次の地震のマグニチュード予測と評価

Magnitude forecasts of the next earthquake and evaluation

統計数理研究所

The Institute of Statistical Mathematics

CSEP の地震予測検証実験が始まって 10 年以上経つ。 その主な取組みは空間領域（例えば、3 ヶ月、1 年、5 年間における予測）および時空領域（日々の予測）における確率的予測を行い、それらの性能を評価することである。CSEP の主な目的は、様々な地震活動モデルの開発を促進し各地の通常の地震活動の標準的な相場を確立することで、異常現象に基づいた大地震の予測の各種提案に対する客観的評価のインフラを整備することである。

これまでのところ、CSEP の殆どの提案モデルの地震マグニチュード（以下 M と記す）予測は実験全域および全期間にわたって同一の *b* 値の Gutenberg-Richter（G-R）則に基づく独立分布系列を仮定している。これは実際には二重の意味で単純であると考える。 第 1 に G-R 則の *b* 値は地域性がある。このような *b* 値モデルは CSEP で唯一検証中である[1)]。第 2 に G-R 則の *b* 値や一般の M 分布は地震活動の履歴に依存する可能性がある。

本報告では前震群，群発地震群，本震余震群の統計的判別による方法[2)]を参考に、CSEP の検証規格に則って過去の震源データから逐次、次の地震の M の確率予測を試み、$b = 0.9$ の G-R 則（以下、基準 G-R 則と記す）と比較し検証した。

先ず M≧4 の気象庁地震カタログから Single-link 法[3] で群分けを行い、 10頁 にある様に群内の M 列がそれまでの最大 M より 0.5 以上の飛躍（$\Delta M \geq 0.5$）がある毎にリセットし予測 M 確率分布を再計算する。すなわち、先頭の地震（孤立地震を含む）に関しては基準 G-R で予測し、 群の 2 番目の地震が $\Delta M \geq 0.5$ の大きな地震である確率は $p_{2|c} = \mu(x_1, y_1)$、$n \geq 3$番目以降の地震が $\Delta M \geq 0.5$ の大きな地震である確率 $p_{n|c}$ は 10頁 の式で計算する。そして各時点での M の予測確率密度分布は 11頁 の式 $\Psi(M \mid M_1, \cdots, M_n)$ で与えられている。

各時点での M 予測の性能は 11頁 最下行にある対数尤度比で比べることができる。大規模なクラスターの地震は殆ど負の情報利得スコアが得られ、小さいサイズのクラスターは一般に正の利得を取る。85%以上のクラスターは高々4 つの地震しか含まないので、クラスターの 5 番目以降の地震については基準 G-R で予測する事にすると、全体としてこの予測は基準 G-R より優位であることが分かる **(14頁)** 。

この様に、様々な前震型アルゴリズムに対応する M 配列の分布を単一の G-R 型から適切に広げることは、大地震の確率利得を高め、有用である。

参 考 文 献

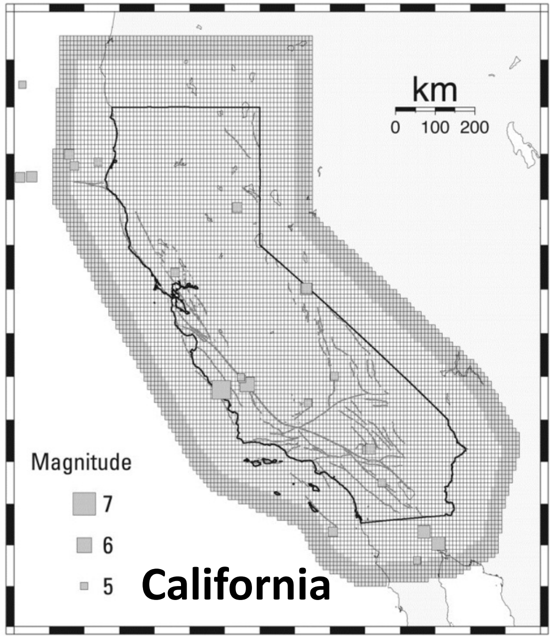
１） Ogata, Y., 2011, *Earth, Planets Space*, **63**, 217.

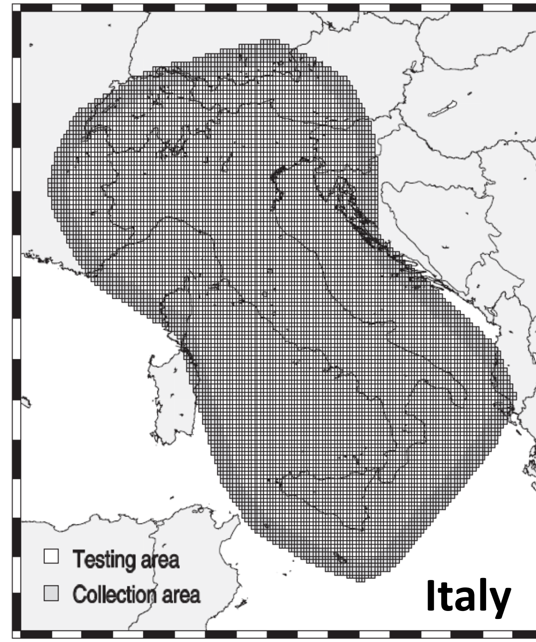
２） Ogata, Y., Utsu, T. and K. Katsura, 1996, *Geophys. J. Int.*, **127**, 17.

３） Ogata, Y., Utsu, T. and K. Katsura, 1995, *Geophys. J. Int.*, **121**, 233.

地震マグニチュードの予測と評価

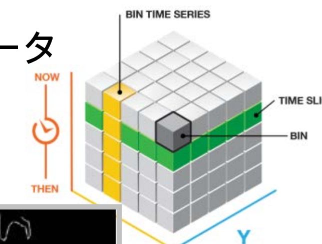
尾形良彦 統計数理研究所





$$\Pr\{an\ event\ in\ a\ bin\ [t,t+\Delta t]\times[x,x+\Delta x]\times[y,y+\Delta y]\times[M,M+\Delta M] \mid H_t,\ F_t\} \approx \lambda(t,x,y,M\mid H_t,F_t)\,\Delta t\,\Delta x\,\Delta y\,\Delta M$$

t 時刻; (x, y) 経度緯度; M マグニチュード; $H_t=\{(t_j,x_j,y_j,M_j);t_j<t\}$ 地震の発生履歴; F_tその他データ

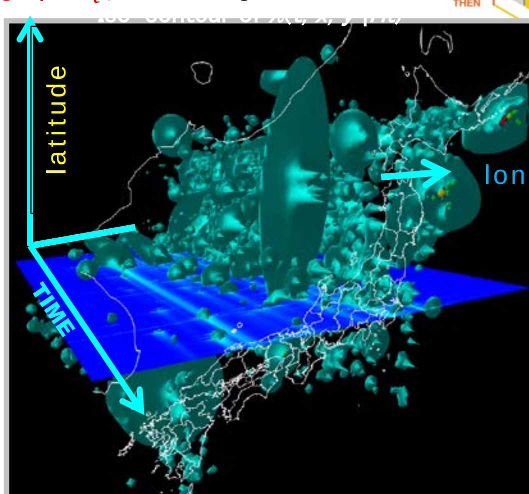


$$\Pr\{an\,event\,in[t,t+\Delta t)\times[x,x+\Delta x)\times[y,y+\Delta y)\mid H_t\} \approx \lambda(t,x,y\mid H_t)\Delta t\Delta x\Delta y$$

Space-Time ETAS model

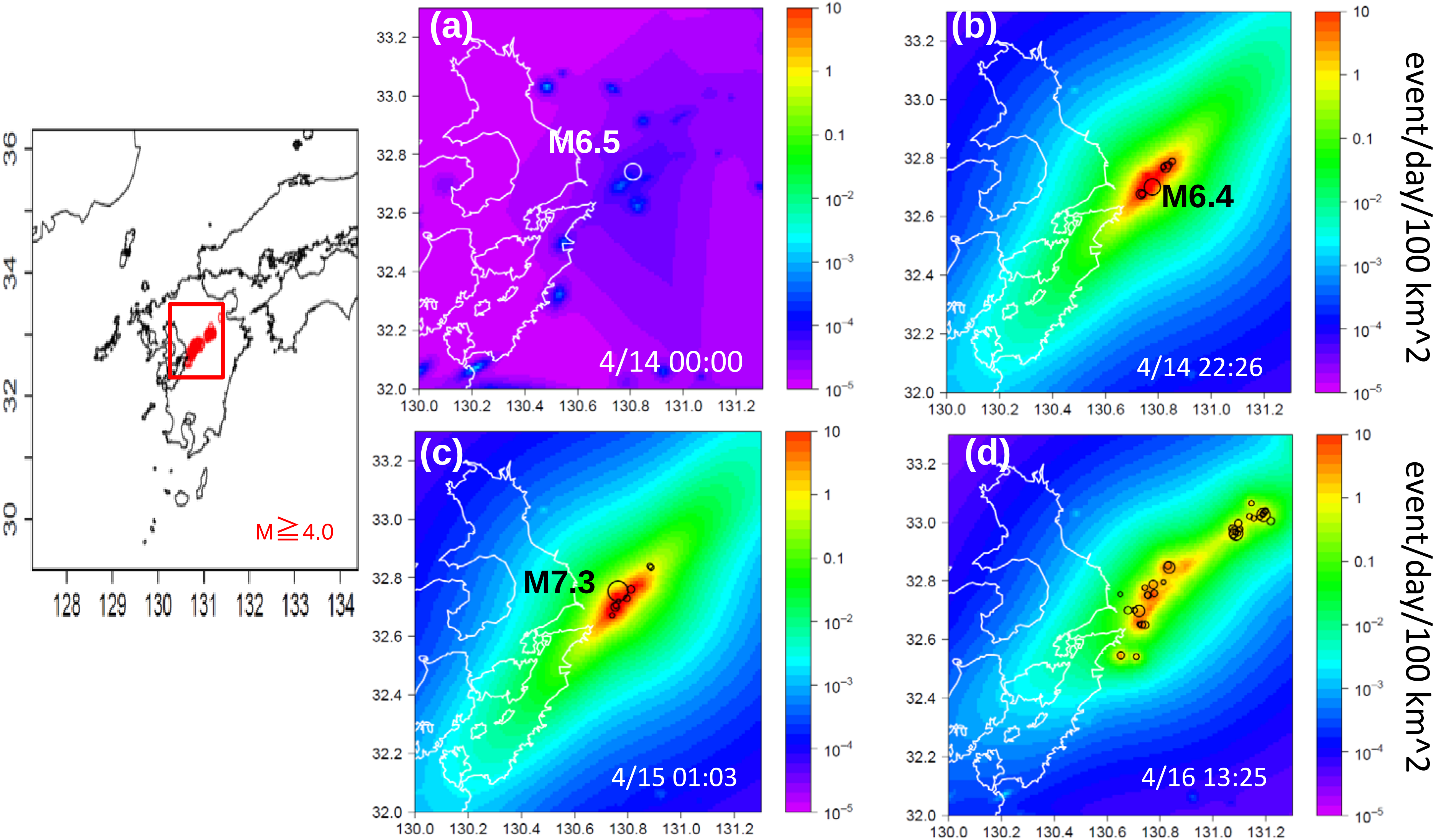
$$\lambda_\theta(t,x,y\mid H_t)=\mu\cdot\nu(x,y)+\sum_{\{j:t_j<t\}}\frac{K}{(t-t_j+c)^p}\left\{\frac{Q_j(x-\bar{x}_j,y-\bar{y}_j)}{e^{\alpha M_j}}+d\right\}^{-q}$$

where $Q_j(x,y)=(x-\bar{x}_j,y-\bar{y}_j)S_j^{-1}\begin{pmatrix}x-\bar{x}_j\\ y-\bar{y}_j\end{pmatrix}$



$$\lambda(t,x,y\,|\,H_t) = \mu(x,y) + \sum_{\{j;t_j<t\}} \frac{K_0(x_j,y_j)}{(t-t_j+c)^{p(x_j,y_j)}} \left[\frac{(x-x_j,y-y_j)\,S_j^{-1}\,(x-x_j,y-y_j)^t}{e^{\alpha(x_j,y_j)(M_j-M_c)}} + d \right]^{-q}$$

Rates of M≧4 event during the 2016 Kumamoto sequence



地震マグニチュードの予測モデル

基準モデル: Gutenberg-Richter 則 （*b*＝定数 ~ 0.9）

$$GR(M) = 10^{a-b(M-M_c)}$$

Gutenberg-Richter 則 （*b*＝位置依存, Ogata, 2011 *EPS*）

$$GR(M \mid x, y) = 10^{a(x,y)-b(x,y)(M-M_c)}$$

履歴に依存するマグニチュード分布

$$\Gamma(M \mid H_t)\,dM = P\left(M < Magnitude \le M + dM \mid H_t\right)$$

但し.

$$H_t = \left\{ (t_j, x_j, y_j, M_j); t_j < t \right\}$$

$$d_{ST} = \sqrt{\Delta^2_{space} + (c\Delta_{time})^2}$$

$$\leq 0.3^{\circ} \text{ (or 33.33km)}$$

$$c = 1^{\circ} / month \approx 1\ km / day$$

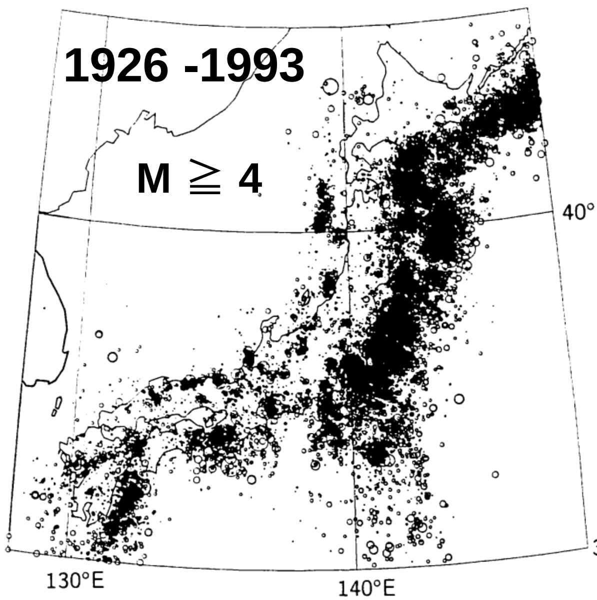
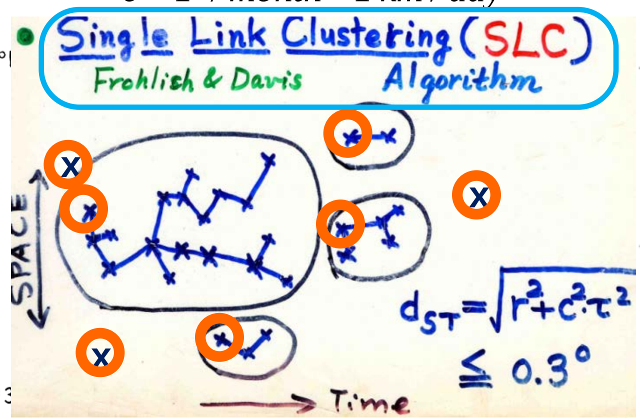
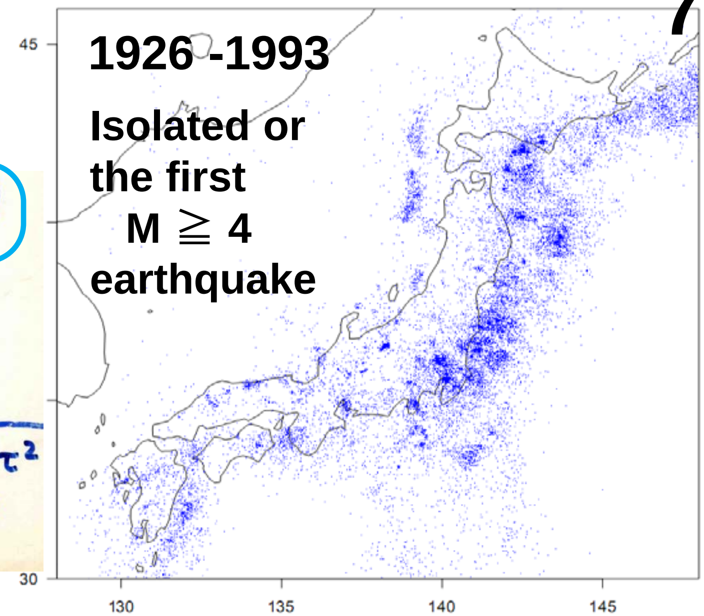
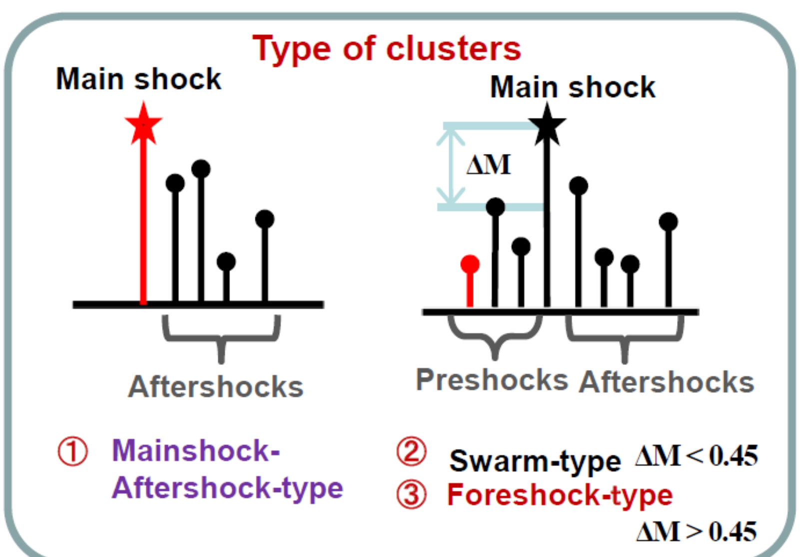


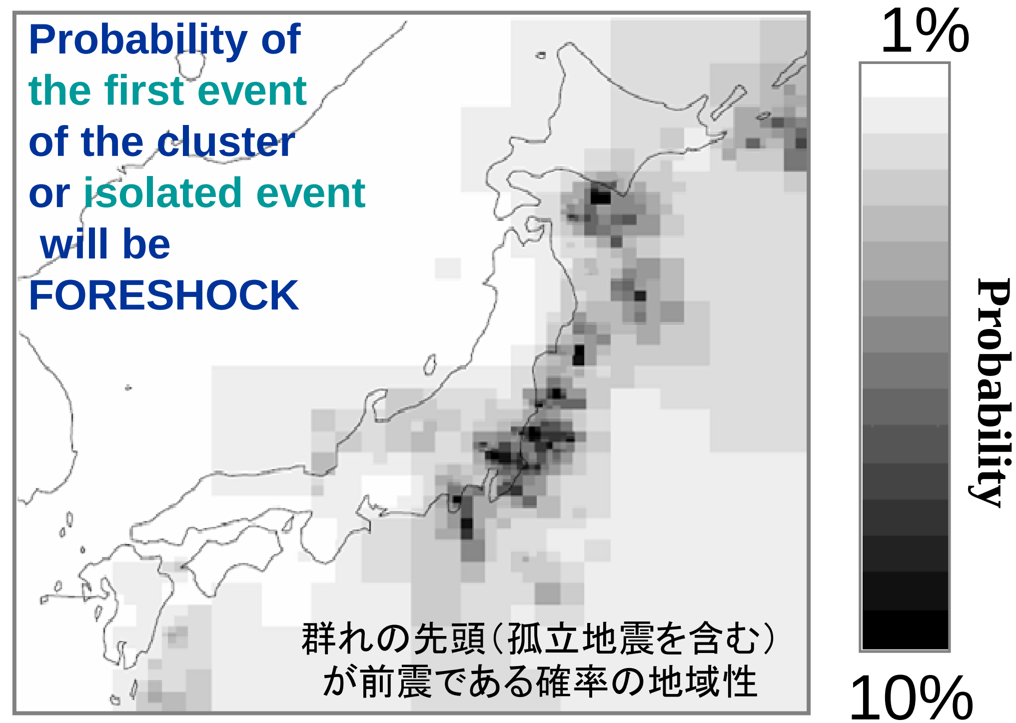
Figure 1. Epicentres of earthquakes ($M_J \geq 4.0$, depth ≤ 100 km) in the JMA catalogue (1926–91).

