### Cooperative Research Symposium Extreme Value Theory and Applications

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

# A discussion of the limit of human longevity based on data for oldest-old survivors or deaths

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### **Background**

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

## Theories of aging in the field of biology

< DAMAGE theories >

We age, because our systems break down over time.

The free-radical theory: organisms age because cells accumulate free radical damage over time.

< PROGRAM theories >

We age, because there is an inbuilt mechanism that tells us to die.

Telomeres theory: telomeres have been shorten with each successive cell division. Shortened telomeres activate a mechanism that prevents further cell multiplication

### **Background**

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

#### Theories of aging

If "DAMAGE theories" are true, we can prevent from aging by avoiding to damage our organism.

If "PROGRAM theories are true, we can not prevent from aging with any effort.



Is there any upper limit in human life distribution?

### **Background**

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

Kaufmann (2001) has analyzed mortality data from West Germany and suggested an improbability that there exist an limit of human longevity.

Aarssen and de Haan (1994) has analyzed mortality data from the Netherlands using statistical methods in the field of extreme value theory and showed that there is a finite age limit in 113–124 years.

### Aims

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

### Our aims in this study are:

- (1) to see if there exists a upper limit of human life distribution based on data for oldest-old survivor or deaths in Japan, and if so
- (2) to estimate that upper limit,
- (3) to see if those upper limits are increasing or decreasing as the year progresses.

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#### **National Census Data**

In the report of national census, the numbers of survivors are indicated in tables where data are totaled for 100 years and older. So the data obtained from the reports of national census is not available.

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### **National List of Aged**

The ministry of health, labors and welfare in Japan is (was) publishing a list of names, addresses and ages of people whose ages are 100 years and older on Respect-for-Senior-Citizens Day every year.

The book has a table of numbers of survivors by age on its first page.

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## (Age, period)-tabulated data (male)

1077	1070	1070	1000	1001	1000	1000	1004	1005	1006	1007	1000	1000	1000	1001	1002	1002	1004	1005	1006	1005	1000	1000	2000	2001	2002
	1978					1983	1984	1985	-, -,	-, .	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	
56	68	101	70	98	108	132	178	157	168	225	298	302	314	373	388	430	509	640	619	722	837	885	945	1173	
31	31	45	52	47	60	68	81	102	97	108	134	171	194	183	223	243	282	302	412	385	457	495	566	623	747
13	16	21	26	32	27	33	48	53	50	65	63	80	93	100	101	143	148	158	188	265	257	292	305	361	401
6	5	4	12	16	23	18	20	27	26	32	31	40	46	47	60	62	84	78	89	96	153	157	169	181	229
8	4	2	1	4	9	14	10	8	12	20	18	13	19	27	26	35	30	47	47	52	54	86	93	108	106
5	2	3	-	1	2	2	8	6	4	6	11	11	6	14	13	15	20	13	26	25	25	30	49	53	65
1	4	-	2	-	1	1	-	4	3	2	5	7	3	4	8	8	12	8	8	18	17	12	14	25	34
1	-	3	-	2	-	-	1	-	1	3	1	4	4 -	-	3	6	4	4	6	4	7	9	7	9	16
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122	132	- 180	165	202	233	260	347	350	361	162	562		680	- 749	822	943	1093	1255	1400	1570	1912	1973	2158	2541	2875
1,44	134	100	103	202	433	209	J+1	339	501	402	302	050	000	147	022	243	1073	1433	1400	1370	1012	1713	2136	2341	2013

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

## (Age, cohort)-tabulated data (male)

					Yea	ar of Bi	irth				
Age	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890
100 Years	70	98	108	132	178	157	168	225	298	302	314
101 Years	47	60	68	81	102	97	108	134	171	194	183
102 Years	27	33	48	53	50	65	63	80	93	100	101
103 Years	18	20	27	26	32	31	40	46	47	60	62
104 Years	10	8	12	20	18	13	19	27	26	35	30
105 Years	6	4	6	11	11	6	14	13	15	20	13
106 Years	3	2	5	7	3	4	8	8	12	8	8
107 Years	3	1	4	4	-	3	6	4	4	6	4
108 Years	-	1	1	1	-	1	3	2	2	2	2
109 Years	-	-	-	-	-	1	2	2	-	2	1
110 Years	-	-	-	-	-	1	-	1	-	2	-
111 Years	-	-	-	-	-	1	-	1	-	1	-
112 Years	-	-	-	-	-	-	-	-	-	1	-
113 Years	-	-	-	-	-	-	-	-	-	1	-

