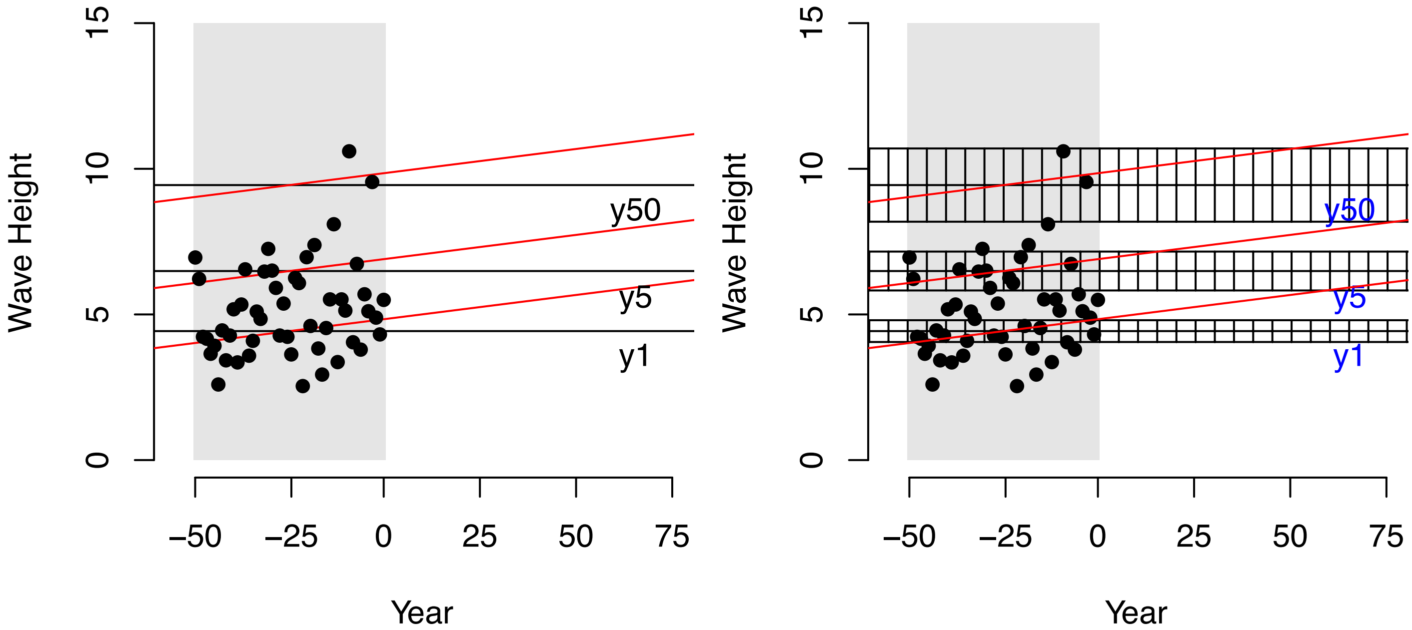
We can show a time history for extremes, and the estimated return levels with CI in conventional way. But we believe it?



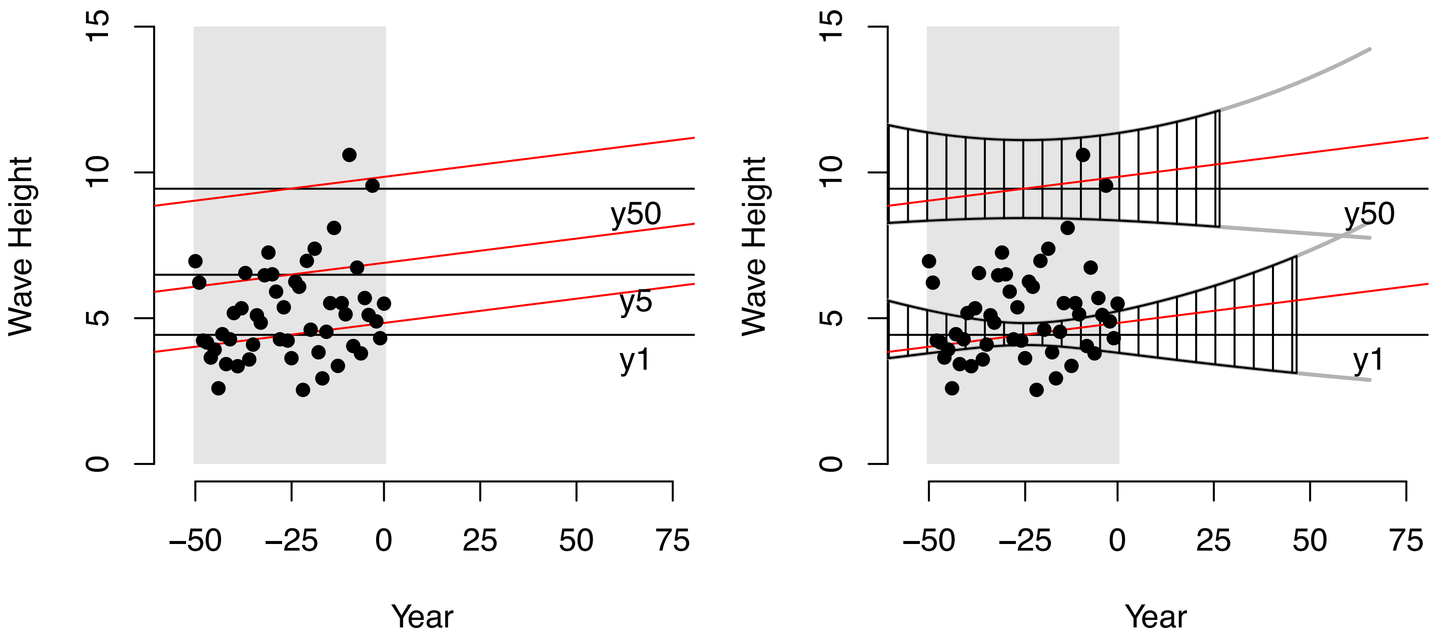
This is NOT tolerant of the probable trend.