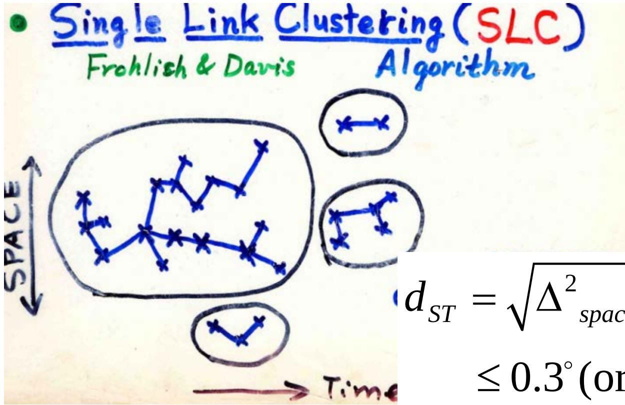




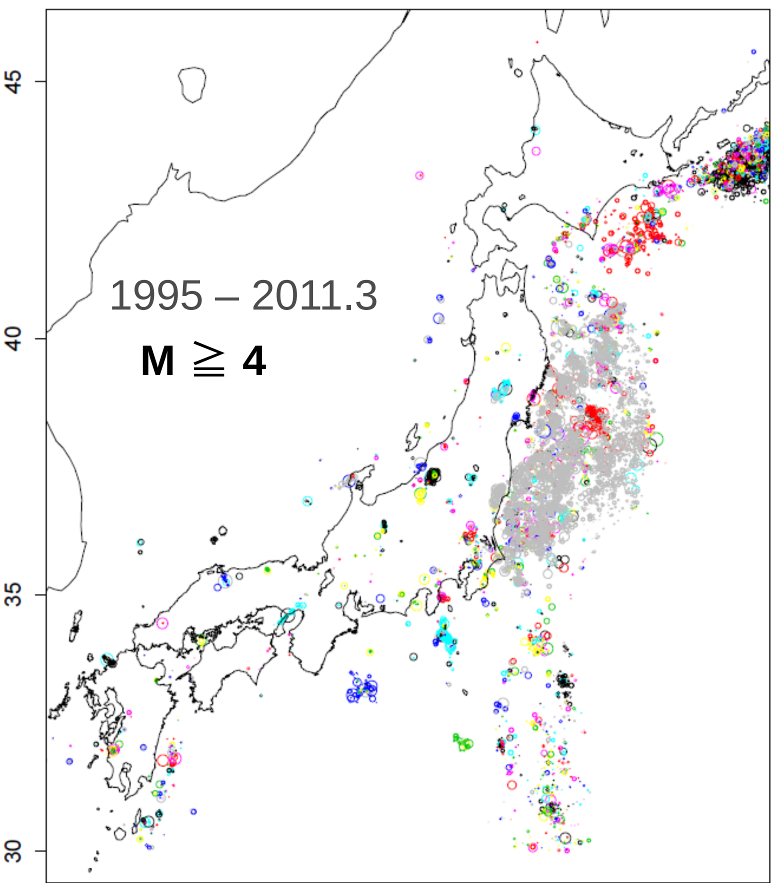
First earthquake

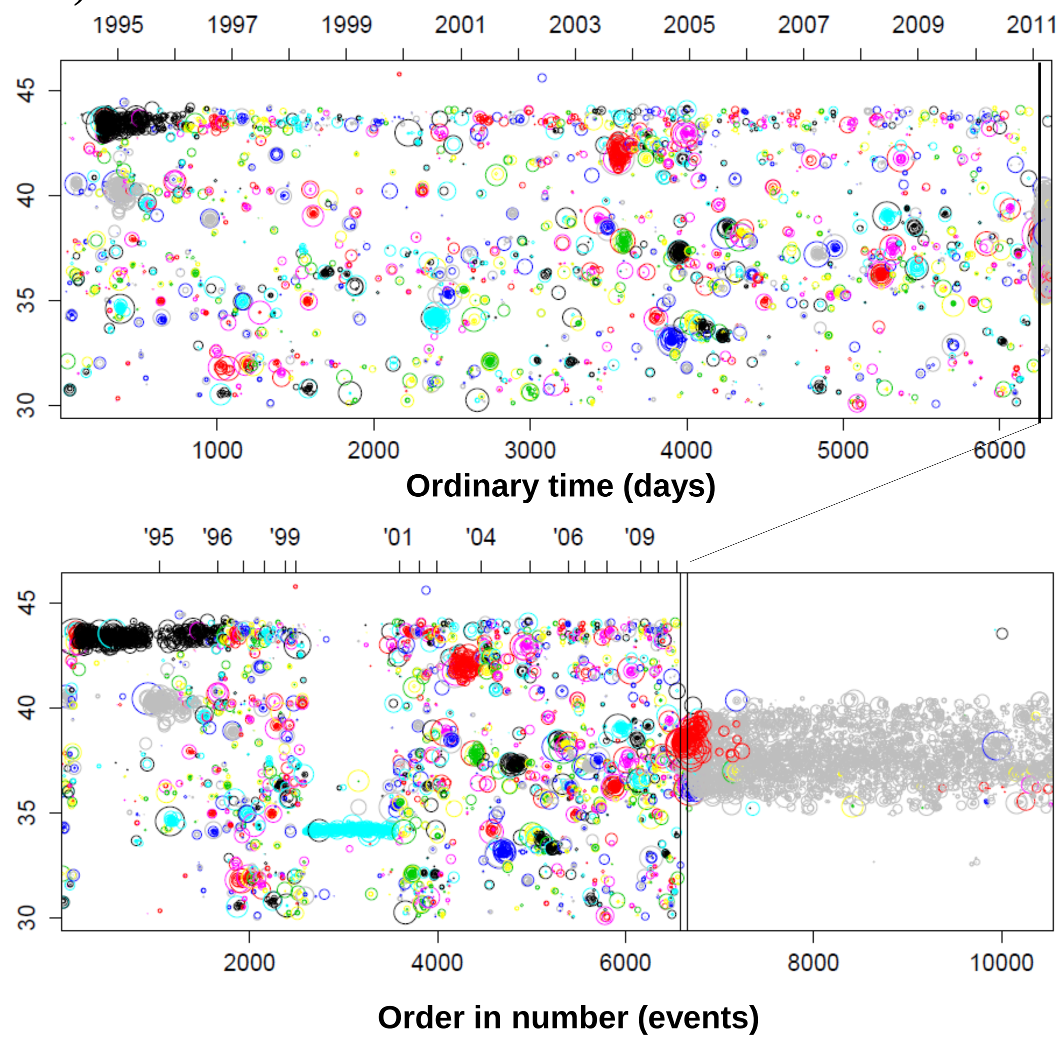


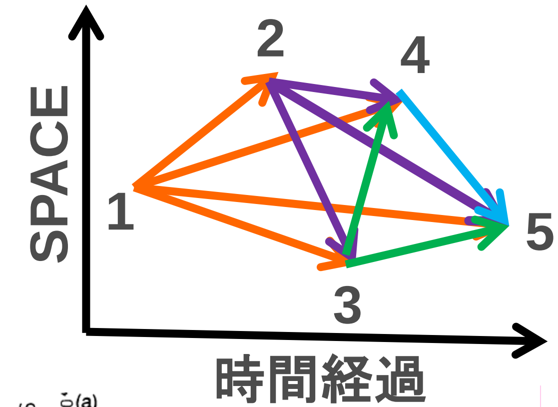


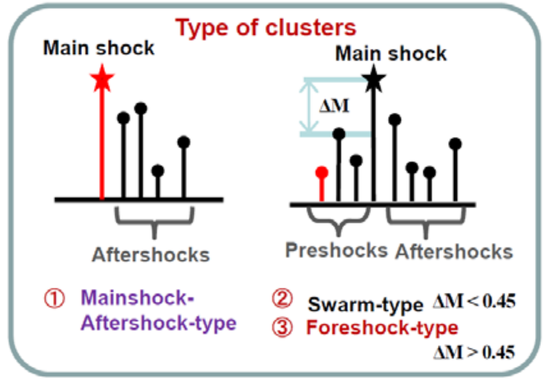


$$d_{ST} = \sqrt{\Delta^2_{space} + (c\Delta_{time})^2} \le 0.3^\circ \text{ (or 33.33km)}$$

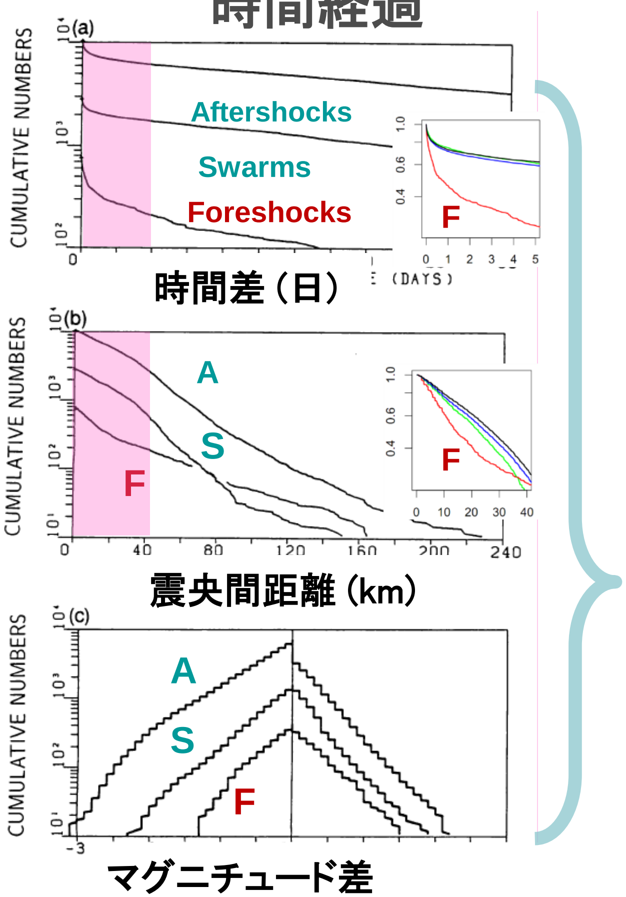


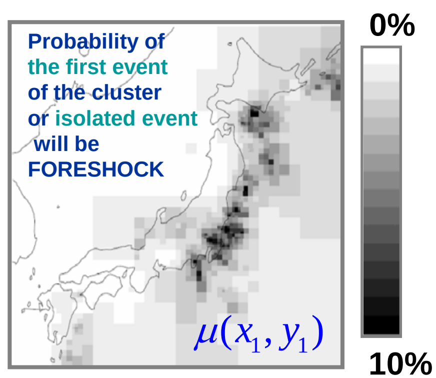


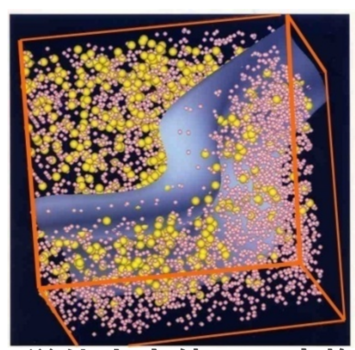




Ogata et al. (1995, 1996; *GJI*)
Ogata & Katsura (2012, *GJI* ; 2014, *JGR*)



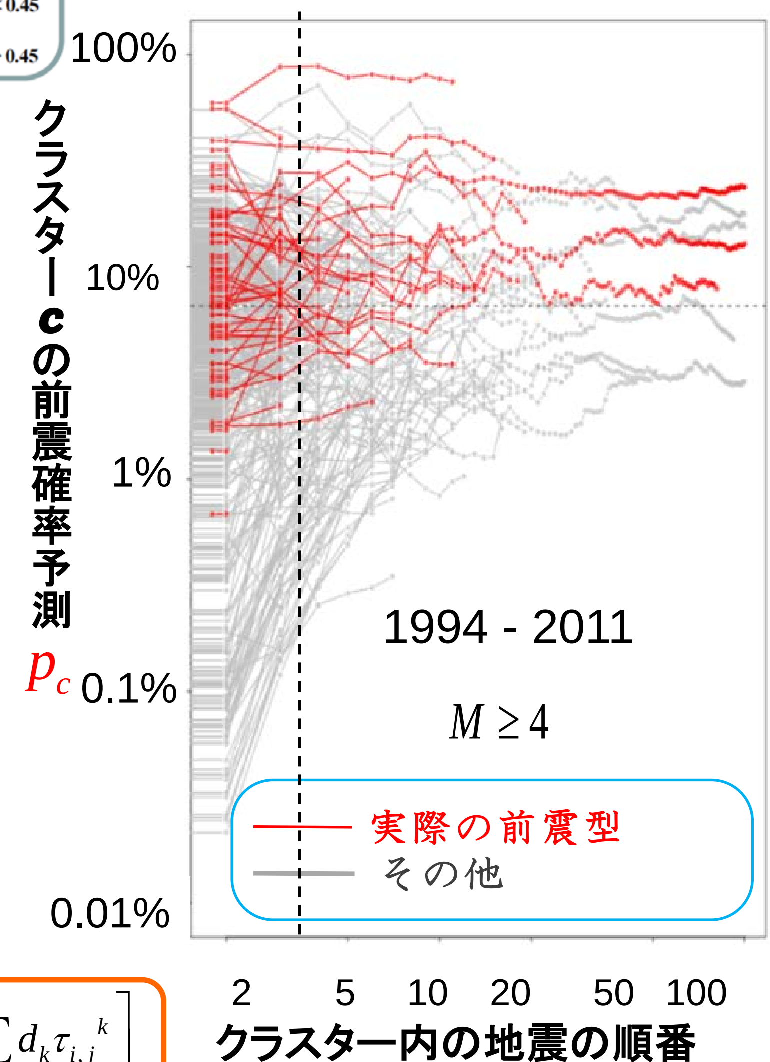




単位立方体への変換

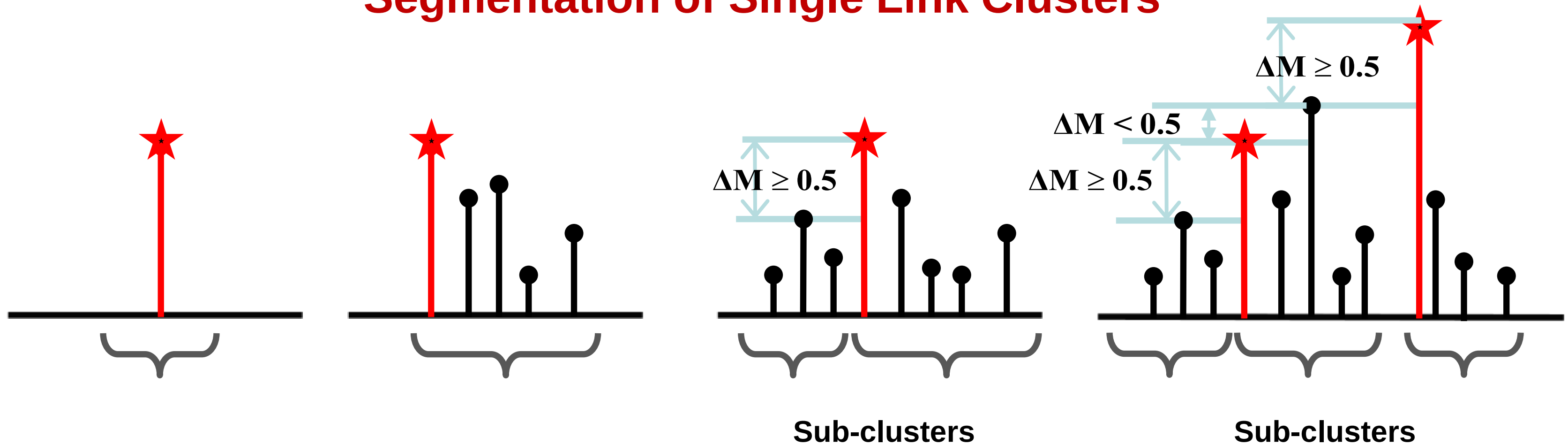
$$\left(\tau_{i,j},\ \rho_{i,j},\ \gamma_{i,j}\right)$$

（1ヶ月）予測と実際の結果



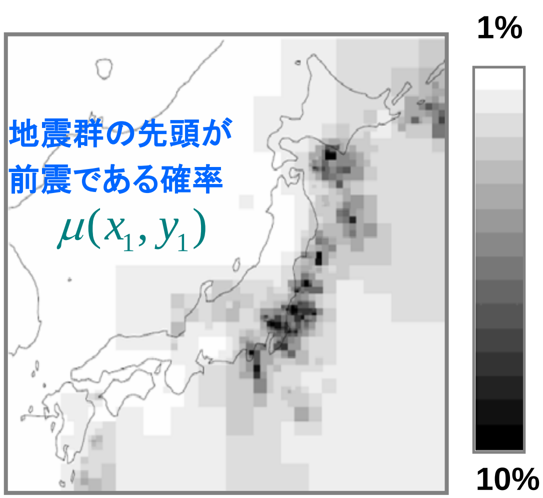
$$\ln\frac{1-p_c}{p_c} = \ln\frac{1-\mu(x_1,y_1)}{\mu(x_1,y_1)} + \frac{1}{\#\{i<j\}}\sum_{i<j}\left[a_1 + \sum_{k=1}^{3} b_k \gamma_{i,j}{}^{k} + \sum_{k=1}^{3} c_k \rho_{i,j}{}^{k} + \sum_{k=1}^{3} d_k \tau_{i,j}{}^{k}\right]$$

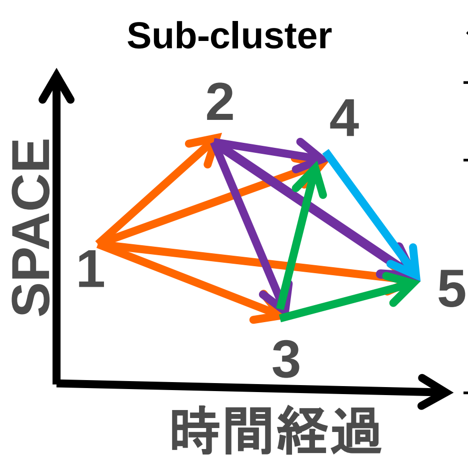
Segmentation of Single Link Clusters



$p_{c|n}$ ＝ 地震群 *c* の *n* 番目の地震でマグニチュードが0.5以上の更新確率

$$\ln\frac{1-p_{c|n}}{p_{c|n}} = \ln\frac{1-\mu(x_1,y_1)}{\mu(x_1,y_1)} + \frac{1}{\#\{i<j\}}\sum_{i<j<n}\left[a_1 + \sum_{k=1}^{3} b_k \gamma_{i,j}{}^{k} + \sum_{k=1}^{3} c_k \rho_{i,j}{}^{k} + \sum_{k=1}^{3} d_k \tau_{i,j}{}^{k}\right]$$





係数 from Ogata, Utsu and Katsura, 1996, *GJI*

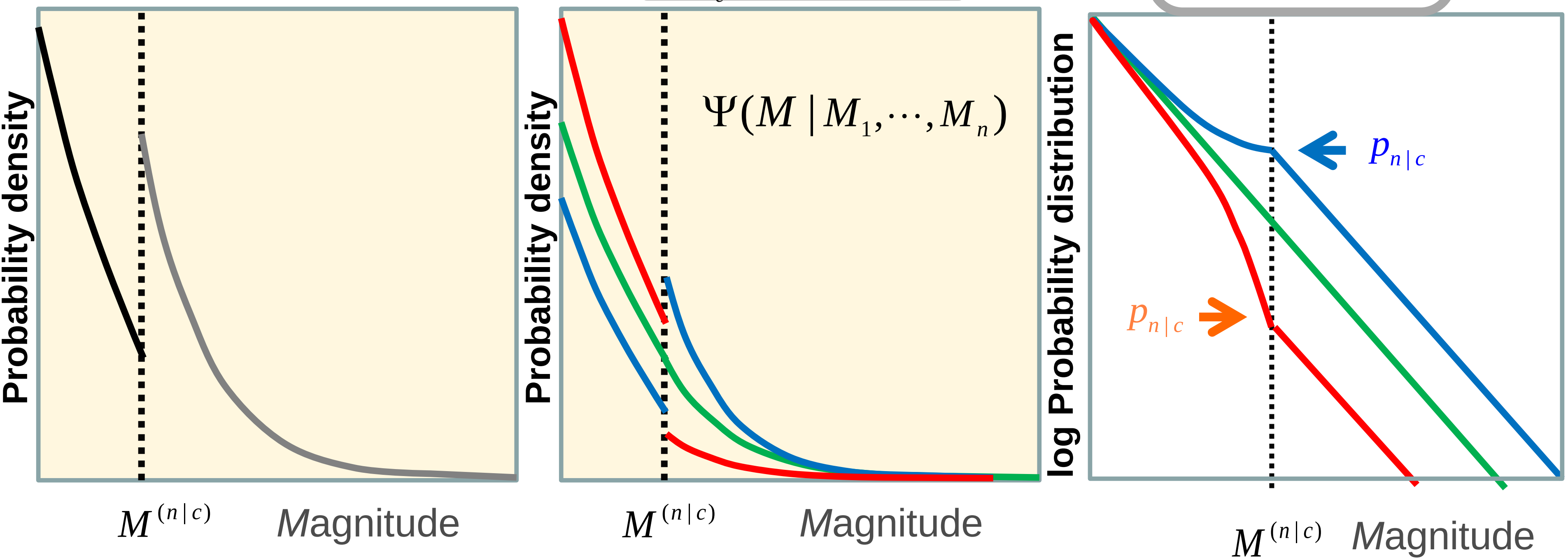
k	a_k	b_k	c_k	d_k
1	8.018	−33.25	−1.490	−10.92
2		62.77	2.805	295.09
3		−37.66	−2.190	−1161.50