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## (Age, period)-tabulated data (female)

1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
271	318	350	332	401	421	492	554	617	628	863	961	1088	1129	1247	1483	1700	1985	2342	2549	2920	3661	3891	4548	5520	6227
155	170	205	203	216	257	259	322	366	402	420	557	652	689	746	843	969	1129	1268	1622	1779	2034	2518	2748	3242	3921
80	95	107	114	128	130	159	152	194	239	253	268	345	390	427	470	555	635	701	850	1069	1220	1332	1707	1857	2225
32	39	54	52	63	89	81	90	93	112	147	149	171	203	227	282	298	359	394	465	559	711	784	886	1147	1240
24	21	21	28	33	39	54	48	56	52	66	85	93	102	115	136	182	183	211	242	313	374	448	488	587	
5	9	11	10	18	21	24	29	24	28	28	42	48	47	58	59	83	115	91	125	148	183	209	272	304	
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-	1	4	1	-	2	2	2	-	3	9	6	5	3	9	11	7	6	17	16	18	13	25	23	29	40
-	-	-	1	1	-	2	2	2	-	1	8	5	3	3	4	8	5	4	10	9	8	9	13	14	18
-	-	-	-	1	-	-	-	1	1	-	1	4	2	1	2	2	5	3	3	3	4	4	5	6	11
-	-	-	-	-	-	-	-	-	-	1	-	1	3	1			-	4	3	3	2	1	3	1	3
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576	660	757	748	870	967	1085	1216	1381	1490	1809	2106	2448	2618	2876	3330	3859	4500	5123	5973	6921	8346	9373	10879	12934	15059

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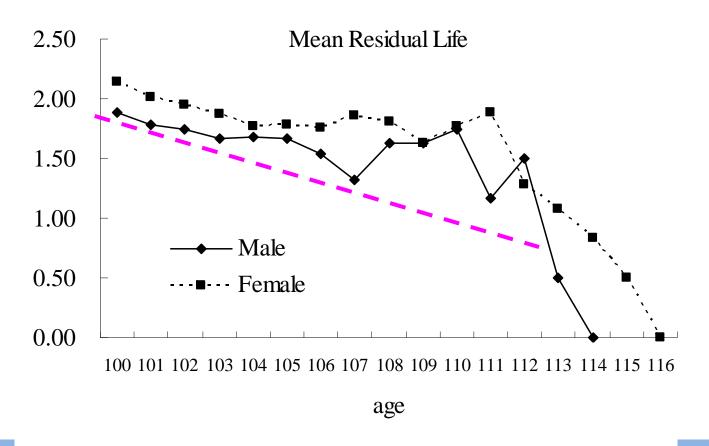
### **Data by cohort (female)**

					Yea	ar of Bi	rth				
age	1880	1881	1882	1883	1884	1885	1886	1887	1888	1889	1890
100 years	332	401	421	492	554	617	628	863	961	1088	1129
101 years	216	257	259	322	366	402	420	557	652	689	746
102 years	130	159	152	194	239	253	268	345	390	427	470
103 years	81	90	93	112	147	149	171	203	227	282	298
104 years	48	56	52	66	85	93	102	115	136	182	183
105 years	24	28	28	42	48	47	58	59	83	115	91
106 years	12	14	19	24	33	22	30	34	54	65	47
107 years	7	9	11	13	17	8	19	22	23	37	22
108 years	6	5	3	9	11	7	6	17	16	18	13
109 years	5	3	3	4	8	5	4	10	9	8	9
110 years	2	1	2	2	5	3	3	3	4	4	5
111 years	1	-	-	-	4	3	3	2	1	3	1
112 years	1	-	-	-	4	2	2	1	1	2	1
113 years	1	-	-	-	3	1	-	1	-	1	-
114 years	1	-	-	-	1	-	-	1	-	-	-
115 years	-	-	-	-	-	-	-	1	-	-	

### Simple analysis of crude data

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### Mean residual life calculated from crude data



### **Extreme Value Theory**

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#### **Generalize Pareto distribution**