Stationary CI is very weak for the probable trend.

A reviewer also pointed out ... the *peculiar* properies ...



Our solution is
stationary estimation with non-stationary CI.
Thus, It is tolerable for the probable trend.

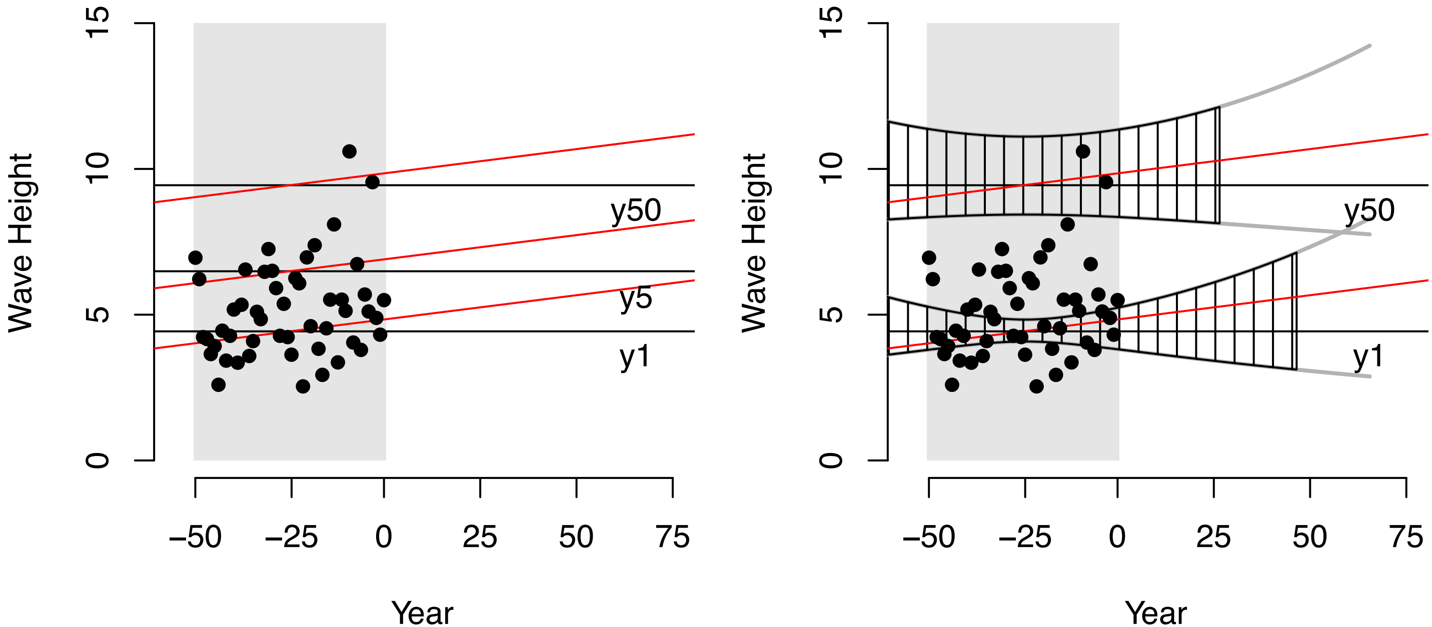


Diffractive effect:

The CI becomes larger along the passage of time.

How to make this CI ? It requires the new concepts:

Degree of experience, and Durability.



Degree of experience (Kitano et al., 2008) defined as:

$$\frac{1}{K} = V(\log \hat{\lambda})$$

2 questions come into our head.

Q1) Why is it for the occurence rate $\hat{\lambda}$,

not for the return level $\hat{y}_R$ directly.

Even if using $V(\hat{y}_R)$ is easy to accept for us engineers, ...

Q2) Why is log transformation adopted?

Somehow log transformed?

No, there are several theoretical bases.

Degree of experience (Kitano et al., 2008) defined as:

$$\frac{1}{K} = V(\log \hat{\lambda})$$

Q1) Why is it for the occurence rate $\hat{\lambda}$,

not for the return level $\hat{y}_R$ directly.

$$V(\hat{y}_R) = \frac{\sigma^2}{N} \text{ funcs.}(\xi)$$

σ = scale and ξ = shape parameters of ann. max. distribution.

We have few idea. $CV(\hat{y}_R) = \dfrac{\sqrt{V(\hat{y}_R)}}{E(\hat{y}_R)}$ sounds no good.

Rule of thumb: whether an amount of $CV(\hat{y}_R)$ is small/big?

It will be difficult to be in connection with any theoretical basis.

Degree of experience (Kitano et al., 2008) defined as:

$$\frac{1}{K} = V(\log \hat{\lambda})$$

Q1) Why is it for the occurence rate $\hat{\lambda}$,

$V(\hat{\lambda})$, or $V\{\lambda(y_R, \hat{\boldsymbol{\theta}})\}$ is also helpless for inference theory.