Magnitude Gap : $M^{(n|c)} = \max\{M_k;\ k=1,\cdots,n\,|\,\text{in cluster } c\} + 0.5$

Probability of M≧Mmax+0.5 of the next magnitude; $p_{n|c} = P\{M_{n+1} > M^{(n|c)}\,|\,in\ c\}$

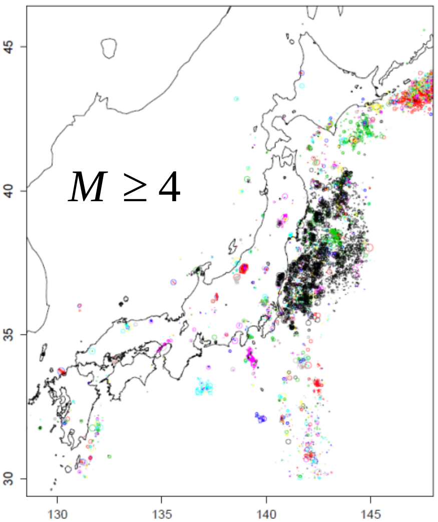
If $(t_{n+1}, x_{n+1}, y_{n+1})$ is connected to c,

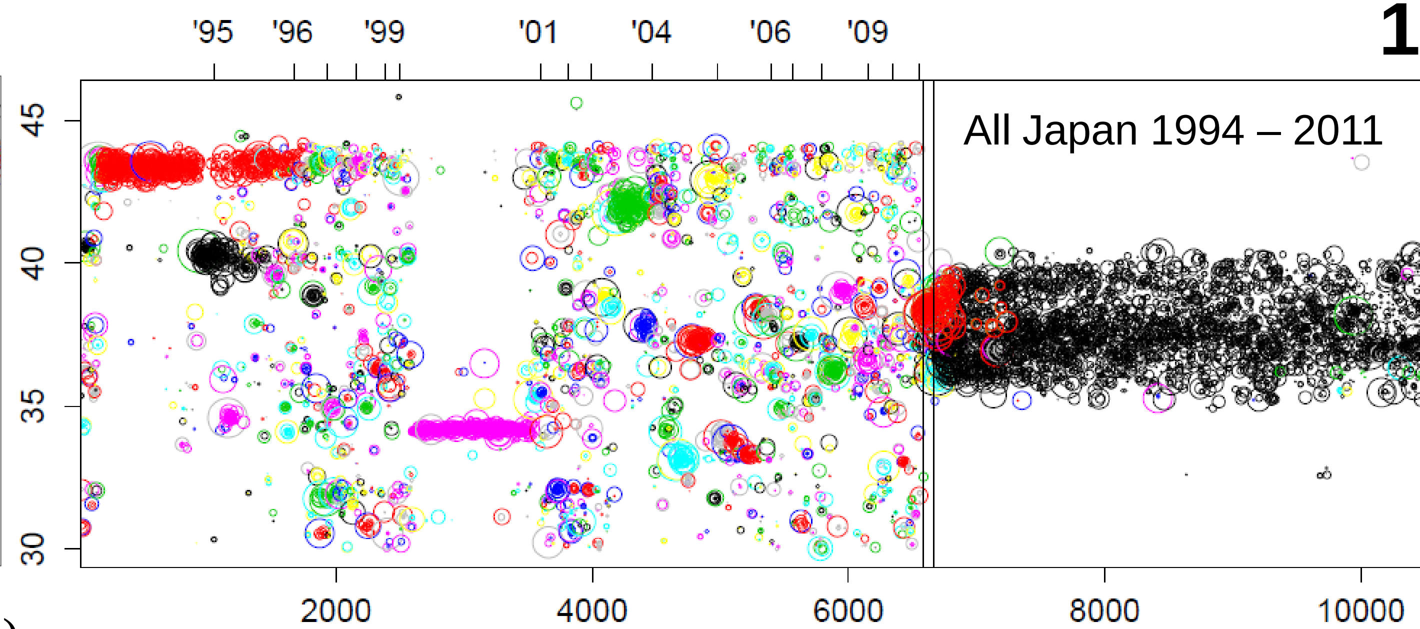
$$\Psi(M\,|\,M_1,\cdots,M_n) = (1-p_{n|c})\frac{1_{(M_c,\,M^{(n|c)})}(M)\cdot 10^{-bM}}{\int_{M_c}^{M^{(n|c)}} 10^{-bM}\,dM} + p_{n|c}(M)\frac{1_{(M^{(n|c)},\ \infty)}\cdot 10^{-bM}}{\int_{M^{(n|c)}}^{\infty} 10^{-bM}\,dM}$$



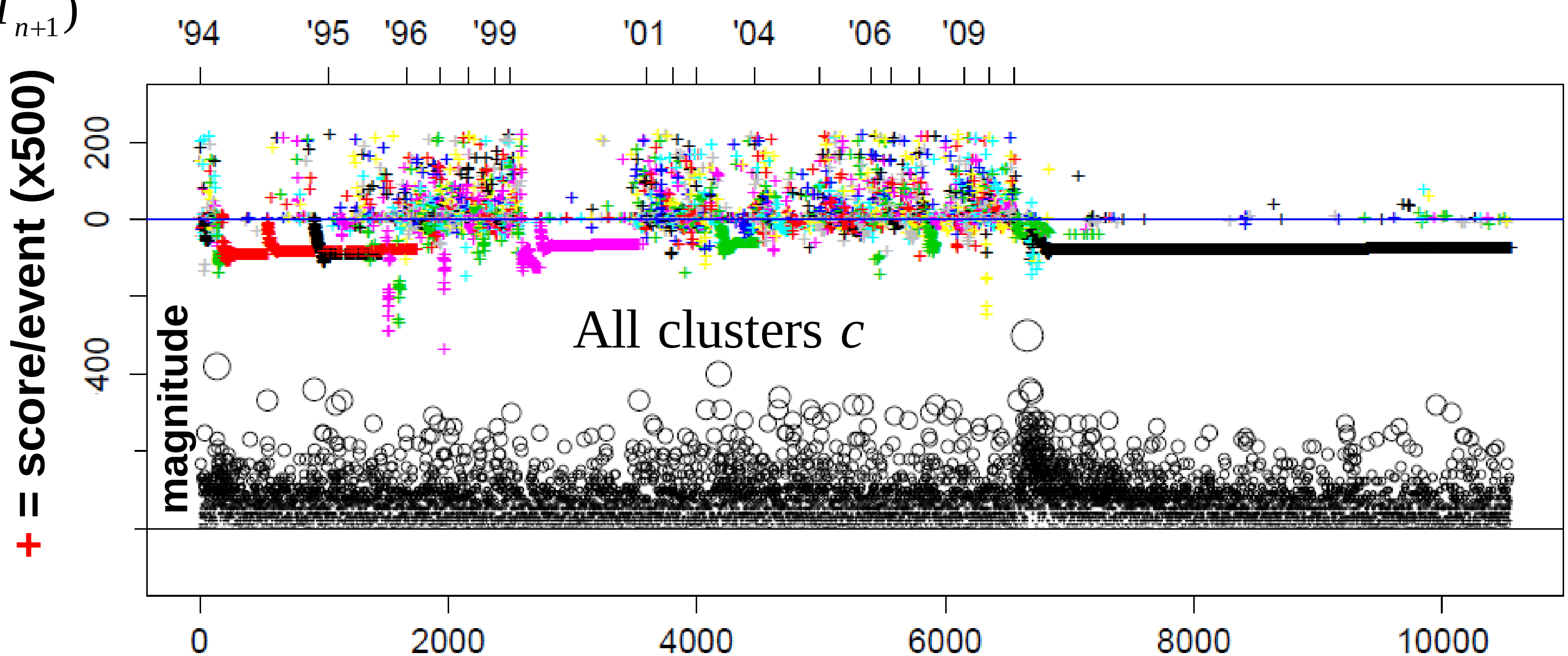
Otherwise, the reference model $\Psi(M) = 1_{(M_c,\infty)}(M)\cdot 10^{-bM} \Big/ \int_{M_c}^{\infty} 10^{-bM}\,dM$

log likelihood-ratio = information gain: $\log L/L_0 = \sum_c \sum_{n=1}^{\#c} \log \frac{\Psi_c(M_{n+1}\,|\,M_c^{(n)})}{\Psi_c(M_{n+1})}$





$$\log \frac{\Psi_c(M_{n+1} \mid M_c^{(n)})}{\Psi_c(M_{n+1})}$$ **= Information gain score per earthquake (+ signs)**



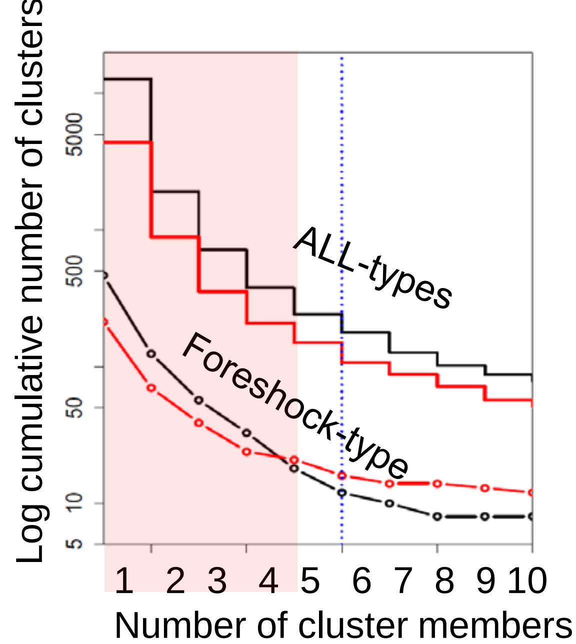
Single-linked clusters used for the learning 1926-1993

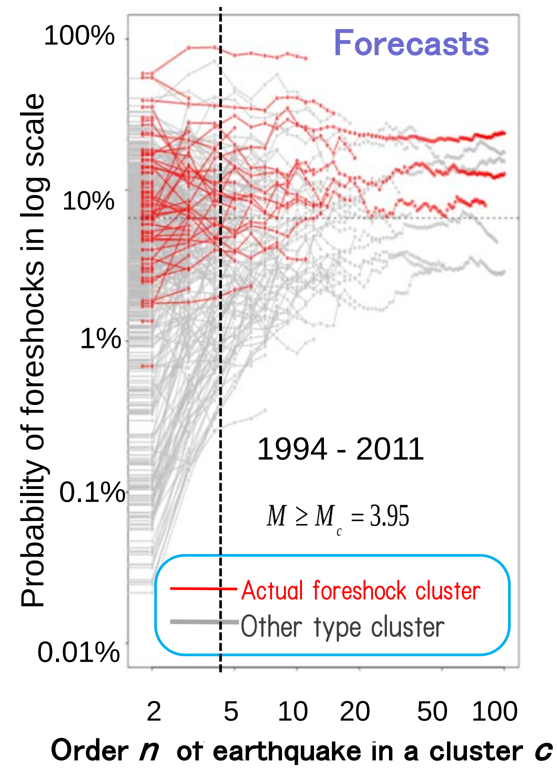
SLC

#c	1926~1993 #cluster	1926~1993 #f	1926~1993 rate (%)	1926~1975 #cluster	1926~1975 #f	1926~1975 rate (%)	1976~1993 #cluster	1976~1993 #f	1976~1993 rate (%)
≥1	12728	467	(3.7±0.2)	8550	306	(3.6±0.2)	4178	161	(3.9±0.3)
≥2	1916	125	(6.5±0.6)	1241	76	(6.1±0.7)	675	49	(7.2±1.0)
≥3	715	57	(8.0±1.0)	468	32	(6.8±1.1)	247	25	(10.1±1.9)
≥4	378	33	(8.7±1.5)	258	20	(7.8±1.8)	120	13	(10.8±2.8)
≥5	242	18	(7.4±1.7)	162	8	(4.9±1.7)	80	10	(12.5±3.7)
≥6	177	12	(6.8±1.9)	120	6	(5.0±2.0)	57	6	(10.5±4.1)
≥7	127	10	(7.9±2.4)	87	5	(5.7±2.5)	40	5	(12.5±5.3)
≥8	102	8	(7.8±2.6)	68	3	(4.4±2.5)	34	5	(14.7±6.1)
≥9	87	8	(9.2±3.1)	55	3	(5.5±3.1)	32	5	(15.6±6.4)
≥10	78	8	(10.3±3.4)	50	3	(6.0±3.4)	28	5	(17.8±7.2)

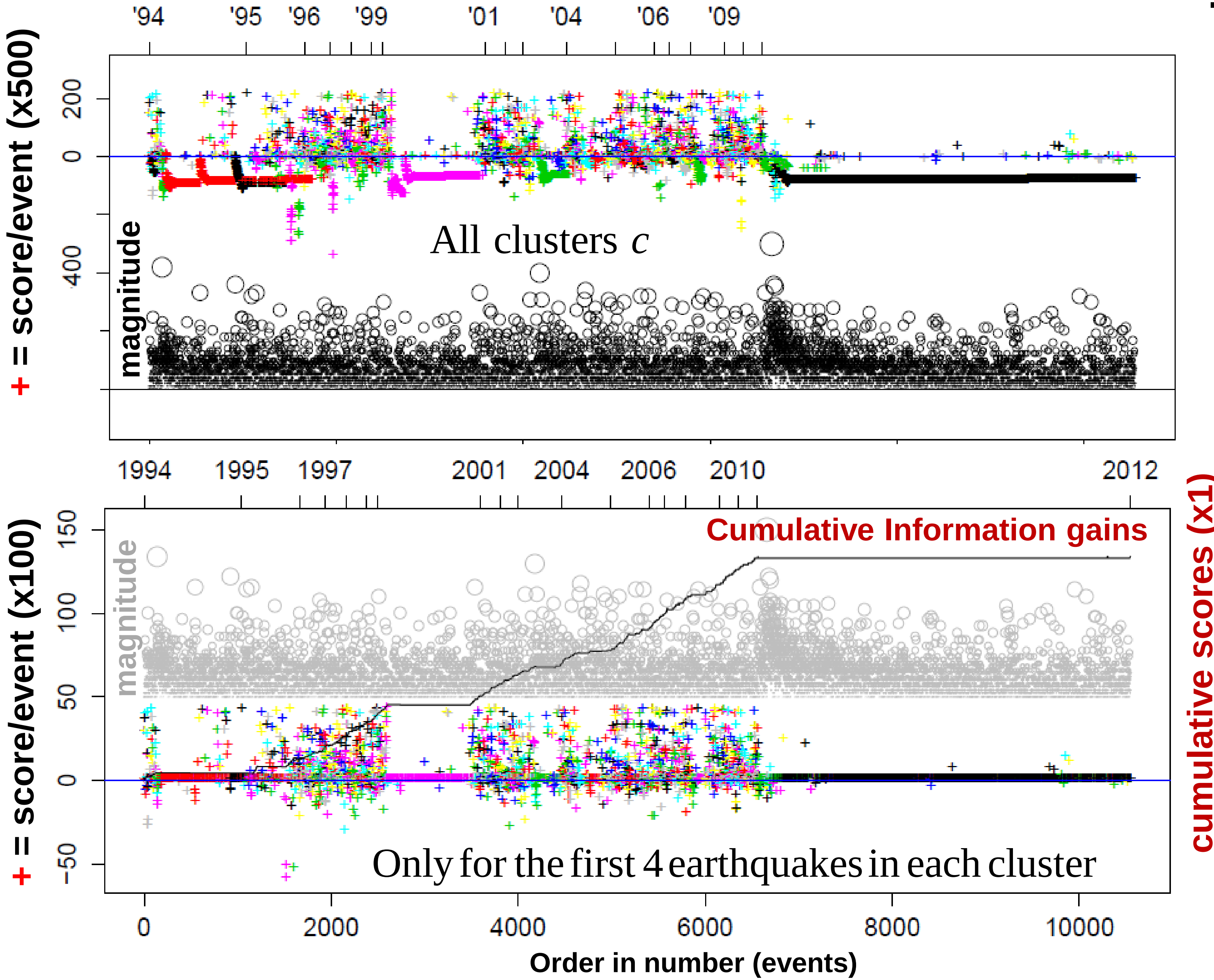
Single-linked clusters used for the experiments1994-2011

#c	#F	Per cent F	+/−	#S	Per cent S	+/−	#MA	#F + #S	#All
≥1	214	4.9	0.3	277	6.3	0.4	3894	491	4385
≥2	70	7.9	0.9	277	31.2	1.6	542	347	889
≥3	39	11.0	1.7	133	37.4	2.6	184	172	356
≥4	24	11.5	2.2	88	42.1	3.4	97	112	209
≥5	21	14.0	2.8	59	39.3	4.0	70	80	150
≥6	16	14.8	3.4	41	38.0	4.7	51	57	108
≥7	14	15.9	3.9	33	37.5	5.2	41	47	88
≥8	14	19.4	4.7	26	36.1	5.7	32	40	72
≥9	13	22.8	5.6	17	29.8	6.1	27	30	57
≥10	12	23.5	5.9	16	31.4	6.5	23	28	51