Let  $X_1, X_2, \cdots, X_n$  be a sequence of independent random variables with common distribution function F. Denote an arbitrary term in the  $X_i$  sequence by X. Then, for large enough u, the distribution function of Y=(X-u), conditional on X>u, is approximately

$$F(y; \gamma, a) = \begin{cases} 1 - (1 + \gamma y/a)^{-1/\gamma} & \gamma \neq 0 \\ 1 - \exp(-y/a) & \gamma = 0 \end{cases}.$$

$$E[Y - v \mid Y > v] = \frac{a + \gamma v}{1 - \gamma}$$

### **Extreme Value Theory**

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$$\gamma < 0$$
 or  $\gamma \ge 0$ 

Under the assumption that Y has the generalized Pareto distribution function, the range of Y is

$$0 < Y < -a/\gamma \equiv \omega$$
 if  $\gamma < 0$ ,

$$0 < Y < \infty$$
 if  $\gamma \ge 0$ .

### **Procedure**

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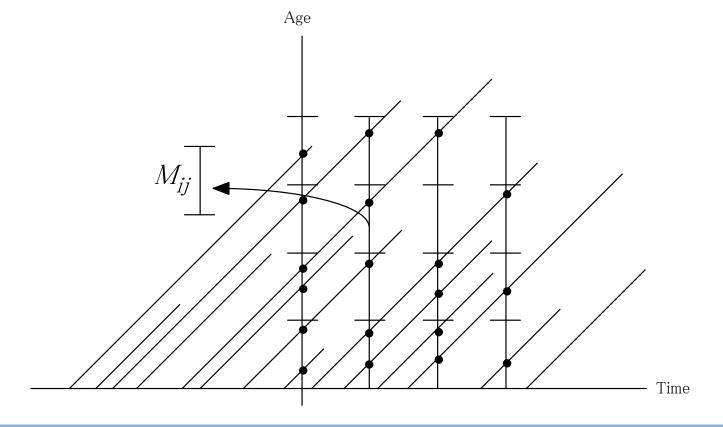
### The steps of this study are:

- 1.to test the hypothesis  $\gamma = 0$ ,
- 2. to estimate the upper limit of life for each cohort,
- 3. to test the hypothesis that the trend in those upper limits equals zero .

### Lexis diagram

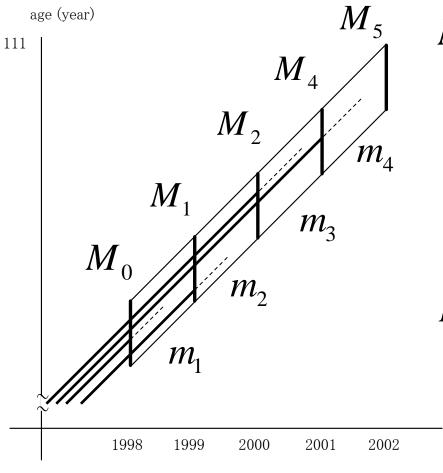
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### To associate continuous survival time with numbers of survivors



### Discrete data

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 $M_i$ : the numbers of survivors whose ages are 100+i.

$$m_i = M_{i-1} - M_i$$

Individuals die in a year with probability

$$p_{i} = \overline{F}(i, \gamma, a) - \overline{F}(i+1, \gamma, a)$$

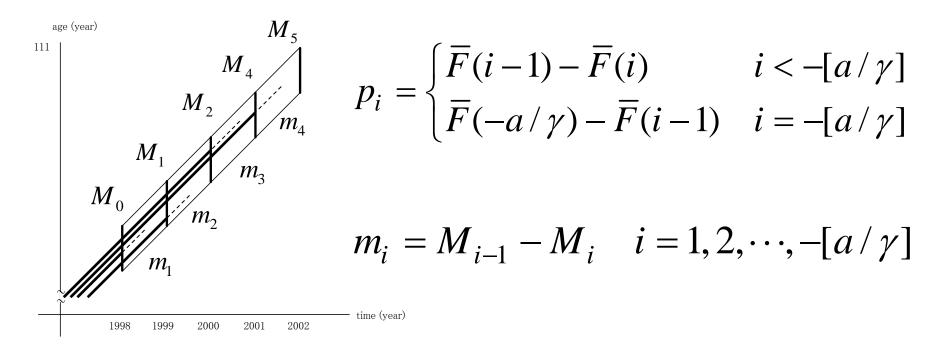
$$Multi-nominal$$

$$distribution$$

### Multinomial model by F

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$$P(m_1, m_2, \dots, m_{-[a/\gamma]}) = \frac{M_0!}{m_1! m_2! \cdots m_{-[a/\gamma]}!} p_1^{m_1} p_2^{m_2} \cdots p_{-[a/\gamma]}^{m_{-[a/\gamma]}}$$