Therefore, we need the log transformation.

Q2) Why is log transformation adopted?

The derivative is $\delta \log \lambda = \dfrac{\delta\lambda}{\lambda}$, thus,

$$V(\log \hat{\lambda}) = E(\delta \log \lambda)^2 = \frac{E(\delta\lambda)^2}{\lambda^2} = \frac{V(\hat{\lambda})}{\{E(\hat{\lambda})\}^2}$$

Occurence Number k is distributed by

a Poisson distibution: $p(k) = \dfrac{(L\lambda)^k}{k!} \exp(-L\lambda)$

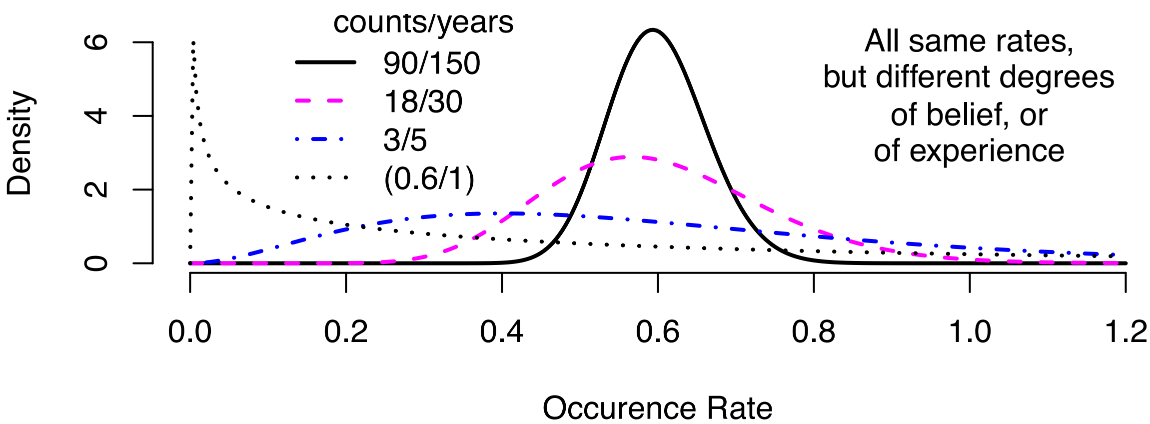
and the natural conjugate distribution is

a Gamma distribution: $f(\lambda) = \dfrac{(L\lambda)^K}{\lambda\,\Gamma(K)} \exp(-L\lambda)$

which is for the Estimated Occurence Rate λ.

a Gamma distribution: $f(\lambda) = \dfrac{(L\lambda)^K}{\lambda \Gamma(K)} \exp(-L\lambda)$

which is for the Estimated Occurence Rate.

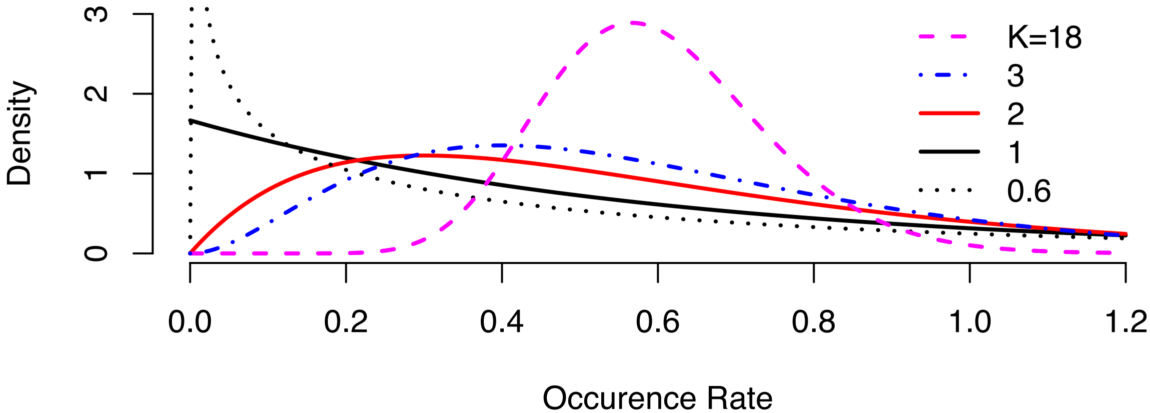


K is the number of counts, L is the length of observed years.

The value of K governs the concentration of the densities.

Rule of thumb:

The critical value of K could be 2.



Proverbs also tell us:

What happened twice will happen three times. (Japanese)

Non c'è due senza tre. (Italian) Non hay dos sin tres. (Spanish)

(please, let me know the others (Greek, ...)

www.isope.org; www.isope2012.org

June 17–22, Rhodes (Rodos), Greece

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3rd Renewable Energy Tech
3rd Sloshing Dynamics & Design
4th Frontier Energy & Clean Materials
4th High-Performance Materials
10th Strain-Based Design

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ISOPE-2012

Rodos Palace Hotel, Rhodes, Greece, June 17

Bad things happen 3 times

Το κακό τρίτωνα.