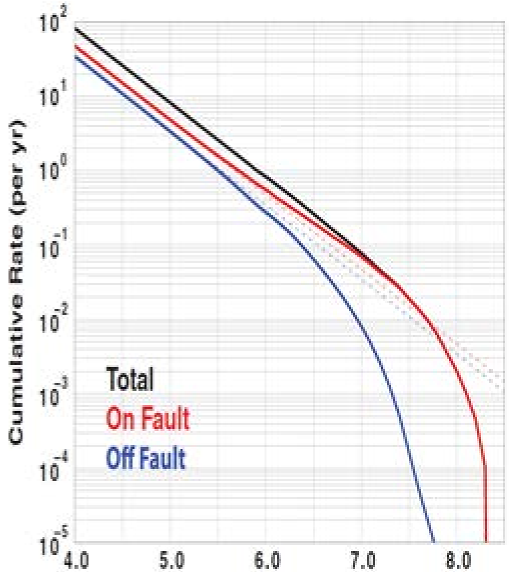


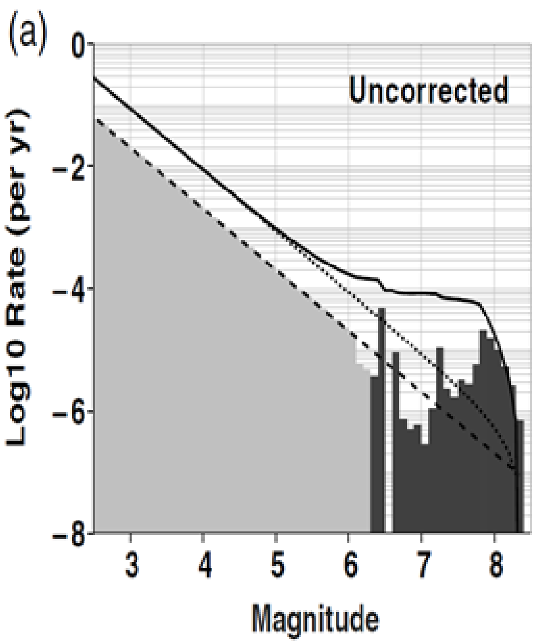


Information gain scores; All Japan 1994 – 2011, $M \geq 4$

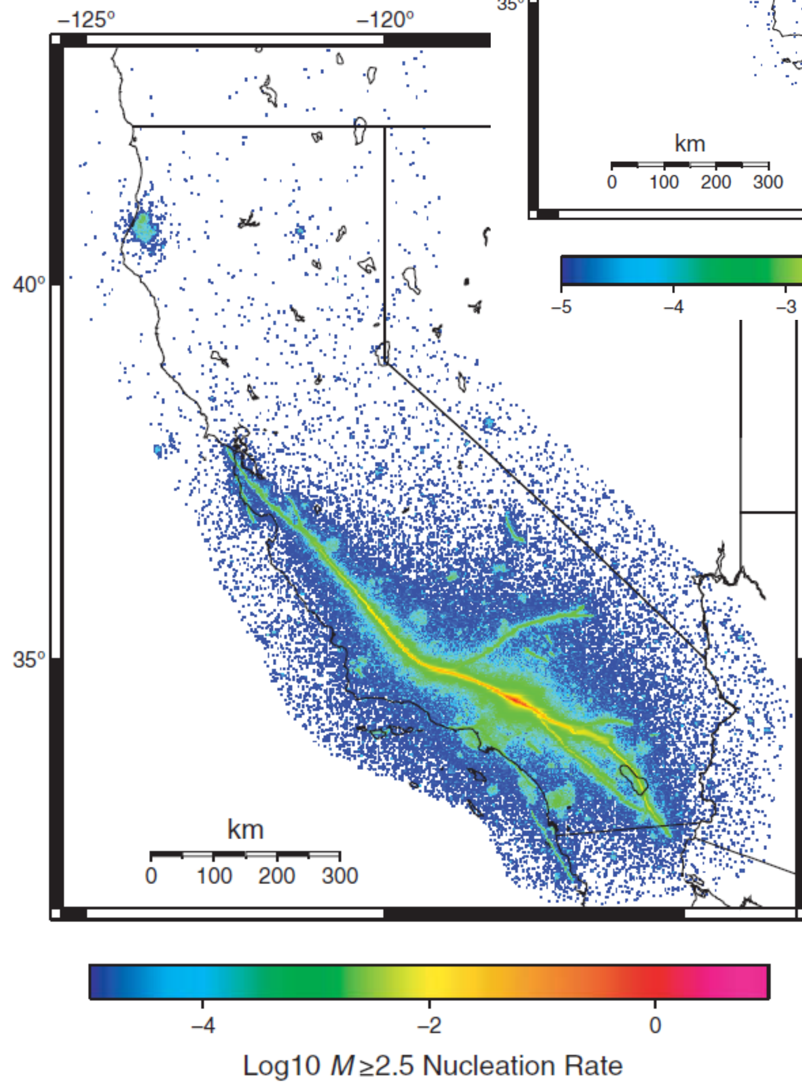


Field et al. (2017, *BSSA***)**

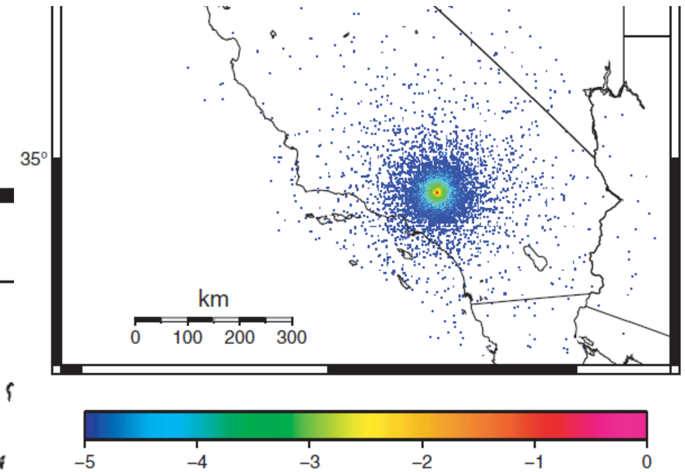




UCERF3-ETAS



ETAS (no fault)

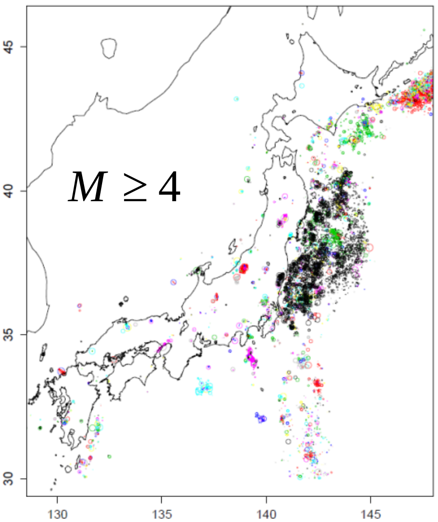


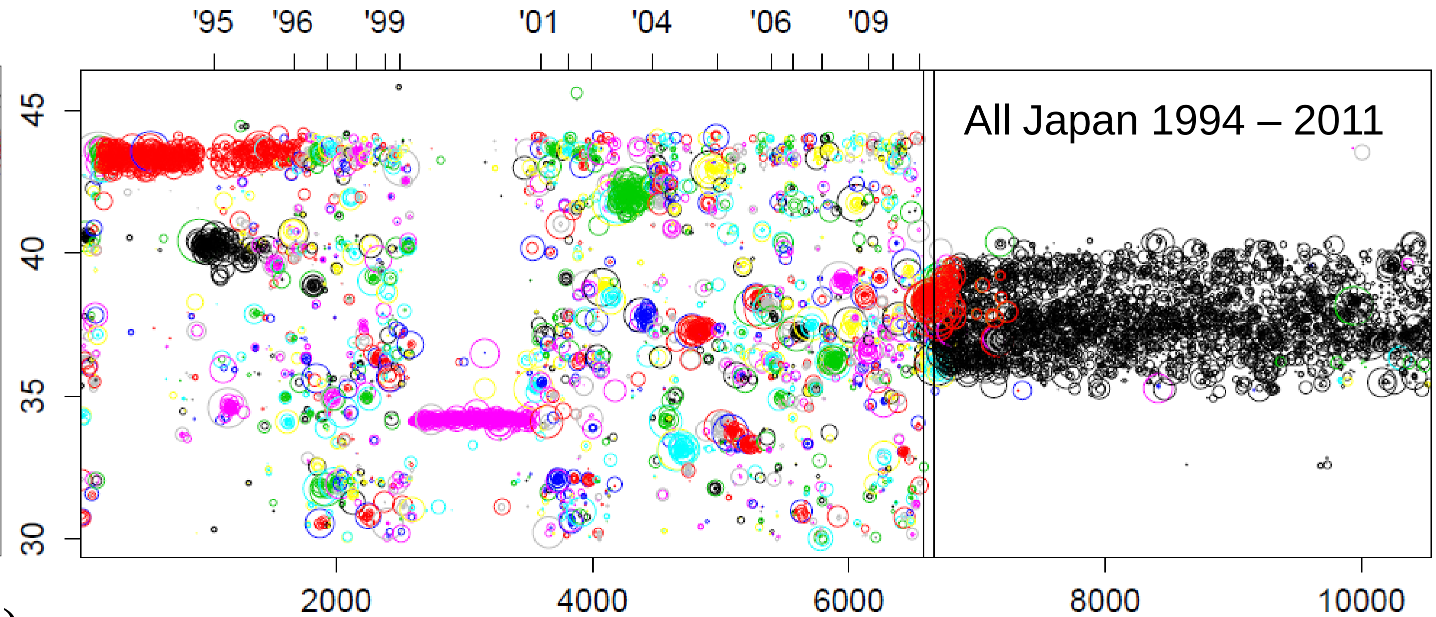
まとめと提案

(1) CSEPプロジェクトの次の課題は、地震発生履歴の特徴および関連地球物理的異常現象に関係するマグニチュード予測モデルを探求することである。
　地震発生特徴には、地震マグニチュード列の変化、前震判別に有効な時空間クラスタリングの集中性の強さ、地震の静穏化と活発化、および先駆的群発地震活動などが含まれる。

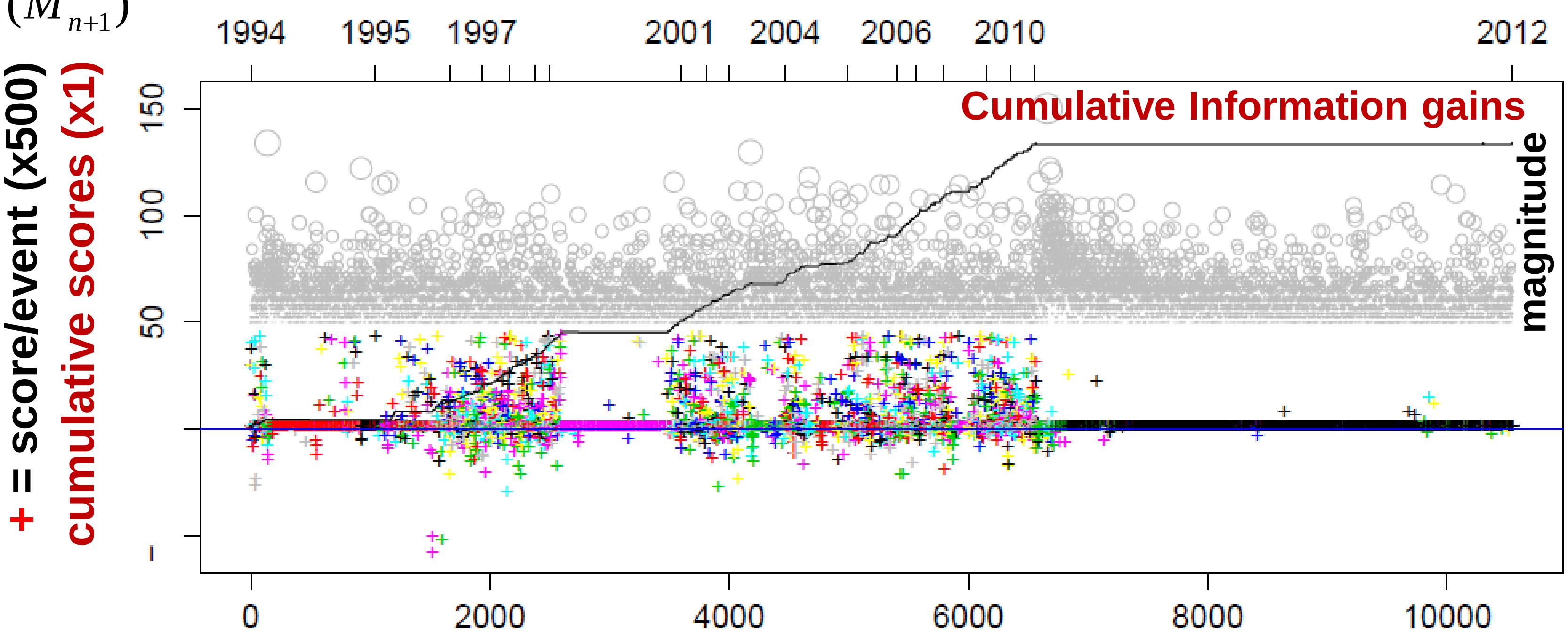
(2) 警報型の大地震の予測は、経験的な成功率の統計を考慮してマグニチュードの分布でモデル化することもできる。
　これらは、前駆的異常情報に基づくマグニチュードの予測アルゴリズムとして提案すれば、それらを独立G-R分布を基準モデルとして情報利得を比較できる。

(3) 既存のCSEPの時間・空間・マグニチュードの対数尤度スコアを用いて試験を総合的に実施すべきである。
　しかし、CSEPで採用されている従来のマグニチュードテストでは、マグニチュード予測の体系的な違いには関係していない。
　テストは、モデルを改善するための診断目的で使用する必要があるため、マグニチュード頻度に関する対数周辺尤度の局所的なスコアまたは対数の条件付き尤度によるテストを実行できる。





$$\log \frac{\Psi_c(M_{n+1} \mid M_c^{(n)})}{\Psi_c(M_{n+1})}$$ **= Information gain score per earthquake (+ signs)**



Algorithm of foreshock probability calculations in case of plural earthquakes in a cluster

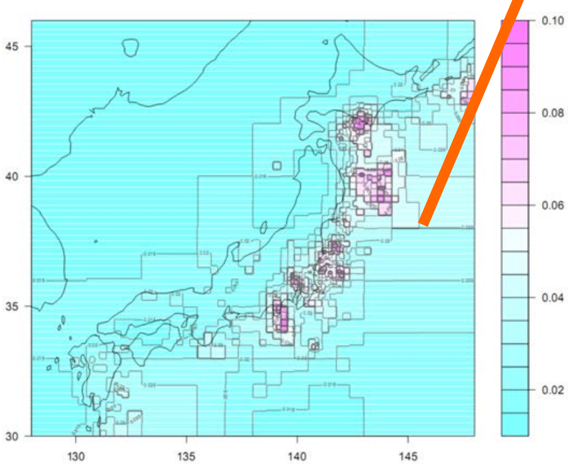
For plural earthquakes in a cluster, time differences $t_{i,j}$ (days), epicenter separation $r_{i,j}$ (km), magnitude difference $g_{i,j}$ are transformed into the unit cube

$$(t_{i,j}, r_{i,j}, g_{i,j}) \to (\tau_{i,j}, \rho_{i,j}, \gamma_{i,j}) \in [0,1]^3$$

$$p = \frac{1}{1+e^f} \Leftrightarrow f \equiv \mathbf{logit}(p) \equiv \ln \frac{1-p}{p}$$

Probability $\boldsymbol{p}_{\mathbf{c}}$ is calculated sequentially

$$\mathbf{logit}(p_c) = \mathbf{logit}\{\mu(x_1, y_1)\} + \frac{1}{\#\{i<j\}} \sum_{i<j} \left[a_1 + \sum_{k=1}^{3} b_k \gamma_{i,j}{}^k + \sum_{k=1}^{3} c_k \rho_{i,j}{}^k + \sum_{k=1}^{3} d_k \tau_{i,j}{}^k \right]$$



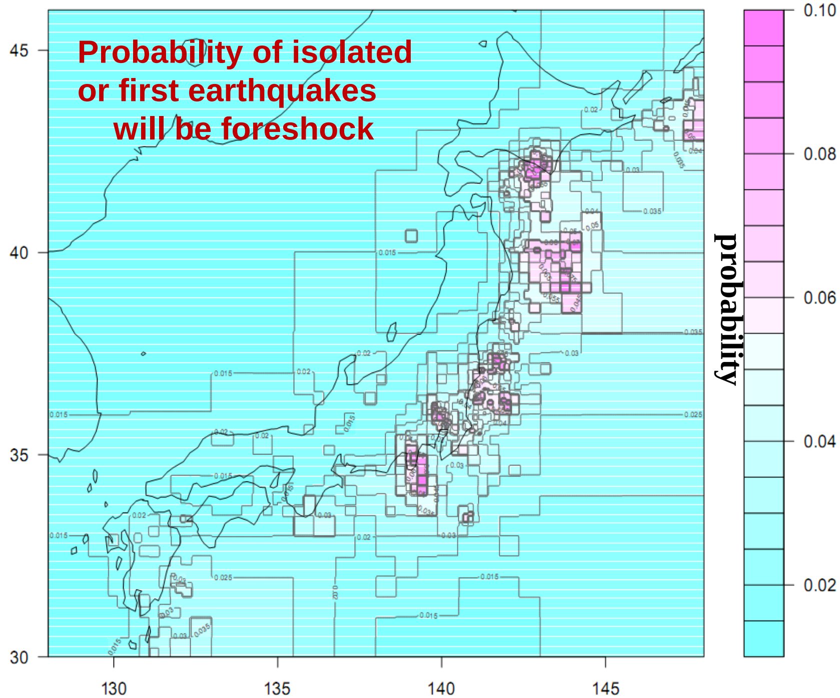
…ability of initial
ı (x,y).

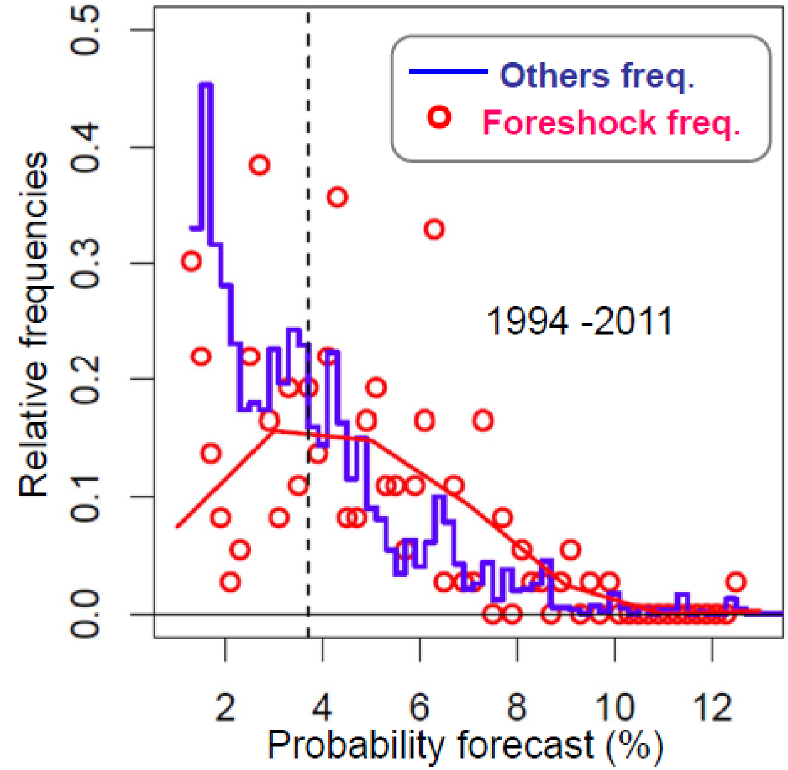
probability

Arithmetic mean of polynomials of the normalized space-time magnitude variables for all pairs of earthquakes $(i < j)$ in a cluster.
The coefficients *a*, *b*, *c*, *d* are estimated by the maximum likelihood method together with the AIC.

Ogata, Utsu and Katsura, 1996, *GJI*)

k	a_k	b_k	c_k	d_k
1	8.018	33:25	-1.490	-10.92
2		62.77	2.805	295.09
3		37:66	-2.190	-1161.5





Forecasted results for 1994 – Mar 2011

Table 2. Contingency table of outcomes against forecast probabilities for the first events of clusters or isolated earthquakes.