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#### For male

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IVIUIC				
Birth Year	Gamma	P-value	Omega (Limit of life)	P-value
1880	$\frac{7}{0.18}$	0.04	(114)	0.00
1881	-0.13	0.08	115	0.01
1882	-0.16	0.03	115	0.00
1883	-0.11	0.08	121	0.01
1884	-0.12	0.04	116	0.01
1885	-0.09	0.12	124	0.01
1886	-0.08	0.14	129	0.01
1887	-0.04	0.27	145	0.00
1888	-0.05	0.19	134	0.01
1889	$\frac{1}{1000}$	0.19	144	0.01
1890	-0.09	0.05	122	0.01

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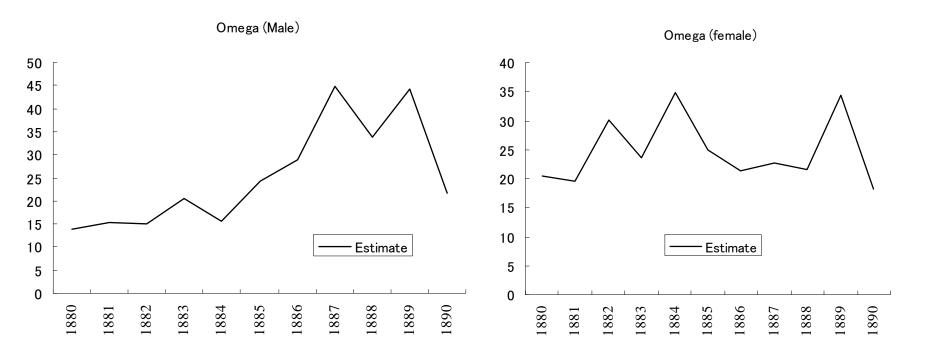
#### For female

Eas	ma	1۵
геі	ma	лe

1 Ciliare				
Birth Year	Gamma	P-value	Omega (Limit of life)	P-value
1880	<del>7</del> 0.11	0.01	120	0.00
1881	-0.12	0.00	120	0.00
1882	-0.07	0.07	130	0.01
1883	-0.10	0.01	124	0.00
1884	-0.07	0.04	135	0.00
1885	-0.09	0.01	125	0.00
1886	-0.12	0.00	121	0.00
1887	-0.10	0.00	123	0.00
1888	-0.11	0.00	122	0.00
1889	-0.07	0.01	134	0.00
1890	-0.14	0.00	118	0.00

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#### **Limit of Life**

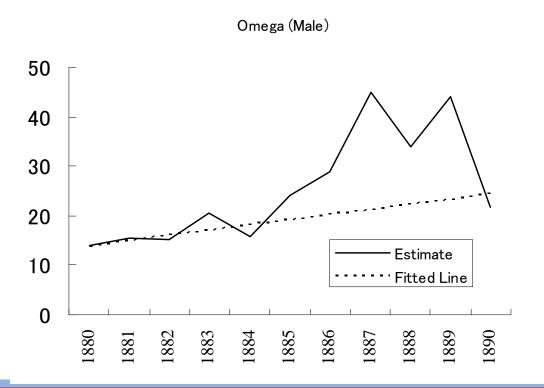


### Is the limit of human life increasing?

### **Regression line**

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### Regression line obtained using weighted-least-square method with asymptotic variances of estimates