For a gamma distributed λ, we can obtain

$$E(\lambda) = \frac{K}{L}; \quad V(\lambda) = \frac{K}{L^2}$$

then, we make it straightforwdly,

$$\frac{V(\lambda)}{\{E(\lambda)\}^2} = \frac{K/L^2}{(K/L)^2} = \frac{1}{K}$$

We remember it (the definition for degree of experience)

$$\frac{1}{K} = V(\log \hat{\lambda}) = \frac{V(\hat{\lambda})}{\{E(\hat{\lambda})\}^2}$$

The evaluation for degree of experience

$$\frac{1}{K} = V(\hat{\lambda}) = V\left\{\log\lambda(y_R, \hat{\boldsymbol{\theta}})\right\}$$
$$= \nabla'_{\boldsymbol{\theta}} \log\lambda(y_R, \boldsymbol{\theta})\, I^{-1}\, \nabla_{\boldsymbol{\theta}} \log\lambda(y_R, \boldsymbol{\theta})$$

where the occurrence rate function is:

$$\lambda(y_R, \boldsymbol{\theta}) = \exp\left\{-\frac{1}{\xi}\log\left(1 + \xi\,\frac{y_R - \mu}{\sigma}\right)\right\}$$

as an implicit function for the relations of return level y_R and the parameters $\boldsymbol{\theta} = \{\mu, \sigma, \xi\}$ of GEV distribution. (i.e. annual max. distribution) **for stationary process model**.

and the inverse of the information matrix I^{-1} is used in place of the estimation errors matrix $V(\boldsymbol{\theta})$.

The evaluation for degree of experience for stationary case

$$\frac{1}{K} = \nabla'_{\boldsymbol{\theta}} \log \lambda(y_R, \boldsymbol{\theta})\, I^{-1}\, \nabla_{\boldsymbol{\theta}} \log \lambda(y_R, \boldsymbol{\theta})$$

$$I = \begin{pmatrix} i_{\mu,\mu} & i_{\mu,\sigma} & i_{\mu,\xi} \\ i_{\sigma,\mu} & i_{\sigma,\sigma} & i_{\sigma,\xi} \\ i_{\xi,\mu} & i_{\xi,\sigma} & i_{\xi,\xi} \end{pmatrix}$$

$$\nabla_{\boldsymbol{\theta}} \log \lambda(y_R; \boldsymbol{\theta}) = \begin{pmatrix} \partial\mu \\ \partial\sigma \\ \partial\xi \end{pmatrix} \log \lambda(y_R; \mu, \sigma, \xi)$$

The evaluation for degree of experience for non-stationary case

$$\frac{1}{K} = \nabla'_{\boldsymbol{\theta}} \log \lambda(y_R, \boldsymbol{\theta})\, I^{-1}\, \nabla_{\boldsymbol{\theta}} \log \lambda(y_R, \boldsymbol{\theta})$$

$$I = \begin{pmatrix} i_{\mu,\mu} & i_{\mu,\sigma} & i_{\mu,\xi} & i_{\mu,\beta_\mu} \\ i_{\sigma,\mu} & i_{\sigma,\sigma} & i_{\sigma,\xi} & i_{\sigma,\beta_\mu} \\ i_{\xi,\mu} & i_{\xi,\sigma} & i_{\xi,\xi} & i_{\sigma,\beta_\mu} \\ i_{\beta_\mu,\mu} & i_{\beta_\mu,\sigma} & i_{\beta_\mu,\xi} & i_{\beta_\mu,\beta_\mu} \end{pmatrix}$$

$$\nabla_{\boldsymbol{\theta}} \log \lambda(y_R; \boldsymbol{\theta}) = \begin{pmatrix} \partial\mu \\ \partial\sigma \\ \partial\xi \\ \partial\beta_\mu \end{pmatrix} \log \lambda(y_R; \mu, \sigma, \xi, \beta_\mu)$$

To distinguish 2 cases, we denote K_0 to
the evaluation for degree of experience for stationary case

$$\frac{1}{K_0} = \nabla'_{\boldsymbol{\theta}} \log \lambda(y_R, \boldsymbol{\theta})\, I^{-1}\, \nabla_{\boldsymbol{\theta}} \log \lambda(y_R, \boldsymbol{\theta})$$

$$I = \begin{pmatrix} i_{\mu,\mu} & i_{\mu,\sigma} & i_{\mu,\xi} \\ i_{\sigma,\mu} & i_{\sigma,\sigma} & i_{\sigma,\xi} \\ i_{\xi,\mu} & i_{\xi,\sigma} & i_{\xi,\xi} \end{pmatrix}$$

$$\nabla_{\boldsymbol{\theta}} \log \lambda(y_R; \boldsymbol{\theta}) = \begin{pmatrix} \partial\mu \\ \partial\sigma \\ \partial\xi \end{pmatrix} \log \lambda(y_R; \mu, \sigma, \xi)$$

By some manipilation techniques of matrix algebra,
The degree of experience for non-stationary case
is decomposed into 2 parts:
the stationary part and the time dependent part.
For the Gumbel type model (shape $\xi = 0$), simply,

$$\frac{1}{K} = \frac{1}{K_0} + \frac{(t-\bar{t})^2}{\sum(t_j-\bar{t})^2}$$

Even for a feneral GEV model (shape $\xi \neq 0$),

$$\frac{1}{K} = \frac{1}{K_0} + \frac{(t-\bar{t})^2}{m\sum(t_j-\bar{t})^2} \quad (m = \text{func. of } \xi)$$

It is notable that the time dependent part does NOT depend
on the magnitude of trend steepness.

As the time dependent part independent from the magnitude of trend steepness, the time dependent part would be included even for stationary case (trend steepness = zero).