Forecast	0–2.5 per cent	2.5–5 per cent	5 per cent	All
Foreshocks	33	84	65	182
Others	1572	1849	770	4191
All types	1605	1933	835	4373
Ratio (per cent)	2.1	4.3	7.8	4.2

Multiple earthquakes in a cluster

Measuring inter-events concentrations in a cluster and magnitude increments

$(t_i^c, x_i^c, y_i^c, M_i^c)$:

Hypocenters in a cluster $c \in C$

① Origin-time differences

$$t_{ij}^c = t_j^c - t_i^c$$

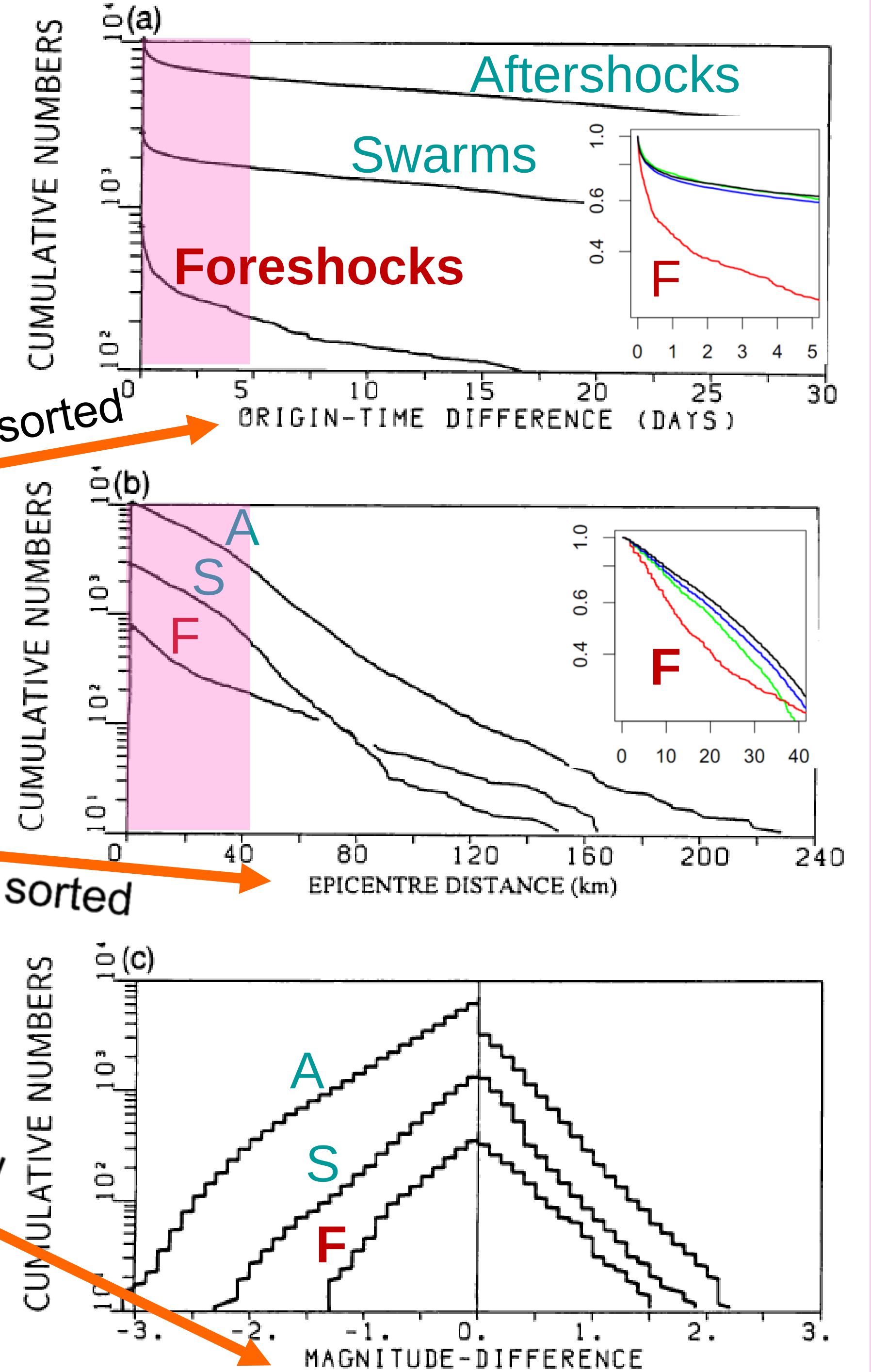
for any pairs i, j such that $i<j$

② Epicenter separations

$$r_{ij}^c = \sqrt{(x_j^c - x_i^c)^2 + (y_j^c - y_i^c)^2} \quad (i<j)$$

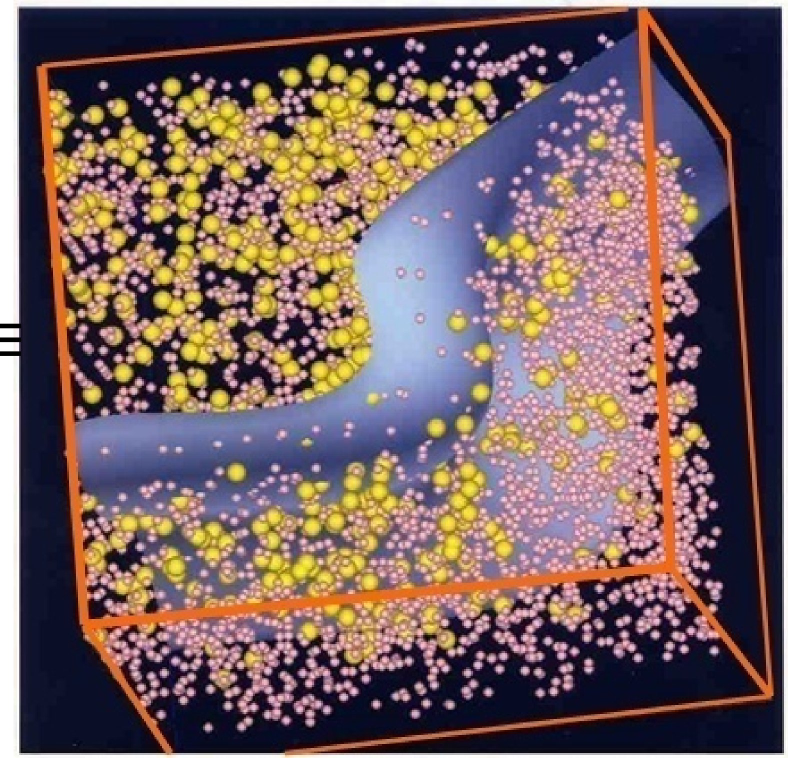
③ Magnitude differences

$$M_{ij}^c = M_j^c - M_i^c \quad (i<j)$$



Normalized time, distance & magnitude difference in unit cube

$$(t, r, g) \rightarrow (\tau, \rho, \gamma) \text{ in } [0,1]^3 \equiv$$



Time Interval Transformation

$$\tau = \begin{cases} 0 & \text{for} \quad t \leq 0.01 \\ \log(100t)/\log(3000) & \text{for } 0.01 < t \leq 30 \\ 1 & \text{for} \quad 30 \leq t \end{cases}$$

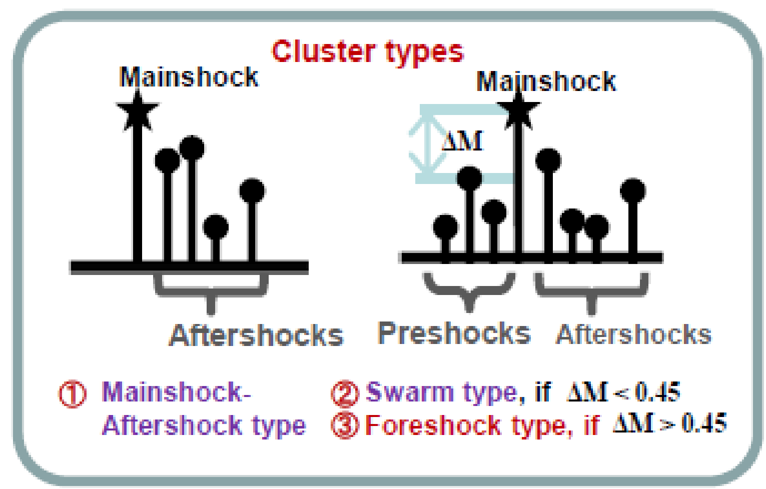
Epicenter Separation Transformation

$$\rho = 1 - \exp\{-\min(r, 50)/20\}$$

Magnitude Difference Transformation

$$\gamma = \begin{cases} (2/3)\exp\{g/\sigma_1\} & \text{for } g \leq 0 \\ (2/3) + (1/3)[1 - \exp\{-g/\sigma_2\}] & \text{for } g > 0 \end{cases}$$

where $\sigma_1 = 6709, \sigma_2 = 0.4456$



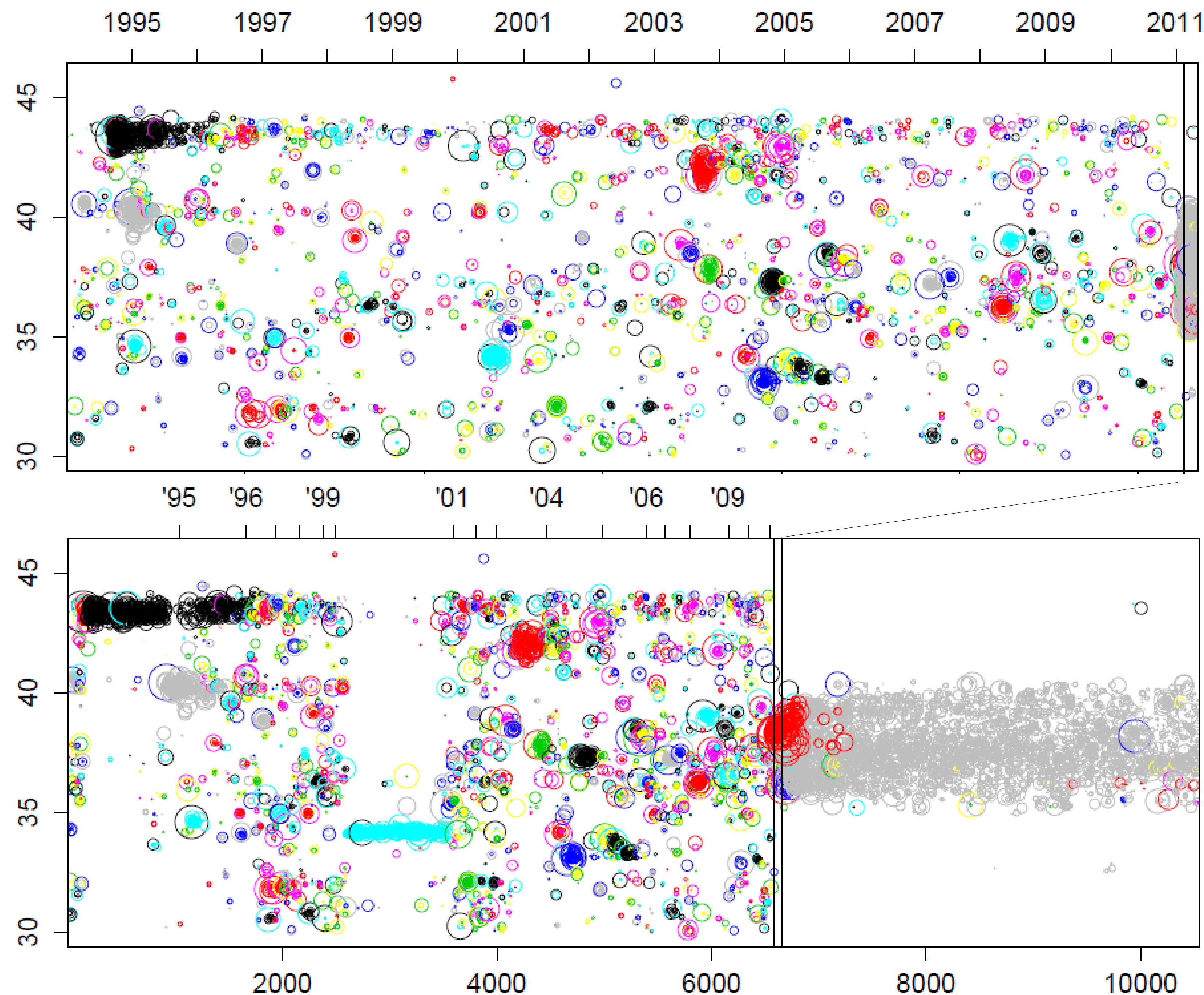
Forecasted sequence and evaluation（1994－2011Mar）

#	F?	#C	Pc	ENTRPY	CU~ENT	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
1	-	1	5.14%	-0.01537	-0.01537	5.14%									
2	-	2	10.06%	-0.06863	-0.08400	7.46%	12.66%								
3	-	1	18.58%	-0.16822	-0.25222	18.58%									
4	-	1	10.71%	-0.07592	-0.32814	10.71%									
5	-	1	0.15%	0.03586	-0.29228	0.15%									
6	-	1	1.70%	0.02028	-0.27200	1.70%									
7	-	4	9.50%	-0.06243	-0.33443	9.14%	11.17%	7.87%	9.82%						
8	-	1	6.03%	-0.02484	-0.35927	6.03%									
9	-	1	1.77%	0.01950	-0.33977	1.77%									
10	+	1	13.14%	1.27605	0.93628	13.14%									
…..															
875	+	80	9.2%	0.923	28.649	6.7%	27.8%	27.7%	20.1%	14.0%	14.2%	13.6%	11.6%	15.7%	11.9%
						10.1%	8.2%	10.1%	11.7%	10.9%	10.6%	11.5%	11.1%	9.9%	8.2%
						7.2%	6.8%	7.6%	7.3%	7.4%	6.7%	7.0%	7.0%	8.0%	8.5%
						8.6%	8.2%	8.0%	8.1%	8.4%	7.8%	7.3%	7.5%	7.8%	8.1%
						8.1%	7.8%	7.4%	7.7%	7.8%	7.6%	7.2%	7.2%	6.9%	6.8%
						6.7%	7.4%	8.0%	7.8%	7.6%	7.7%	8.3%	9.0%	8.7%	8.5%
						8.6%	8.3%	8.4%	8.2%	8.2%	8.0%	7.9%	7.9%	8.4%	8.4%
						8.6%	8.5%	8.6%	8.4%	8.2%	8.4%	8.3%	8.3%	8.1%	7.9%
…..															
880	-	11	2.44%	0.01266	31.60644	4.69%	4.77%	6.21%	3.42%	1.74%	1.24%	1.04%	0.90%	0.83%	0.97%
						1.03%									
881	-	16	2.11%	0.01604	31.62248	0.03%	0.25%	0.51%	0.83%	2.77%	2.21%	2.02%	3.19%	2.78%	2.50%
						2.43%	3.07%	2.92%	2.74%	2.84%	2.68%				
882	-	7	1.47%	0.02259	31.64507	0.06%	0.79%	1.70%	2.06%	1.90%	1.90%	1.88%			
883	-	1	4.51%	-0.00878	31.63629	4.51%									
884	-	1	3.84%	-0.00178	31.63451	3.84%									
885	+	7	5.04%	0.31698	31.95149	6.89%	7.42%	4.88%	3.98%	3.56%	4.05%	4.49%			
886	-	1	2.84%	0.00853	31.96002	2.84%									
887	-	1	7.00%	-0.03518	31.92483	7.00%									
888	-	1	7.65%	-0.04219	31.88264	7.65%									
889	-	1	7.83%	-0.04419	31.83845	7.83%									

M7.3 Foreshock of 9 Mar 2011

M9.0

2*Entropy0 = 523.96;　2*Entropy = 460.29:　　2*ΔEntropy = –63.68


1995
1997
1999
2001
2003
2005
2007
2009
2011
45
40
35
30
'95
'96
'99
'01
'04
'06
'09
45
40
35
30
2000
4000
6000
8000
10000

$\lambda_{ETAS}(t, x, y)$ Conditional intensity function of the ETAS model

$$\phi_{c|n}(t, x, y) = \sum_{k=1}^{n} a_k p_{k|c} \nu(t - t_k) \rho(x - x_k, y - y_k), \qquad n \le \#c, \ t_n \le t, \ \sum_{k=1}^{n} a_k = 1$$

where, in Ogata et al. (GJI,1995);
$\nu(t)$ is normalized density of foreshock survival function of foreshocks in Fig. 5a, and
$\rho(x,y)$ is normalized density of foreshock survival function of foreshocks in Fig. 5b.
Moreover, $p_{k|n}$ is defined in the paragraph including equation (18) of Ogata et al. (GJI,1996),

Manitude frequency for the next event after the n-th earthquake in the cluster *c*

$$GRdensity(m) = \psi^{small}(m \mid M) + \psi^{large}(m \mid M)$$

$$\psi_0^{small}(m \mid M),\ \psi_0^{large}(m \mid M);\ normalized$$

$$M^{(n)} = \max\{M_k, k = 1, \cdots, n\} + 0.45$$

$$\Psi(m \mid M^{(n)}) = p_{n|c}\psi_0^{large}(m \mid M^{(n)}) + (1 - p_{n|c})\psi_0^{small}(m \mid M^{(n)}) \quad \text{if } (t, x, y) \text{ is connected to } c|n$$

$$GRdensity(m) \quad \text{otherwise}$$

If $\psi(t)$ is normalized density of magnitude-differences between foreshocks in Fig. 5c of Ogata et al. (GJI,1995),

$$\psi_k(m) = GRdensity\{m \mid \text{truncated@}\max(M_j, j = 1, \cdots, k) + 0.45\}$$

$$\Psi(m \mid n+1) = GRdensity(m) - \prod_{k=1}^{n}\{(1 - a_k)\psi_k(m)\}$$

Algorithm of foreshock probability calculations in case of plural earthquakes in a cluster

For plural earthquakes in a cluster, time differences t_{ij} (days), epicenter separation r_{ij} (km), magnitude difference g_{ij} are transformed into the unit cube

$$(t_{i,j}, r_{i,j}, g_{i,j}) \to (\tau_{i,j}, \rho_{i,j}, \gamma_{i,j}) \in [0,1]^3$$

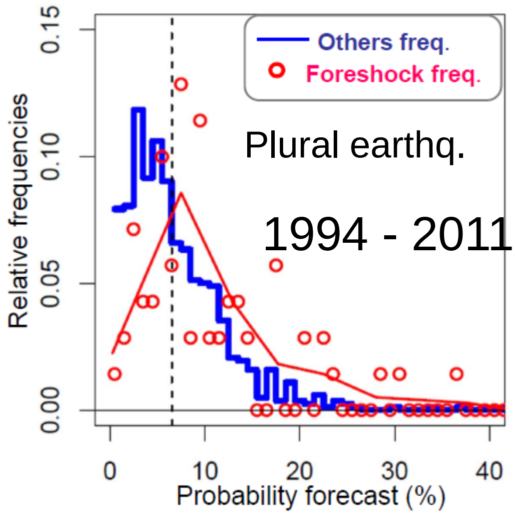
Probability p_c is calculated sequentially

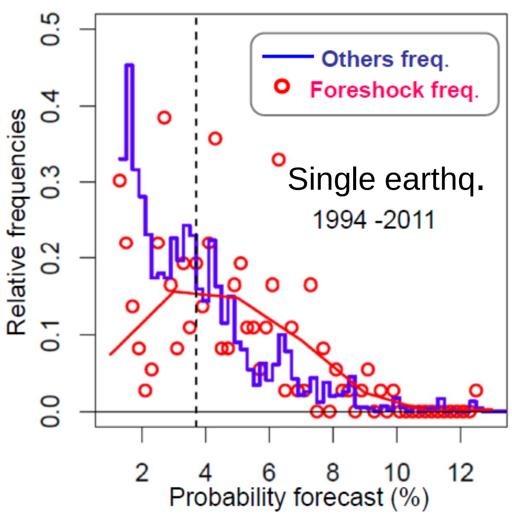
$$\operatorname{logit}(p_c) = \operatorname{logit}\{\mu(x_1, y_1)\} + \frac{1}{\#\{i < j\}} \sum_{i<j} \left[a_1 + \sum_{k=1}^{3} b_k \gamma_{i,j}{}^k + \sum_{k=1}^{3} c_k \rho_{i,j}{}^k + \sum_{k=1}^{3} d_k \tau_{i,j}{}^k \right]$$

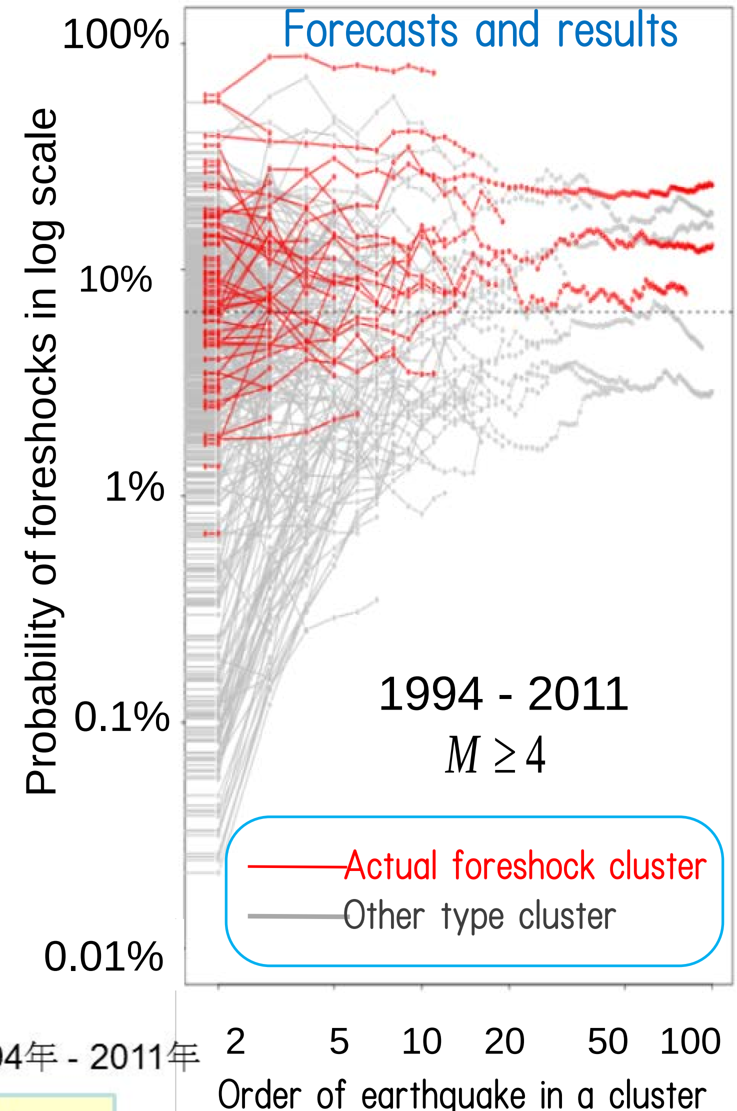
Here $\mu(x, y)$ indicates probability of initial earthquake at location (x,y), and the 2nd term calculates arithmetic mean of polynomials of the normalised space-time magnitude variables for all pairs of earthquakes $(i < j)$ in a cluster, where the coefficients a, b, c, d are as follows.