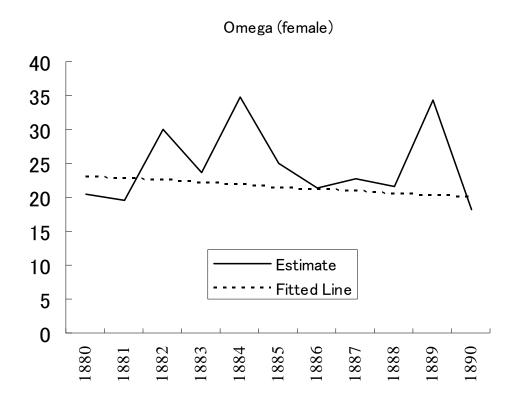


	Constant	Trend
Estimate	13.83	1.05
P-value	0.00	0.23

### **Regression line**

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

### Regression line obtained using weighted-least-square method with asymptotic variances of estimates



	Constant	Trend
Estimate	23.03	-0.31
P-value	0.00	0.29

### Conclusions based on data on survivors

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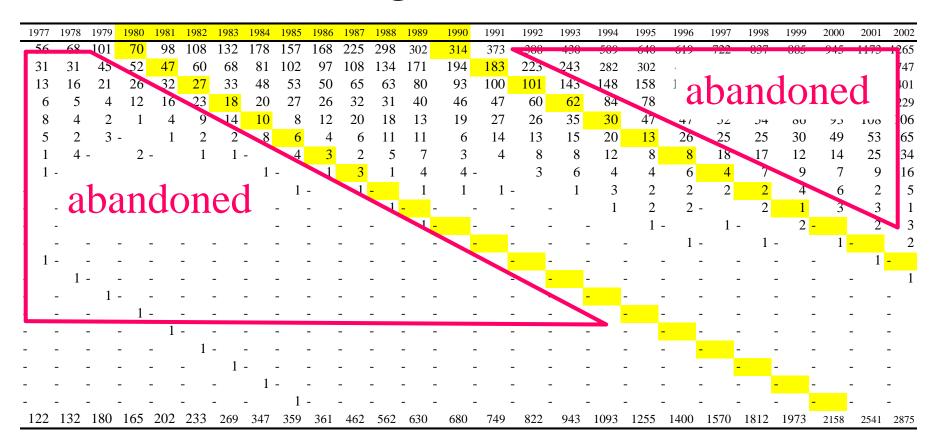
(1) The limit of human life is significantly estimated finite for some male cohort and all the female cohort.

(2) The hypothesis "the increasing rate of the limits of life is zero" has been rejected.

### A point

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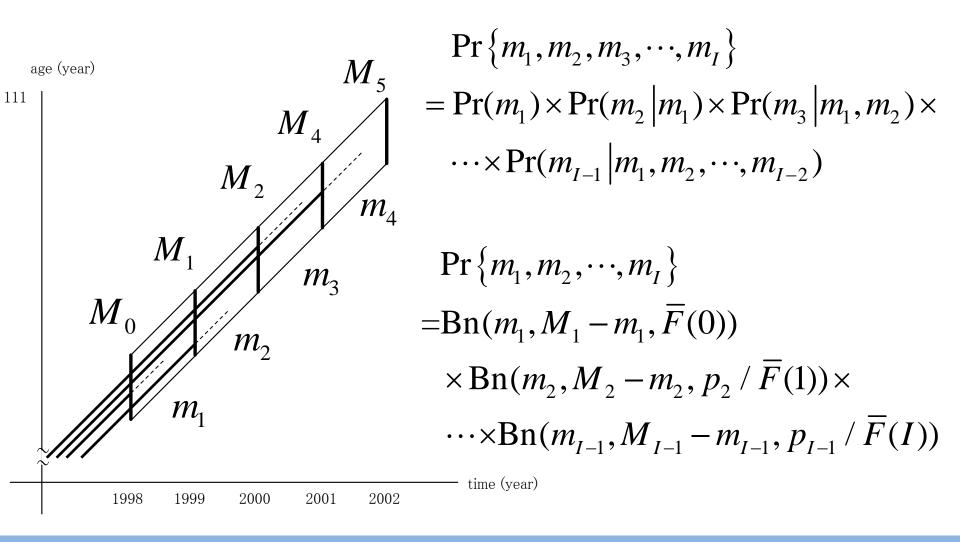
### It has been pointed out that not a few data are abandoned to make (age, cohort)-table.