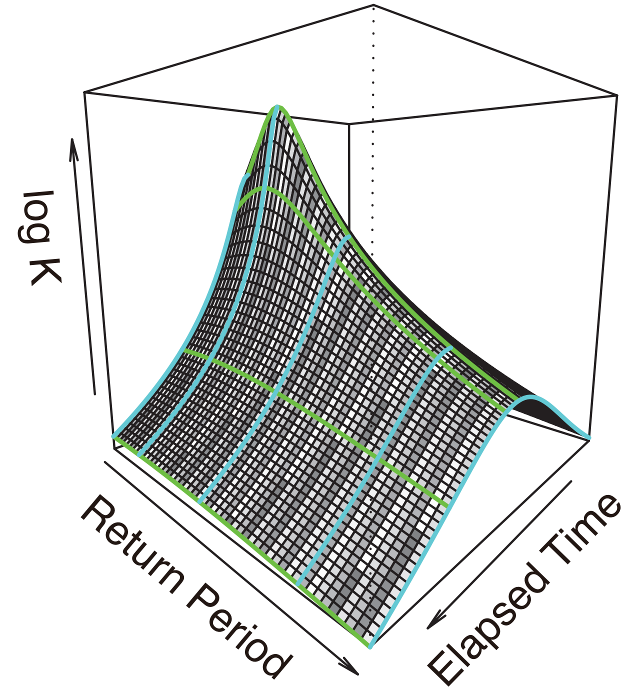
Durability is defined as the composition of the degree of experience and the time dependent part.

$$\frac{1}{\boxed{K}} = \frac{1}{\boxed{K_0}} + \boxed{\frac{(t-\bar{t})^2}{m\sum(t_j-\bar{t})^2}}$$

As seen before in the figure, the CI of return levels become enlarged along the time progressing. So, it should be named as diffractive effect term.

Durability decreases, as the degree of experience decreases, or as the elapsed time increases.

$$\frac{1}{K} = \frac{1}{K_0} + \frac{(t - \bar{t})^2}{\sum (t_j - \bar{t})^2}$$

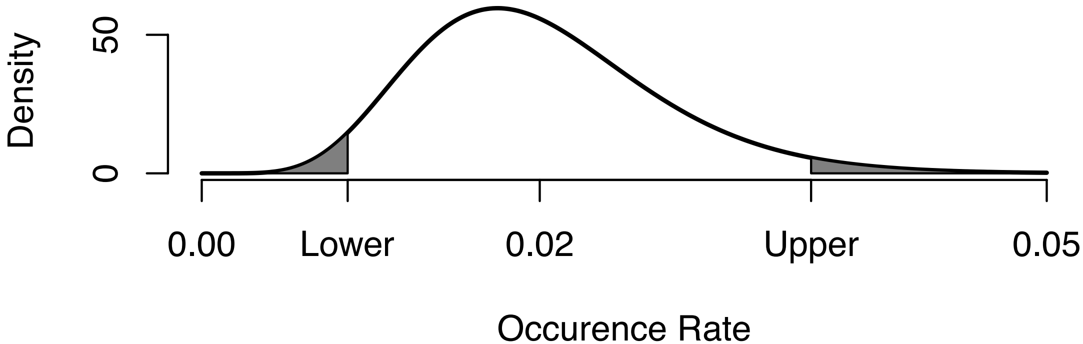


After the value of durability K is given,

the CIs of return levels are obtained in an easy manner.

The upper/lower value for the occurence rate are

throgh the gamma distribution governed by K.



$$\lambda_L \le \lambda(y_R; \hat{\boldsymbol{\theta}}) \le \lambda_U$$

Then, simply solve it.

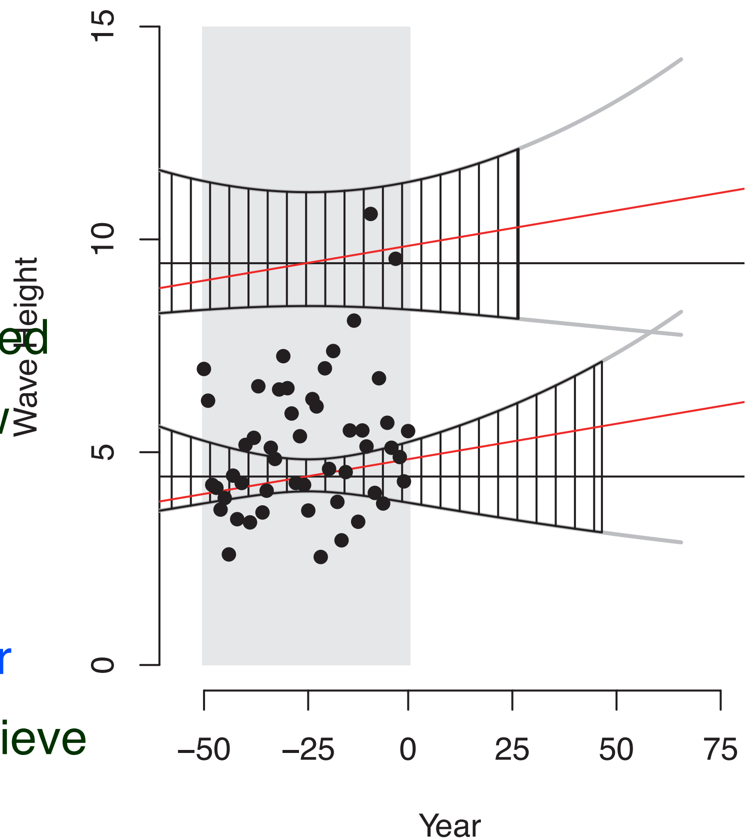
$$y_R(\lambda_U; \hat{\boldsymbol{\theta}}) \le y_R \le y_R(\lambda_L; \hat{\boldsymbol{\theta}})$$

Conclusions:

1) The degree of experience
is introduced to show
the limitation to the extrapolation
for rare occurence rates.