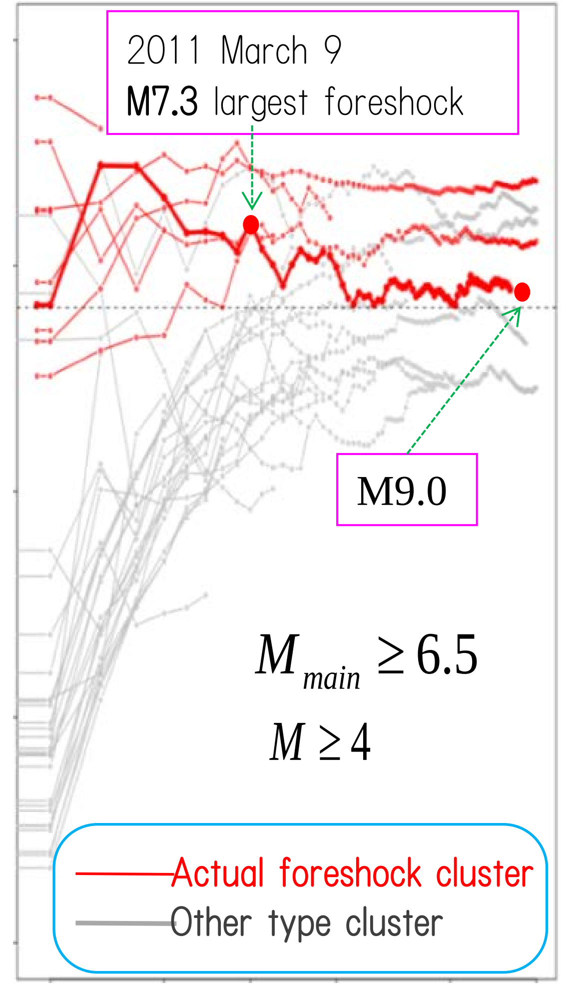
Ogata, Utsu and Katsura, 1996, *GJI*)

k	a_k	b_k	c_k	d_k
1	8.018	−33.25	−1.490	−10.92
2		62.77	2.805	295.09
3		−37.66	−2.190	−1161.5



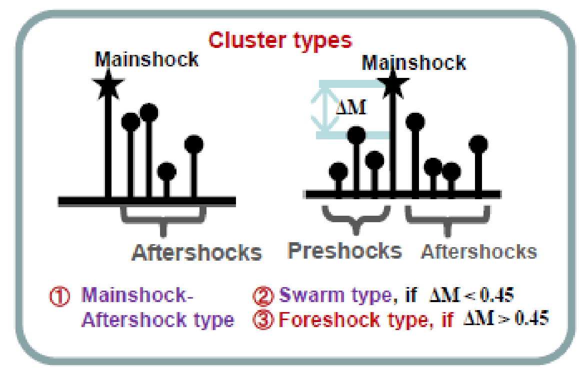




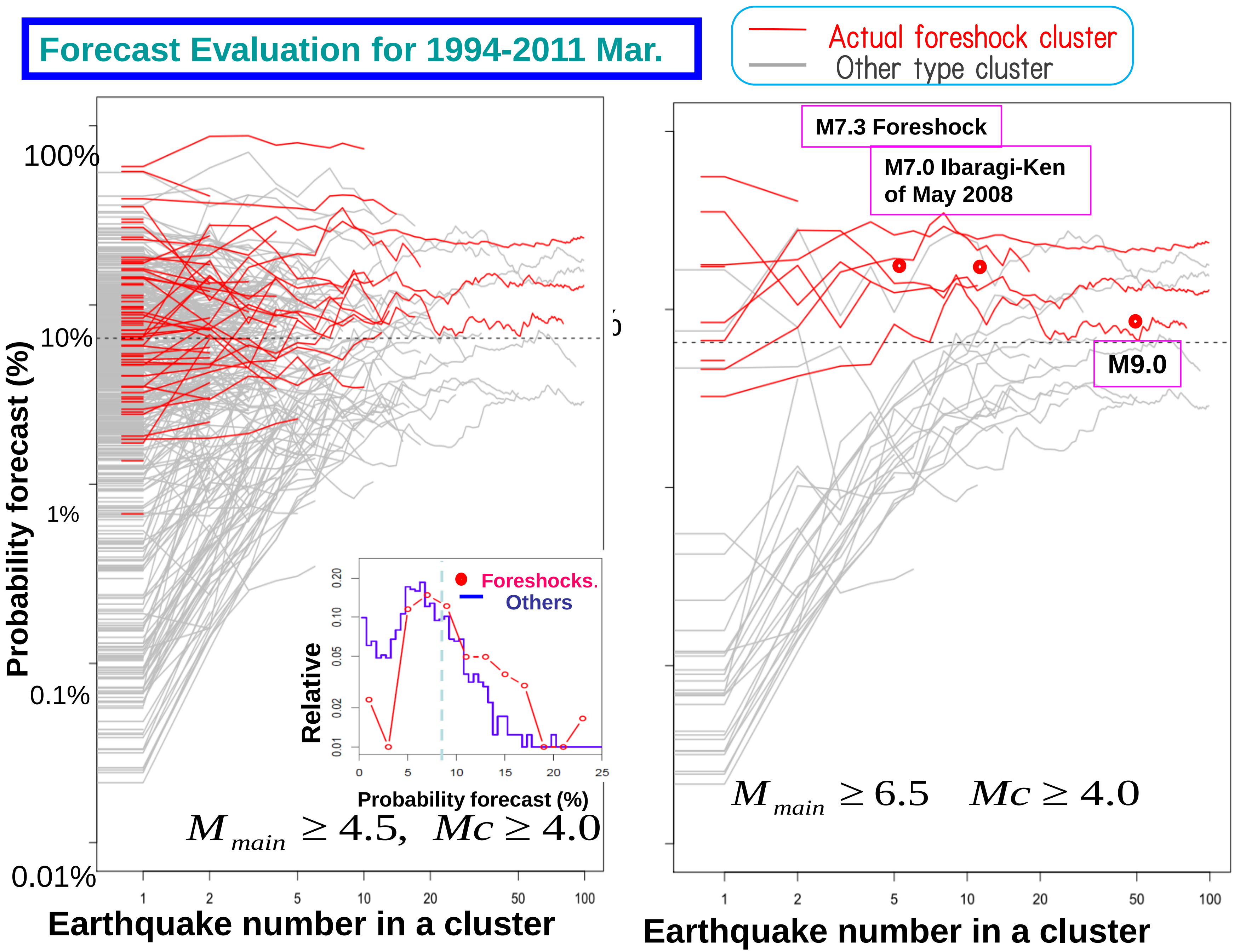


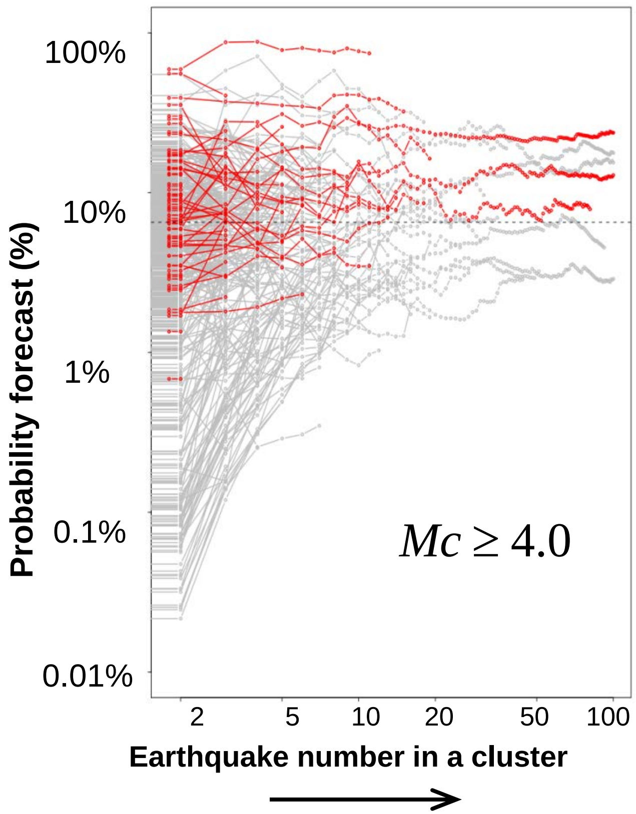
Forecast & performance 1994年 - 2011年

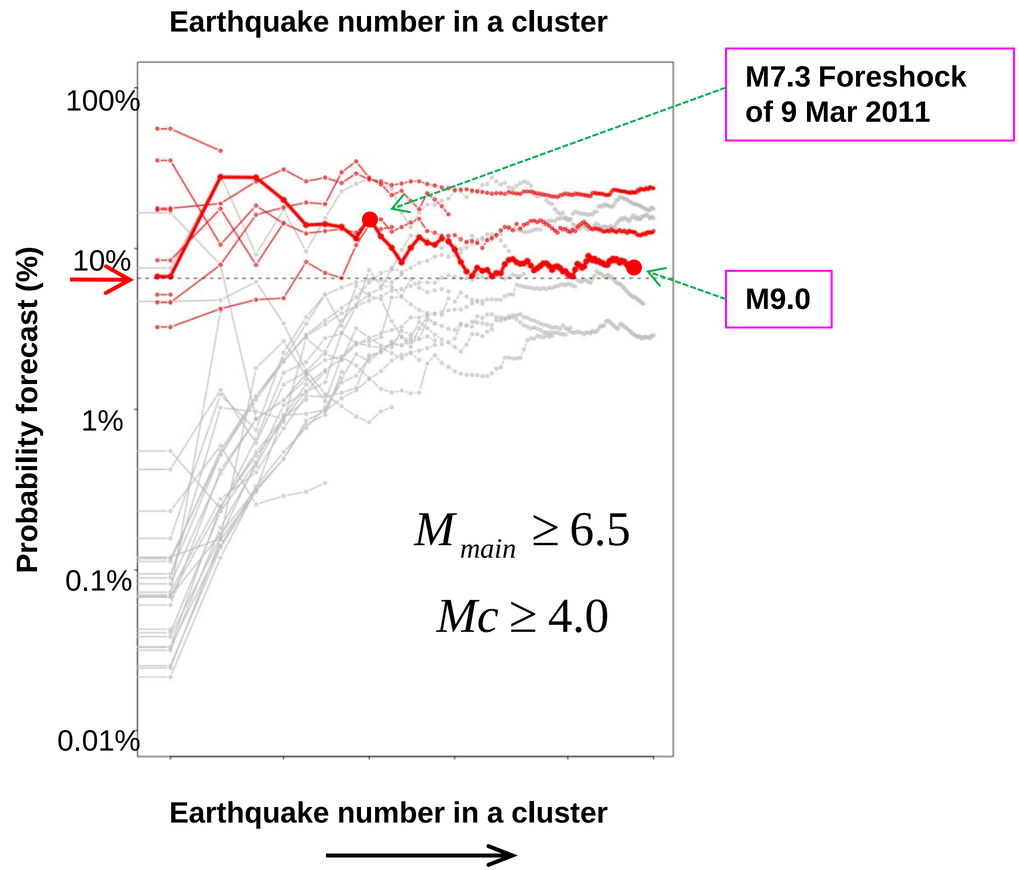
Predicted probability	2.5	5%	10%	15%		
Foreshock	4	10	30	12	14	70
Others	179	211	263	115	51	819
	183	221	293	127	65	889
Relative Frequency	2.2	4.5	10.2	9.4	21.5	7.9

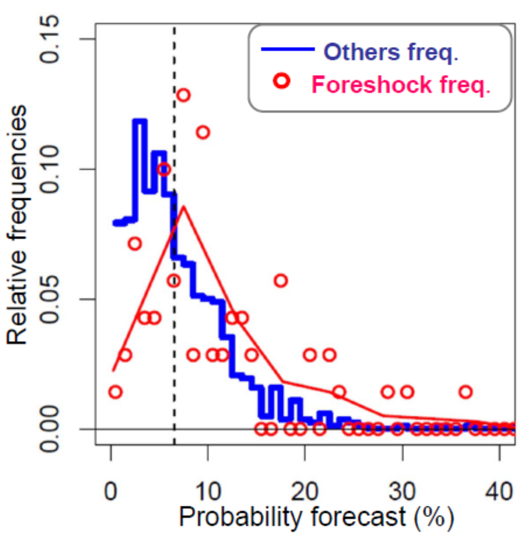


Forecast Evaluation for 1994-2011 Mar.









2*Entropy0 = 523.96 / 2*Entropy = 460.29
2*ΔEntropy = −63.68

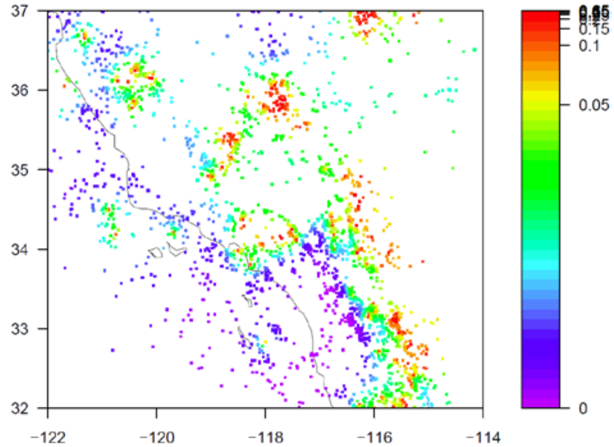
Forecast & performance 1994年 - 2011年3月

2.5	5%	10%	15%		|
4	10	30	12	14	70
179	211	263	115	51	819
183	221	293	127	65	889
2.2	4.5	10.2	9.4	21.5	7.9

aic0 = 3178.62 aic1 = 3157.15
Δaic = −21.47

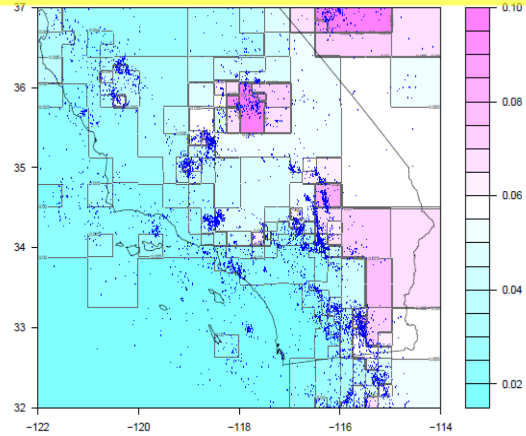
Southern California

1932—2006 M>=3.5



Single-link-clustering

$$d_{ST} = \sqrt{\Delta^2_{space} + (c\Delta_{time})^2} \le 0.3^{\circ}\,(\text{or } 30\text{km})$$



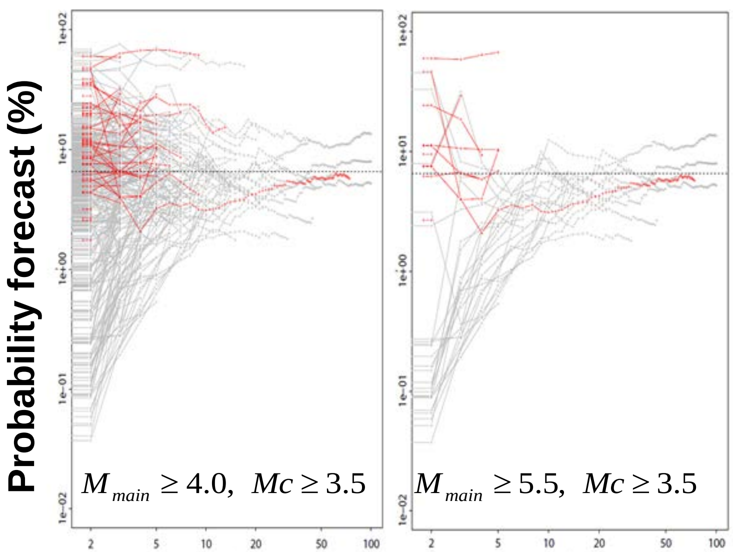
ent0 = 235.0 ent = 236.8

Predicted Foreshock probability

	0-2.5	2.5-5.	5.-	all
Other	678	1296	655	2629
Fore	8	41	66	115
All	686	1337	721	2744
ratio%	1.2	3.1	9.2	4.2

aic0 = 6712.6 aic1 = 6656.8

#	#fore	%	(+/-)	#sw	%	(+/-)	#Maft	#f+#s	All
1	115	4.2	(0.4)	200	7.3	(0.5)	2429	315	2744
2	44	7.8	(1.1)	200	35.3	(2.0)	322	244	566
3	23	8.3	(1.7)	110	39.7	(2.9)	144	133	277
4	16	9.6	(2.3)	67	40.1	(3.8)	84	83	167
5	13	10.8	(2.8)	51	42.5	(4.5)	56	64	120
6	6	6.7	(2.6)	40	44.4	(5.2)	44	46	90
7	5	7.6	(3.3)	28	42.4	(6.1)	33	33	66
8	3	5.9	(3.3)	23	45.1	(7.0)	25	26	51
9	3	6.8	(3.8)	19	43.2	(7.5)	22	22	44
10	2	4.9	(3.4)	17	41.5	(7.7)	22	19	41



Earthquake number in a cluster

Earthquake number in a cluster

2*ent0 = 374.09 2*ent = 325.05

2*Δentropy =−49.0 (Information gain)

Predicted Foreshock probability

0.0	2.5	5.0	10.0	15.0	ALL
1	6	17	7	20	51
96	149	226	65	54	590
97	155	243	72	74	641
1.0	3.9	7.0	9.7	27.0	8.0

aic0 = 2278.19 aic1 = 2247.61

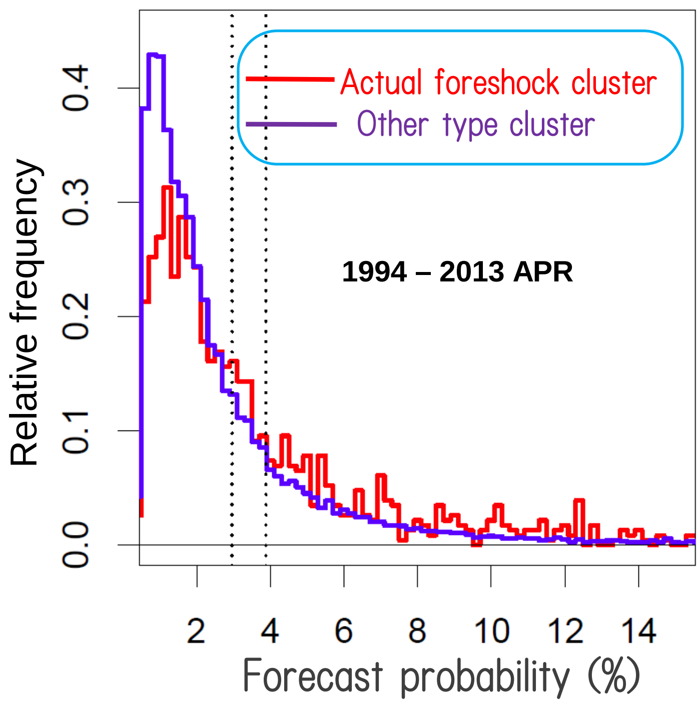
Δaic = −30.58

Global Forecast Result using NEIC-PDE catalog ($M \geqq 4.7$)