N. Hanayama, Shobi University, Japan

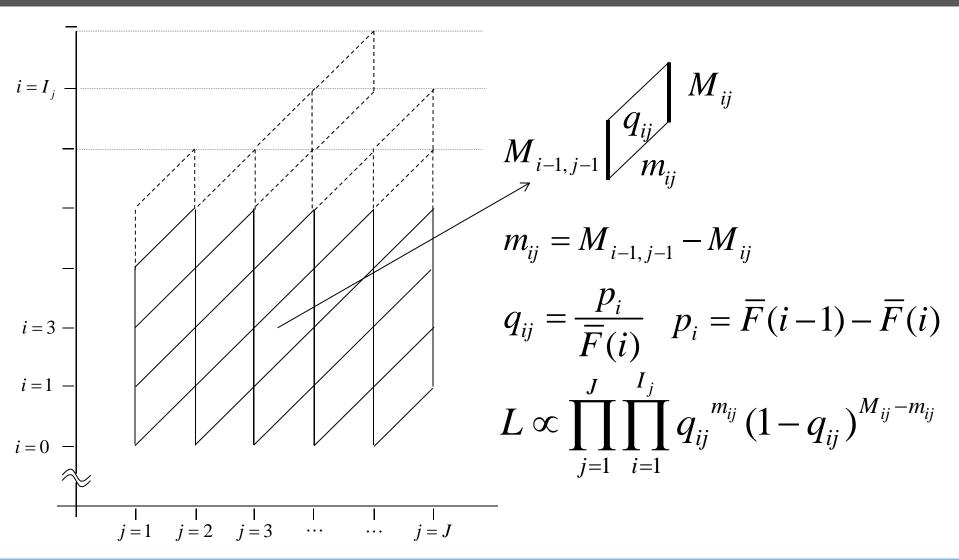
### Binomial model by GPD

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### Binomial model by the GPD

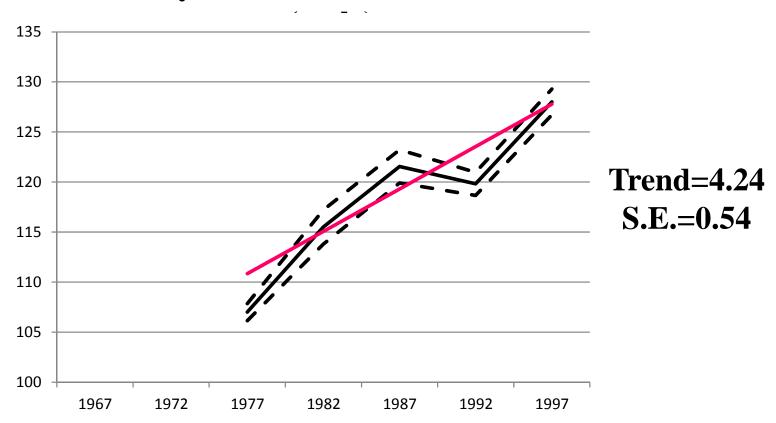
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### **Regression line**

A discussion of limit of human longevity based on statistical analysis of data on oldest -old survivors or deaths

### Regression line obtained using weighted-least-square method by variances of estimates

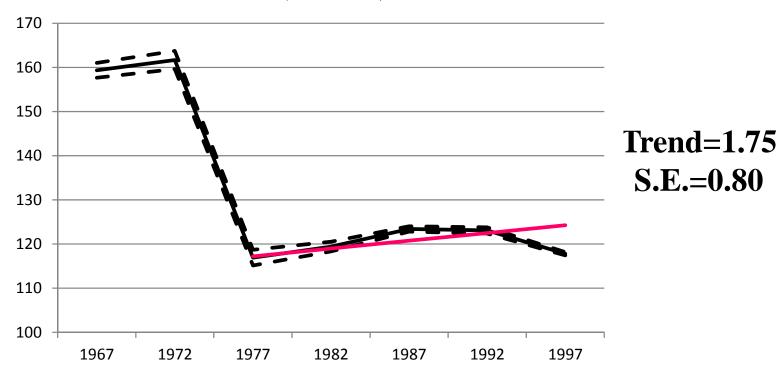


### **Regression line**

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

## Regression line obtained using weighted-least-square method by variances of estimates





### Conclusions based on data on survivors

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The trends in the limit of human life is significantly estimated positive for male and female after 1977.