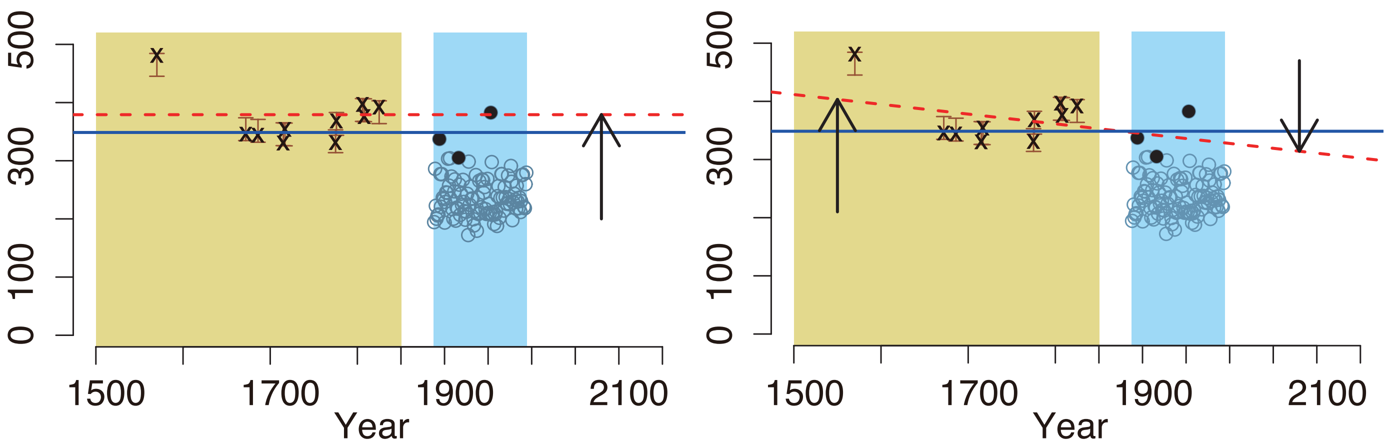
2) Durability is defined by modefied
the degree of experience to show
the limitation to the extrapolation
for the time progressing.

3) The resultant CI is tolerable for
a probable trend, and we can believe
it a feasible solution,
rather than the conventional ones.



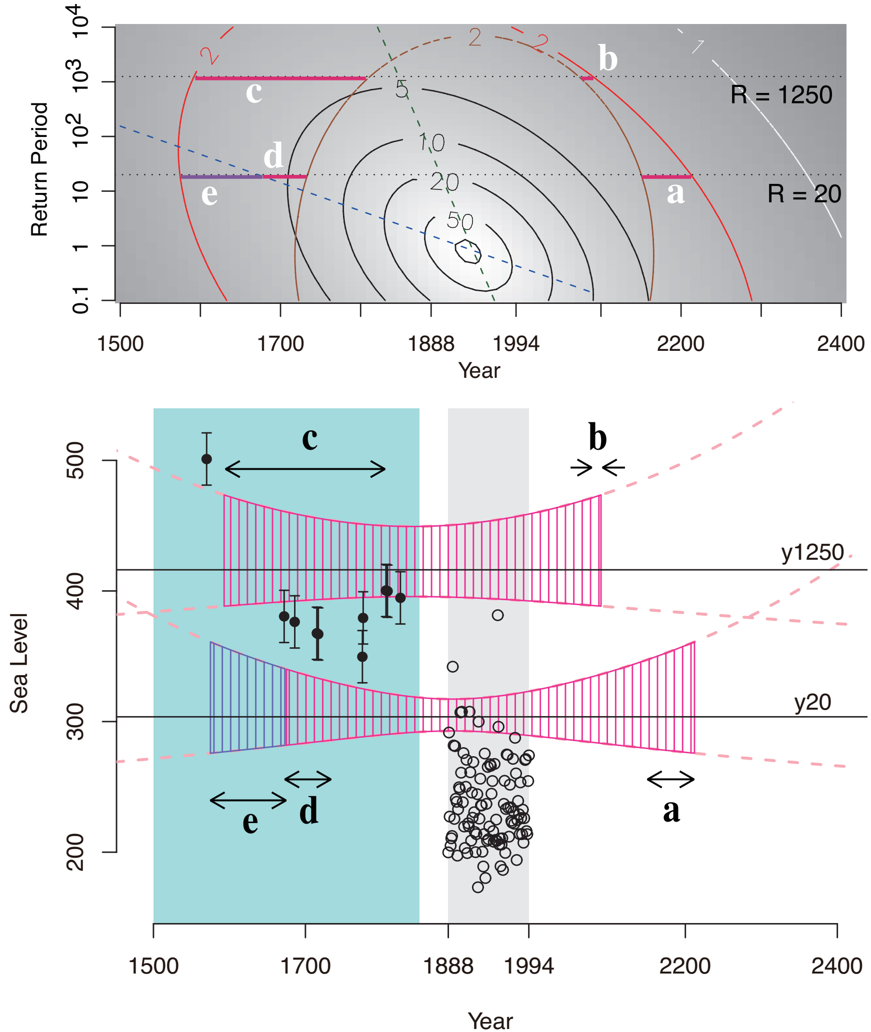
Future works:

Applcation to the analysis for Hook of Holand sea level data
(= modern records + ambiguous historic data).
A dilemma is arised in this case, due to additional historic data.
The modification in the future sea level is positive, or negative?