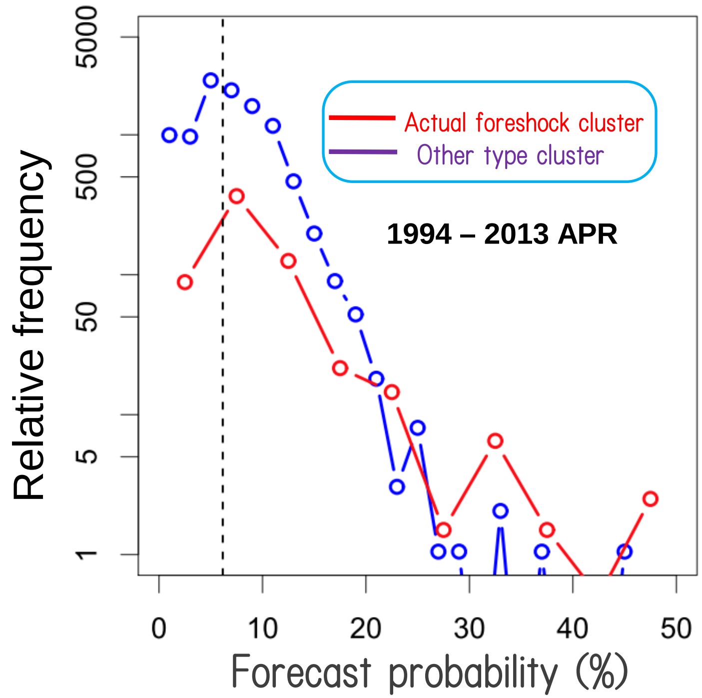
1973 ~ 1993: learning period, calibrating the forecasting parameters in Ogata et al. (1993, GJI)

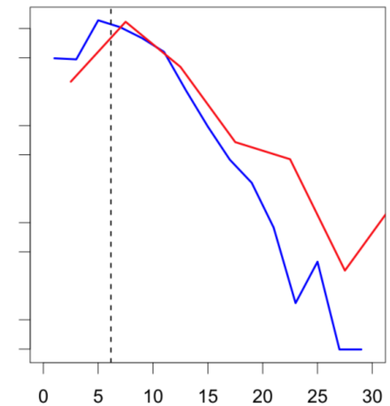
1994 ~ 2013 April: forecasting period

Isolated or 1st quake in a cluster



Plural earthquakes within a cluster





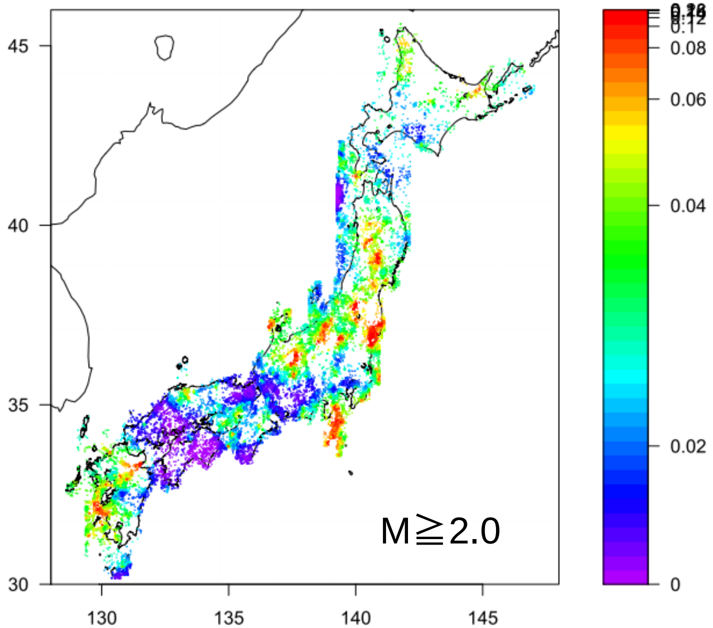
Predicted probability	2.5%	5.0%	+	all
Foreshock	18610	6154	3721	28485
Others	580	304	267	1151
	19190	6458	3988	29636
Frequency ratio	3.0	4.7	6.7	3.9

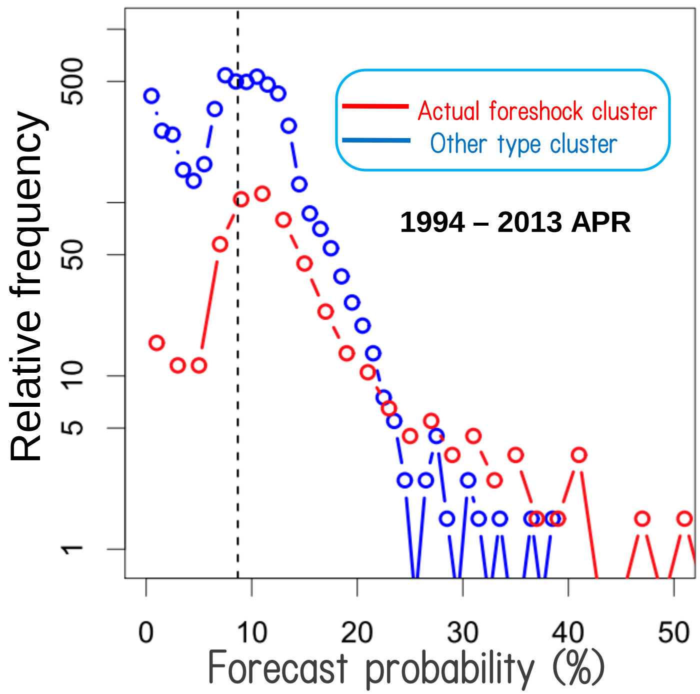
2.5	5.0	10.0	15.0		All
14	73	365	125	45	622
1222	1873	4999	1763	239	10096
1236	1946	5364	1888	284	10718
1.1	3.8	6.8	6.6	15.8	5.8

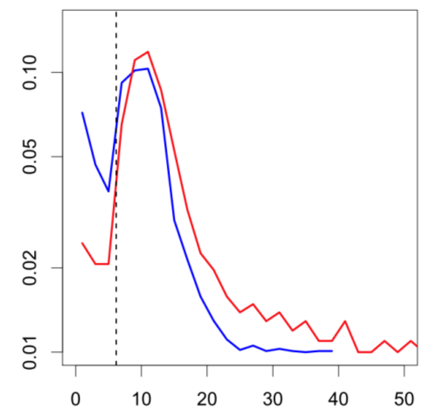
2*ΔLL = –121.1

Δaic = –129.6

1998 - 2010







	2.5	5.0	10.0	15.0	All
16	14	168	214	104	516
788	416	2043	1845	333	5425
804	430	2211	2059	437	5941
2.0	3.3	7.6	10.4	23.8	8.7

2*ΔLL = −195.4

Δaic = -176.70

Global Forecasting using NEIC-PDE catalog ($M \geqq 4.7$)

Single-link-clustering by connecting the space-time distance $d_{ST} = \sqrt{\Delta^2_{space} + (c\Delta_{time})^2} \leq 0.45^\circ$ (or 50km)

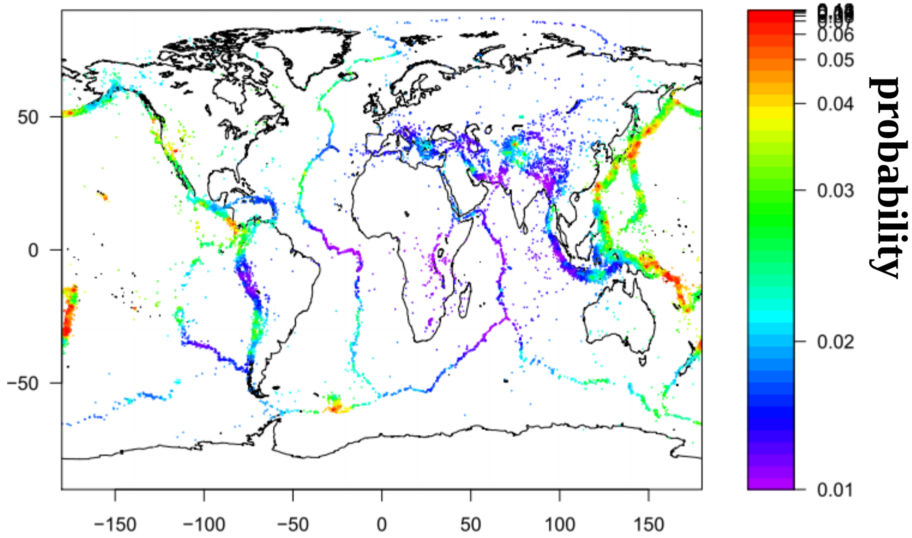
1973 ~ 1993: learning period, calibrating the forecasting parameters in Ogata et al. (1993, GJI)

1994 ~ 2013 April: forecasting period

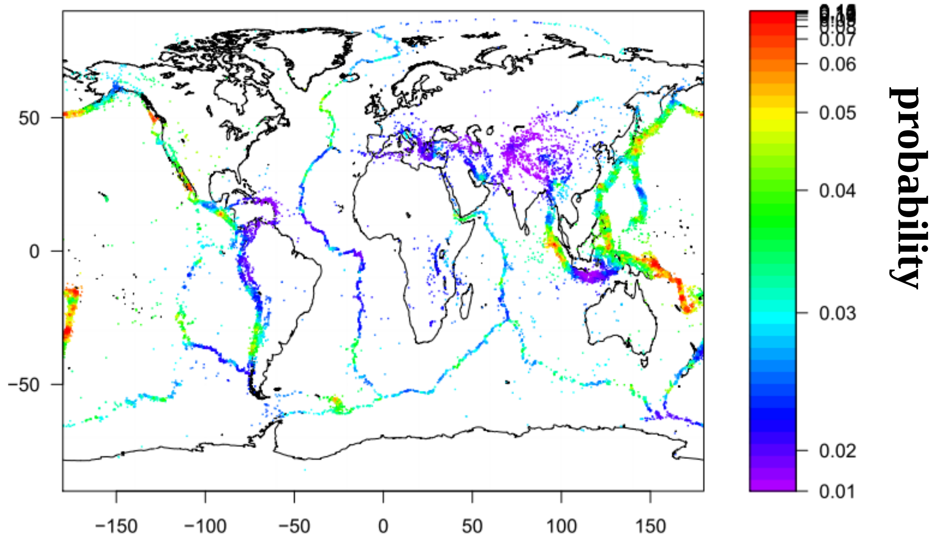
Foreshock probability for isolated or the 1st quake estimated from the NEIC data from 1973 – 1993

Given location of a future earthquake, probability is calculated by the interpolation using the including Delaunay triangle.

1973 – 1993



1994 – 2013 April

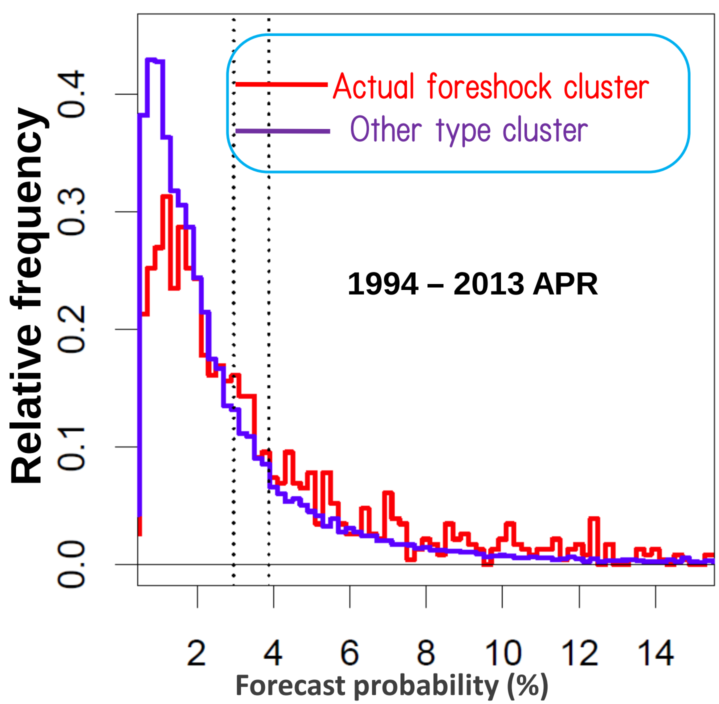


Global Forecast Result using NEIC-PDE catalog ($M \geqq 4.7$)

1973 ~ 1993: learning period, calibrating the forecasting parameters in Ogata et al. (1993, GJI)

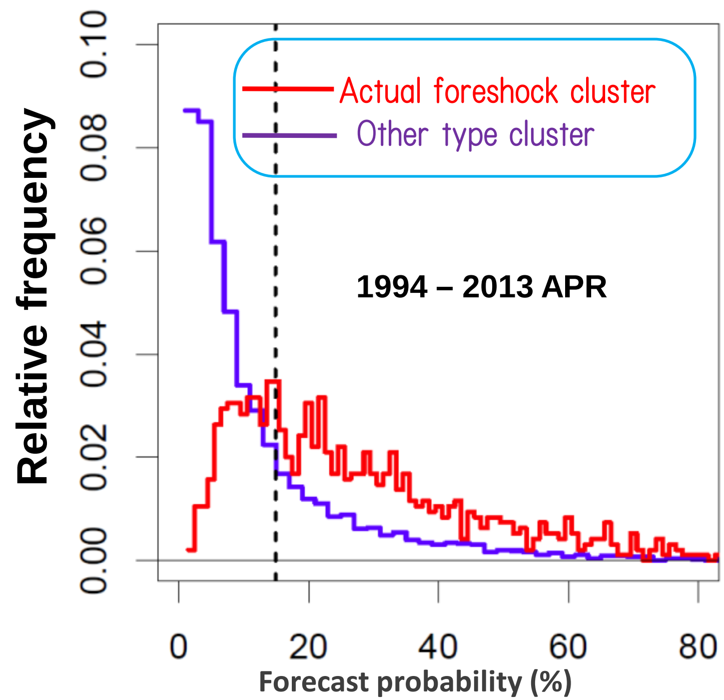
1994 ~ 2013 April: forecasting period

Isolated or 1st quake in a cluster



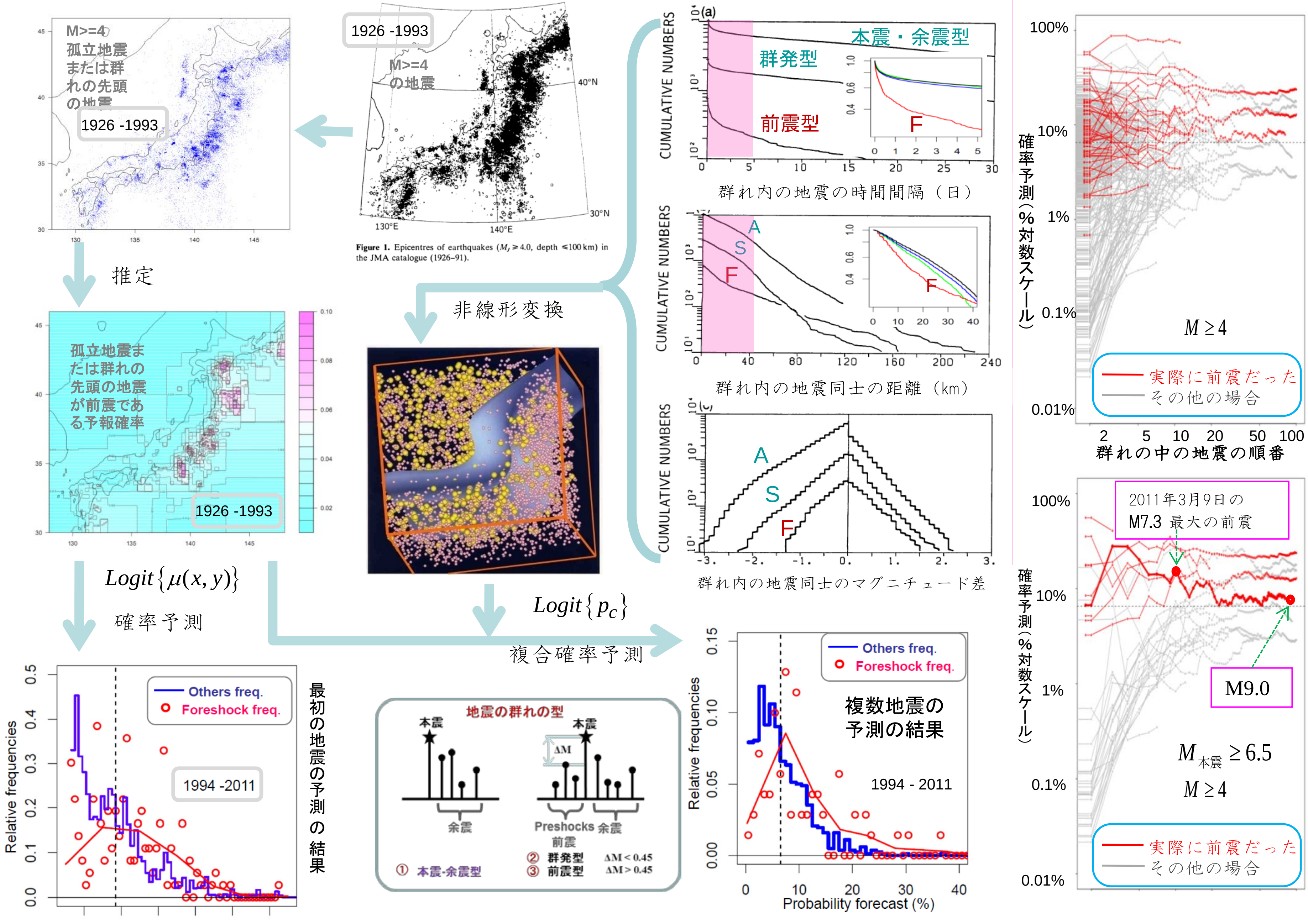
Predicted probability	2.5%	5.0%	+	all
Foreshock	18610	6154	3721	28485
Others	580	304	267	1151
	19190	6458	3988	29636
Frequency ratio	3.0	4.7	6.7	3.9

Plural earthquakes within a cluster



Predicted probability	5%	10%	20%	30%	+	all
Foreshock	32	115	207	156	440	950
Others	1684	1237	1246	552	707	5426
	1716	1352	453	708	1147	6376
Frequency ratio	1.9	8.5	14.2	22.0	38.4	14.9

前震の確率予報



Ogata, Y. and K. Katsura (2012) Prospective foreshock forecast experiment during the last 17 years, *Geophys. J. Int.* (in press)

Summary and suggestions

It is conceivable that the *b* value of the G-R rule depends on the earthquake location when the earthquakes are small.

When the earthquakes are small, such location-dependent b-value model performs a slightly better forecast performance than the reference model of $b = 0.9$ through out entire regions.

But, there are many outlyingly negative information gain score which causes total predictive performance worse; this is clearly seen inland Japan experiments.

We need to pursue the physics of aftershocks and elaborate the magnitude frequency models.

FORMLATION OF THE ISSUES

Prediction models are based on the ***conditional intensity function*** of point process,

$$\lambda(t,x,y,M \mid H_t,F_t) \approx \frac{P\left\{an\ event\ in\ a\ bin\ [t,t+\Delta t]\times[x,x+\Delta x]\times[y,y+\Delta y]\times[M,M+\Delta M] \mid H_t,\ F_t\right\}}{\Delta t\,\Delta x\,\Delta y\,\Delta M},$$

for calculating probability of an earthquake occurring at a ***time t***, a ***location (x, y)***, and a ***magnitude M***, that conditional on ***history of occurrence records*** $H_t = \left\{ (t_j, x_j, y_j, M_j);\ t_j < t \right\}$ and can further depend on relevant information F_t as ***exogenous records***.