### **Updated data**

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### **National List of Aged**

The List of Oldest Old survivors has halted its publication since 2003. Instead the name and of oldest old survivors who deserve to receive tribute are reported on their web site.

### **Updated data**

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### The numbers of oldest old survivors who deserve to receive tribute

Though numbers of survivors who are 100 and over are given in the reports, those are totaled in one age group, that is, "100-", in the reports after 2005.

### Missing centenarians

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10 September 2010 Last updated at 09:53 GMT



### More than 230,000 Japanese centenarians 'missing'

More than 230,000 elderly people in Japan who are listed as being aged 100 or over are unaccounted for, officials said following a nationwide inquiry.

An audit of family registries was launched last month after the remains of the man thought to be Tokyo's oldest were found at his family home.

Relatives are accused of fraudulently receiving his pension for decades.



Japanese citizens are famous for their longevity

### **Data on deaths**

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## **Statistics of population movement** (The ministry of health, labors and welfare )

平成21年	人口動態記	周査																
4IC 保管	第3表 死	亡数,性	•年齢(	各歳)•/	学生-	中学生	(再掲)・	死因(列	E因簡単	分類)別								
		100歳	101歳	102歳	103歳	104歳	105歳	106歳	107歳	108歳	109歳	110歳	111歳	112歳	113歳	114歳	115歳	116歳~
総	総数	5537	3715	2341	1314	864	500	336	175	82	38	29	10	5	2	1	_	_
	男 M.	961	662	343	199	105	62	30	14	6	3	3	1	_	1	_	_	_
	女 F.	4576	3053	1998	1115	759	438	306	161	76	35	26	9	5	1	1	-	_

## The number of deaths is more confident than that of survivors?

### **Data on deaths**

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	100歳	101歳	102歳	103歳	104歳	105歳	106歳	107歳	108歳	109歳	110歳	111歳	112歳	113歳	114歳	115歳	116歳
1980年	155	102	59	37	27	8	11	1	1	3	0	1	0	0	0	0	(
1981年	186	125	75	46	25	19	8	2	1	5	1	0	0	0	1	0	(
1982年	171	111	94	56	32	18	9	11	3	0	0	2	0	0	0	0	(
1983年	235	140	97	55	32	13	13	9	3	2	4	2	0	0	0	0	
1984年	219	149	93	80	35	26	10	15	4	1	2	1	0	1	3	0	
1985年	280	150	107	87	52	30	17	6	2	1	0	0	1	1	0	0	
1986年	251	172	123	72	52	26	16	9	6	3	1	1	2	1	0	0	_
1987年	294	249	142	106	74	43	15	10	7	4	1	1	0	0	0	_	
1988年	380	255	197	114	57	36	21	13	8	8	3	2	0	1	_	_	- /
1989年	407	328	180	123	85	55	30	15	10	3	1	1	0	_	1	_	-/
1990年	460	310	203	132	86	63	27	24	13	4	4	2	1	_	-		/_
1991年	465	371	210	179	122	54	32	16	12	5	4	1	1	1	1	_ /	_
1992年	498	372	272	185	98	75	54	29	13	13	4	_	3	1	-	- /	_
1993年	710	445	301	168	132	91	58	27	19	5	6	4	5	_	1	_/	_
1994年	719	549	331	216	156	110	73	46	14	11	6	5	1	1	-	/-	
1995年	865	534	387	264	212	136	84	45	25	22	6	4	_	2	/1		