Our answer is simple: no change!

It is because the diffractive effect is considered, which includes the possible trend (increase/decrease). Rather, it is of our interest to know how long the limitations are postponed by adding the historic data.

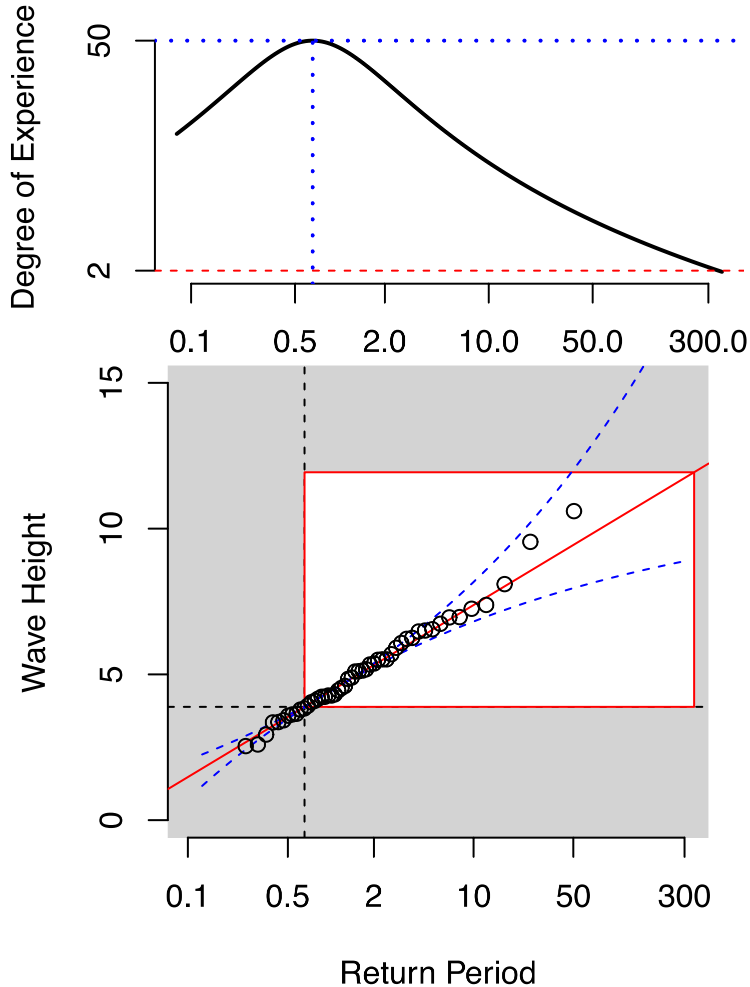


Presumed Question:
Why are the data
at lower levels
outside of the frame?

Our answer is that
the larger extremes
are only of our interest.

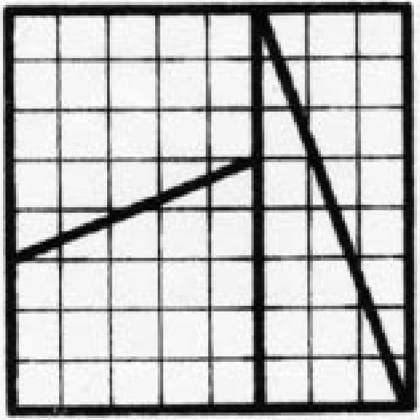
So, we cut the peak of
degree of experience.

It is a kind of thresold.

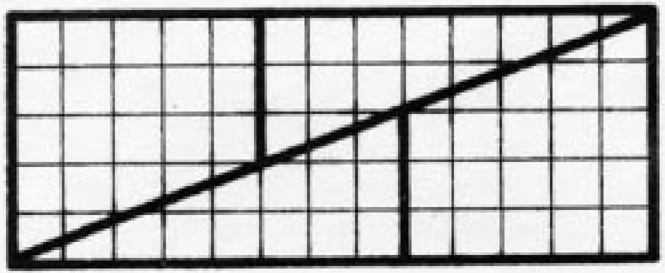


（オマケ） 以下の喩えは，ちょっと過ぎるかな？

問い：　どうして，１マス増えたの？



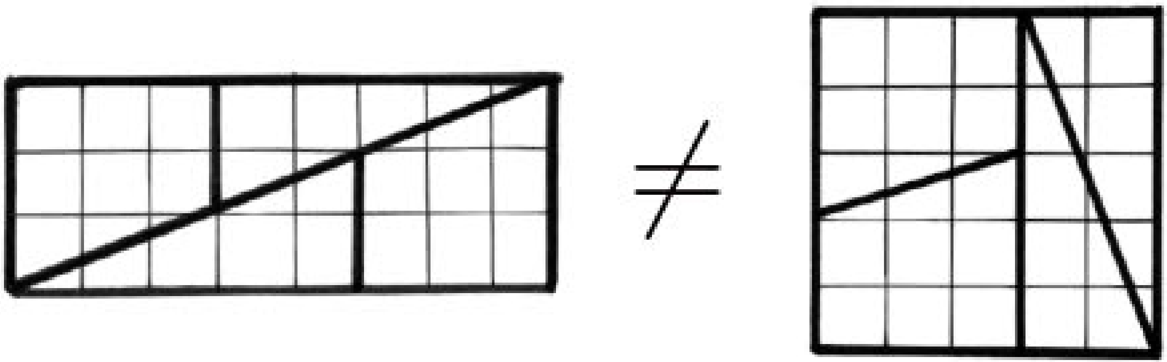
$\neq$



$$8 \times 8 \neq 5 \times 13$$

極値解析の本質的なルールがこのパズルの答えでもある．

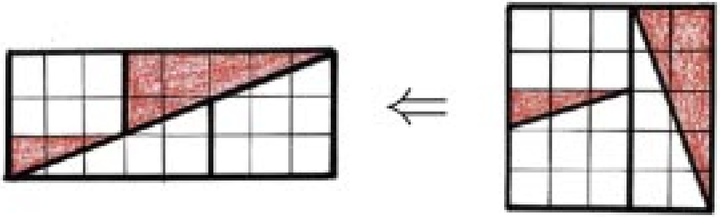
問い：　どこが，おかしいのでしょうか？



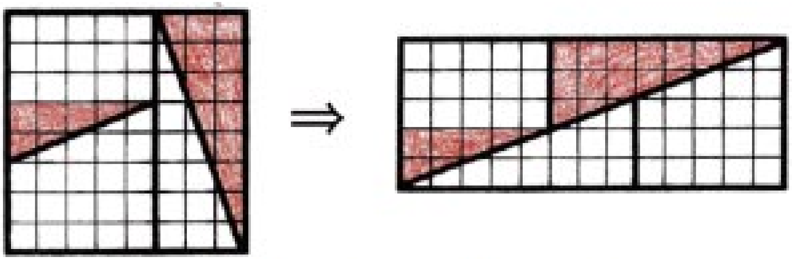
$$3 \times 8 \neq 5 \times 5$$

極値解析も，このようなルールはまかり通りません

答え：　比例関係が成り立っていません.



$$1 : 3 \neq 2 : 5?$$



$$2 : 5 \neq 3 : 8?!$$

Proportionality in Crisis!

比例関係の成立（＝ 再現期間の本質と考える）

３：６０ ＝ ５：１００

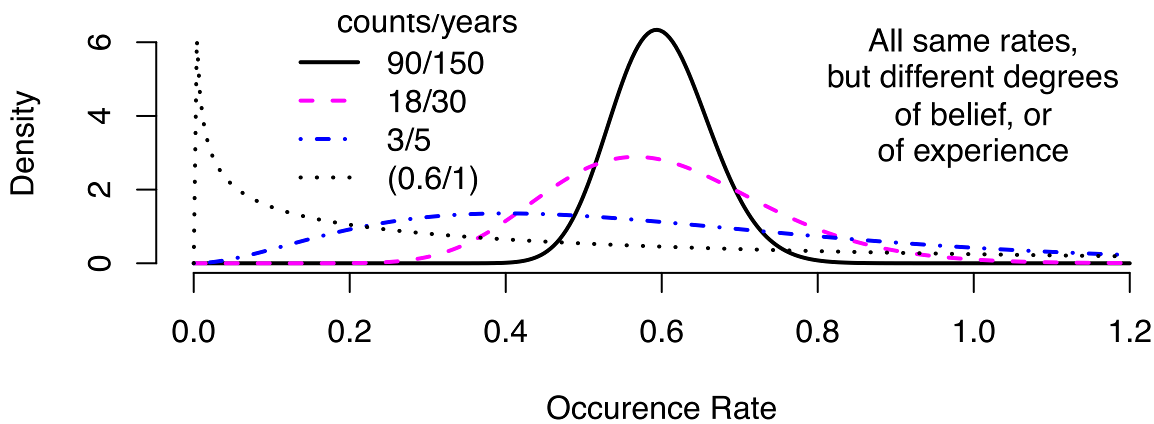
６０年に３回の頻度＝１００年に５回の頻度

0.3：３０ ？＝ １：１００

３０年に 0.3 回の頻度＝１００年に１回の頻度

a Gamma distribution: $f(\lambda) = \dfrac{(L\lambda)^K}{\lambda\,\Gamma(K)} \exp(-L\lambda)$

which is for the Estimated Occurence Rate.



K is the number of counts, L is the length of observed years.

The value of K governs the concentration of the densities.

発生率と再現期間

$$\lambda = \frac{1}{R}$$

cf. スペクトル解析でおなじみの関係

$$f = \frac{1}{T}$$

発生率と再現期間

$$\lambda = \frac{1}{R}$$

比例関係の成立

$$1 : R = K : L$$

$$\lambda = \frac{1}{R} = \frac{K}{L}$$

$$\hat{\lambda} = \lambda(y, \hat{\boldsymbol{\theta}})$$

$$E(\hat{\lambda}) = \frac{K}{L}$$

$$f(\hat{\lambda}) = \frac{L^K}{\Gamma(K)} \hat{\lambda}^{K-1} \exp(-L\hat{\lambda})$$

$$V(\hat{\lambda}) = \frac{K}{L^2}$$

変動係数
という視点？

$$C^2 = \frac{V(\hat{\lambda})}{E^2(\hat{\lambda})} = \frac{K/L^2}{(K/L)^2} = \frac{1}{K}$$

対数変換
の適用！（なぜ）

$$\frac{1}{K} = V\left(\log \hat{\lambda}\right) \left(= \nabla' \log \lambda \ V(\hat{\lambda}) \ \nabla \log \lambda = \frac{V(\hat{\lambda})}{\lambda^2} \right)$$