Ms. Madoka Hongo 本郷かまと(ほんごう・かまと)さん

### **Data on deaths**

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

### Can it be supposed that

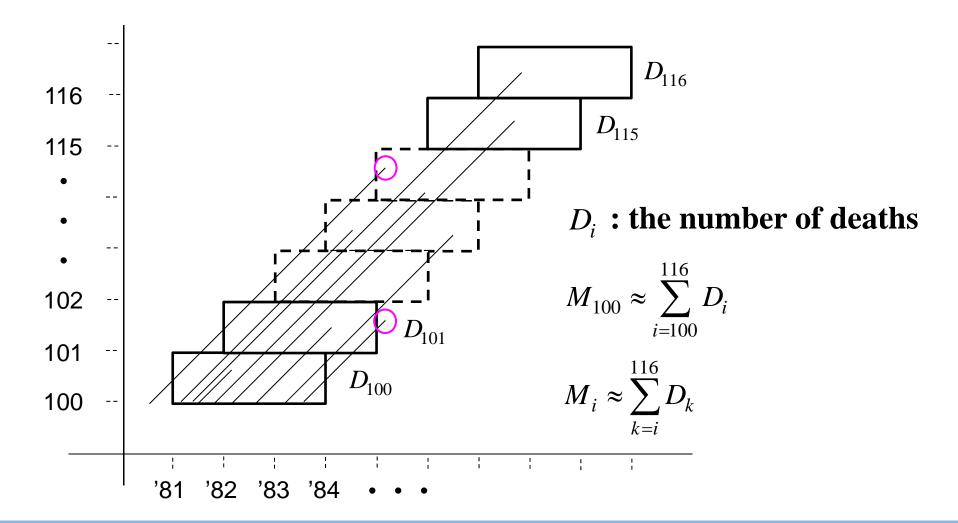
elderly people go missing independently from how old they are?



Sums of deaths who are i (>100) years old and over have the same distribution function as the distribution function of the numbers of survivors.

### Data aggregated for each 3 years

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#### For male

Birth Yeear	gamma	P-value	omega	P-value
1881	-0.08	0.06	27.78	0.05
1884	-0.09	0.01	22.11	0.01
1887	-0.06	0.03	32.77	0.02
1890	-0.06	0.02	34.44	0.01
1893	-0.11	0.00	20.11	0.00

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

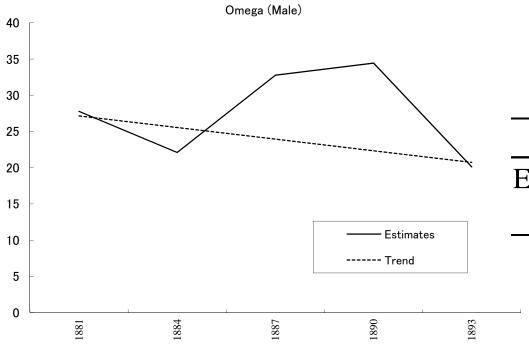
#### For female

Birth Yeear	gamma	P-value	omega	P-value
1881	-0.06	0.01	35.87	0.01
1884	-0.07	0.00	34.74	0.00
1887	-0.10	0.00	23.06	0.00
1890	-0.12	0.00	21.11	$\setminus 0.00$
1893	-0.11	0.00	24.27	0.00

### Regression line on the limit of life

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### Regression line obtained using weighted-least-square method by asymptotic variances of estimates



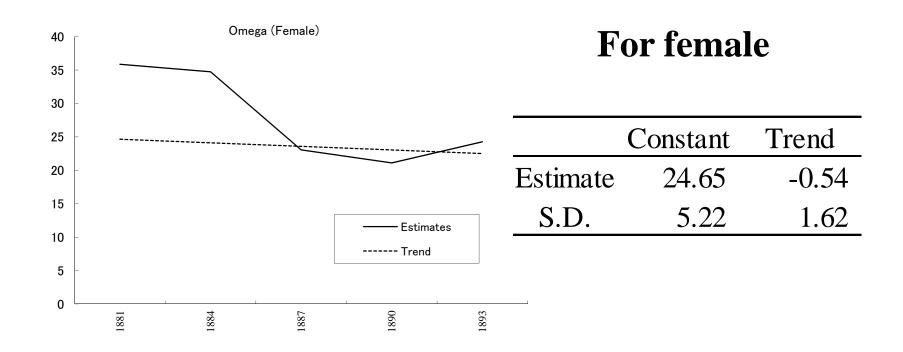
#### For male

	Constant	Trend
Estimate	27.15	-1.60
S.D.	8.58	2.41

### Regression line on the limit of life

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

### Regression line obtained using weighted-least-square method by asymptotic variances of estimates



### **Conclusions based on data on deaths**

A discussion of limit of human longevity based on statistical analysis of data on oldest -old survivors or deaths

- (1) The limit of human life is significantly estimated finite for all the cohort for male and female.
- (2) The hypothesis "the increasing rate of the limits of life is zero" has not been rejected.
- (3) The above result are contradict to the results based on the data on survivors.
- (4) More deliberate studies are required.

### References

A discussion of limit of human longevity based on statistical analysis of data on oldest –old survivors or deaths

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