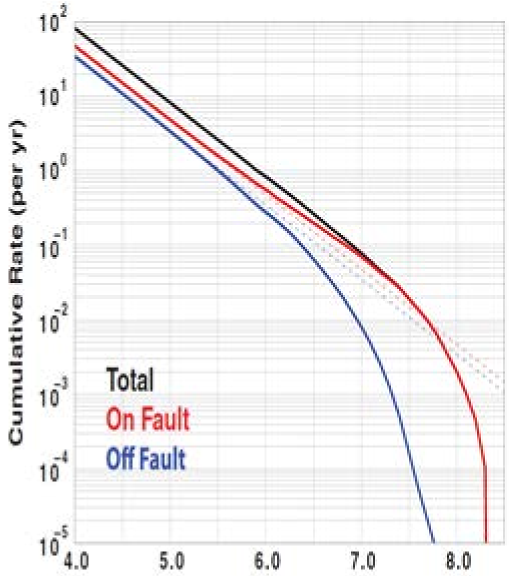
Then we assume the ***separablity*** between space-time and magnitude components.

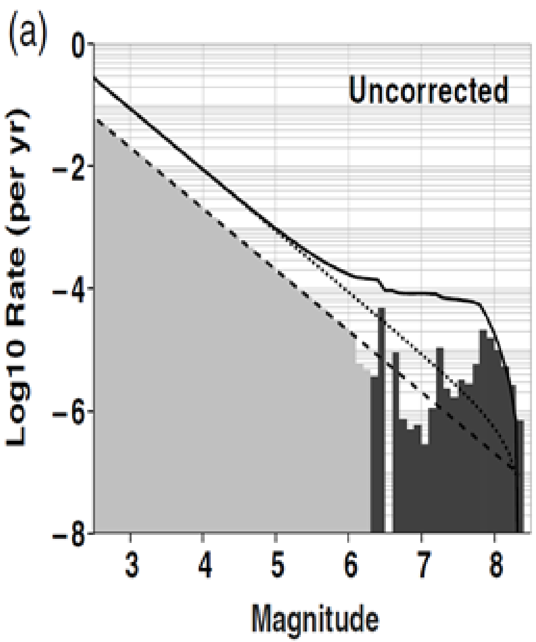
$$\lambda(t,\ x,\ y, M \mid H_t,\ F_t) \approx \lambda(t,\ x,\ y \mid H_t,\ F_t)\ \gamma(M \mid t,\ x,\ y, H_t, F_t)$$

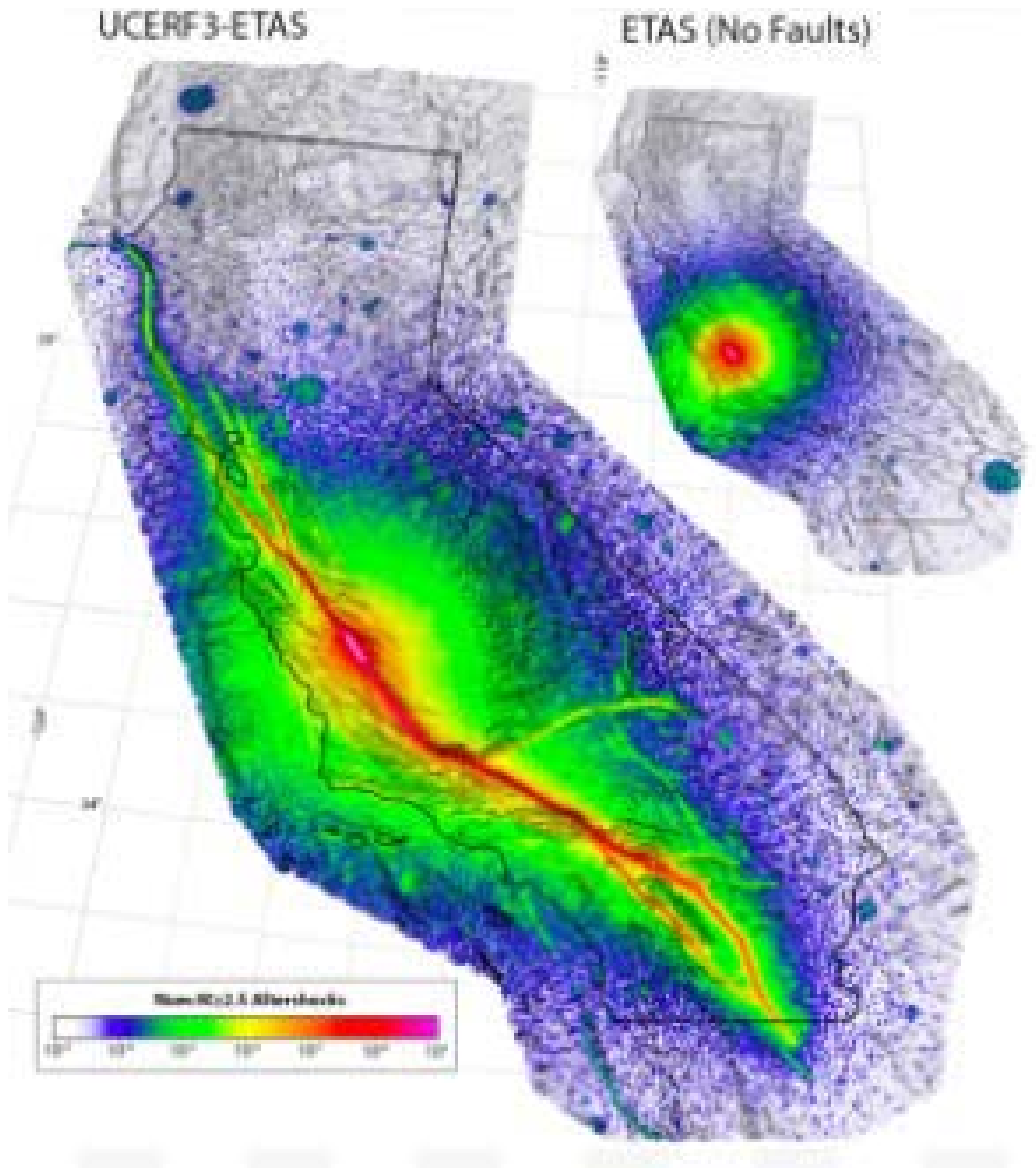
where $\gamma(M \mid t,x,y,H_t,F_t)\,dM = P\left(M < Magnitude \le M + dM \mid t,x,y,H_t,F_t\right)$

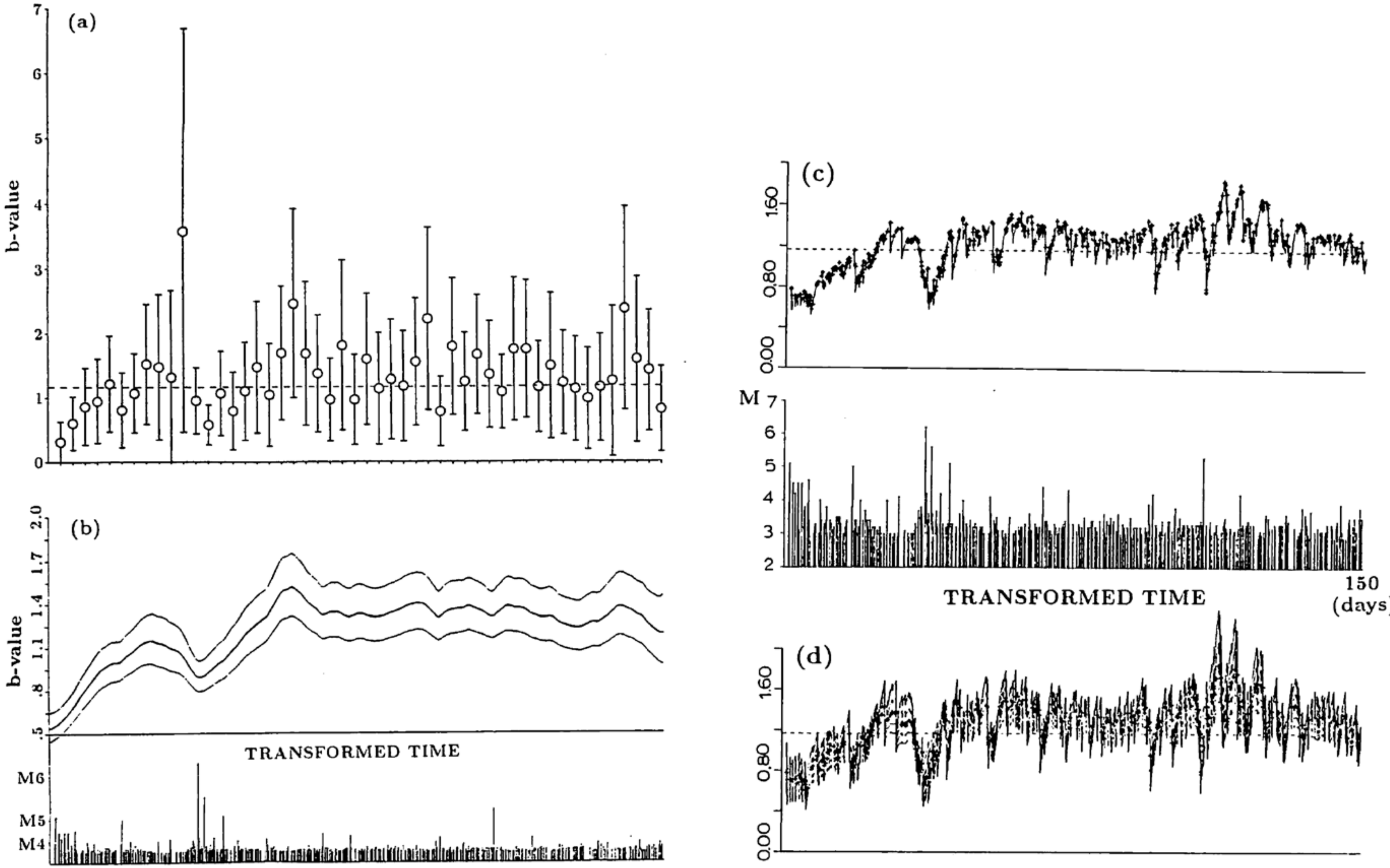
Our task is to model $\gamma(M \mid t,x,y,H_t,F_t)$ ***and evaluate the probability and information gains relative to the reference model***, $\gamma_0(M \mid t,x,y,H_t,F_t) = 10^{a-b(M-M_c)}$

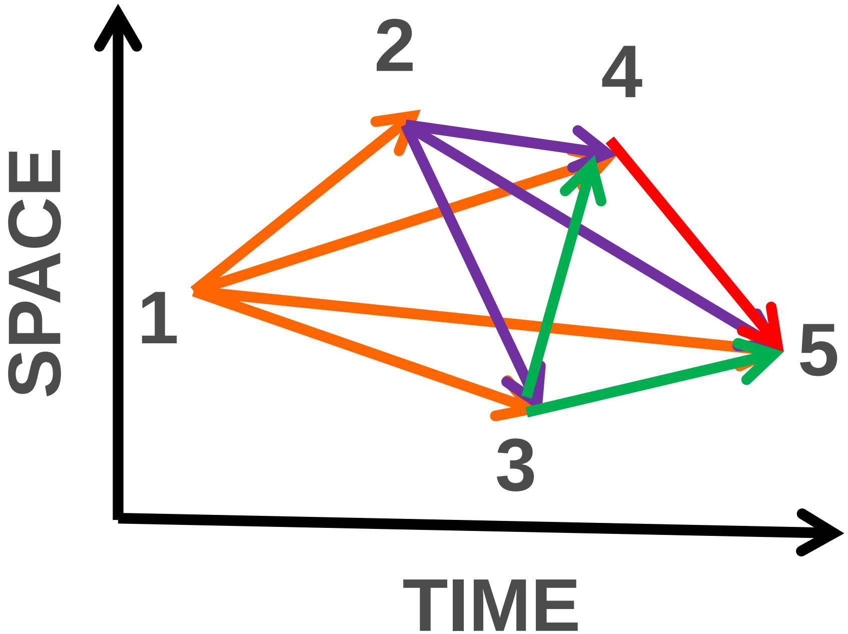
Field et al. (2017, *BSSA*)








(a)
b-value
0
1
2
3
4
5
6
7
(b)
b-value
.5
.8
1.1
1.4
1.7
2.0
TRANSFORMED TIME
M6
M5
M4
(c)
0.00
0.80
1.60
M
2
3
4
5
6
7
150
(days)
TRANSFORMED TIME
(d)
0.00
0.80
1.60


SPACE
TIME
1
2
3
4
5

Magnitude Gap : $M^{(n|c)} = \max\{M_k;\ k=1,\cdots,n \,|\, \text{in cluster } c\} + 0.5$

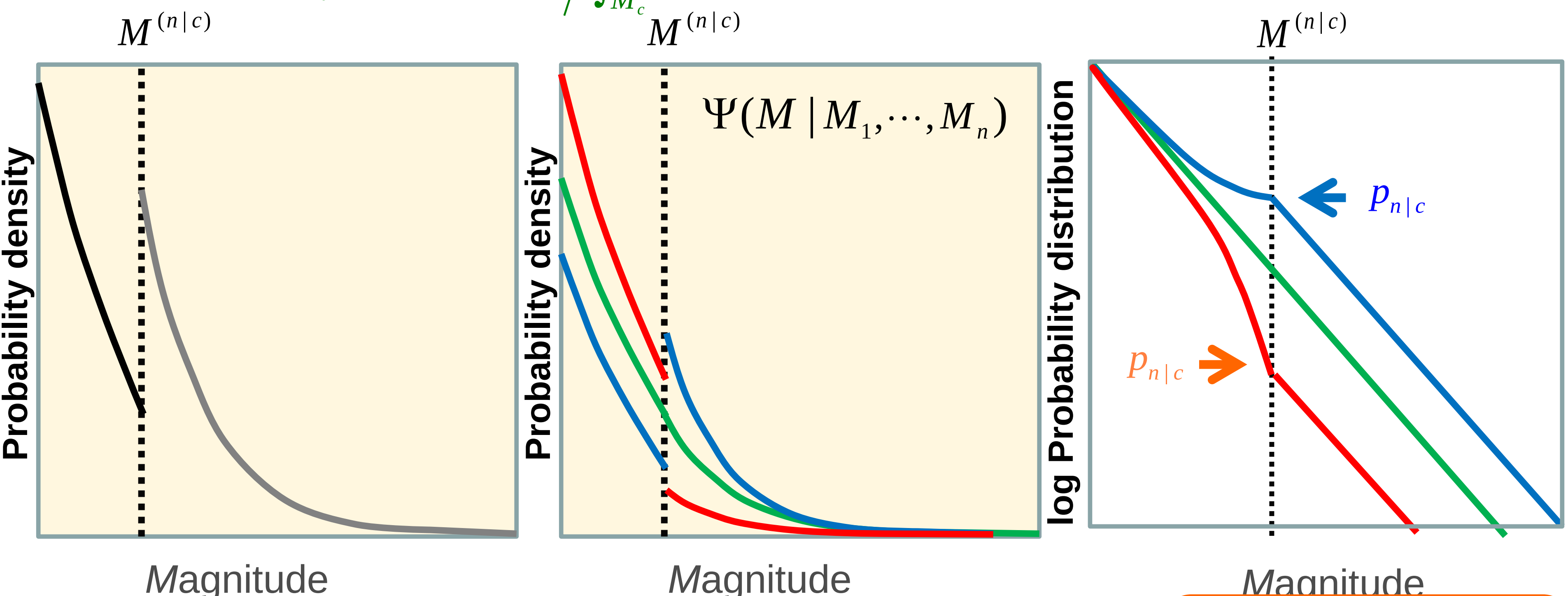
Probability of M≧Mmax+0.5 of the next magnitude; $p_{n|c} = P\{M_{n+1} > M^{(n|c)} \,|\, in\ c\}$

If $(t_{n+1}, x_{n+1}, y_{n+1})$ is connected to c,

$$\Psi(M \,|\, M_1,\cdots,M_n) = (1-p_{n|c}) \frac{1_{(M_c,\,M^{(n|c)})}(M)\cdot 10^{-bM}}{\int_{M_c}^{M^{(n|c)}} 10^{-bM}\,dM} + p_{n|c}(M) \frac{1_{(M^{(n|c)},\,\infty)}\cdot 10^{-bM}}{\int_{M^{(n|c)}}^{\infty} 10^{-bM}\,dM}$$

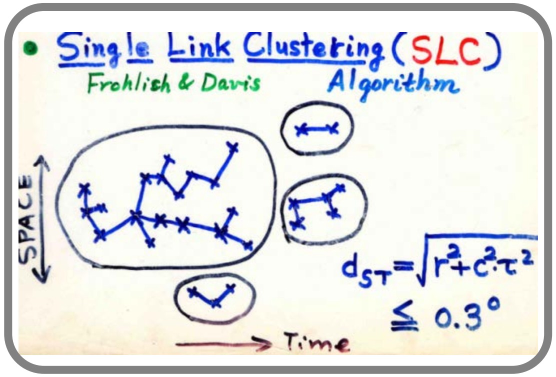
Otherwise

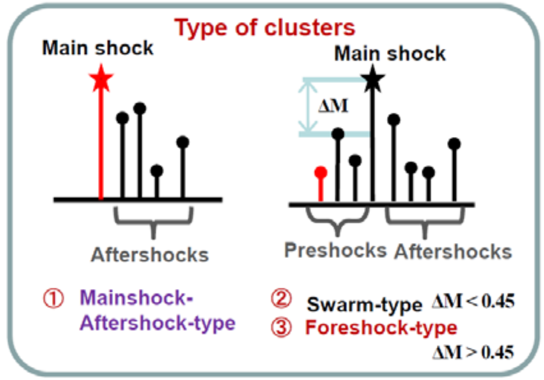
$$\Psi(M) = 1_{(M_c,\infty)}(M)\cdot 10^{-bM} \Big/ \int_{M_c}^{\infty} 10^{-bM}\,dM$$

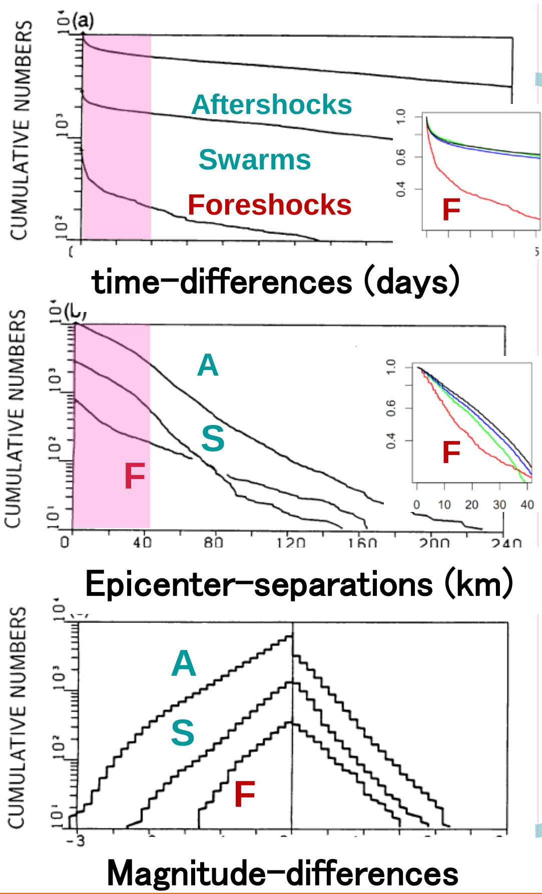


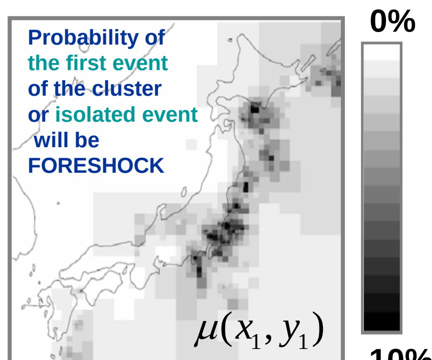
log likelihood-ratio = information gain: $\log L/L_0 = \sum_c \sum_{n=1}^{\#c} \log \frac{\Psi_c(M_{n+1} \,|\, M_c^{(n)})}{\Psi_c(M_{n+1})}$

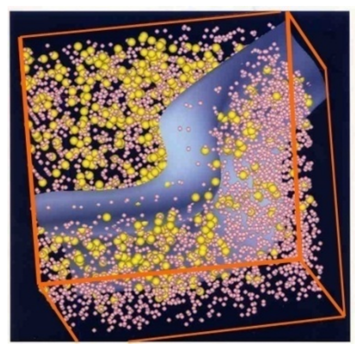
Ogata et al. (1995, 1996; *GJI*)
Ogata & Katsura (2012, *GJI* ; 2014, *JGR*)





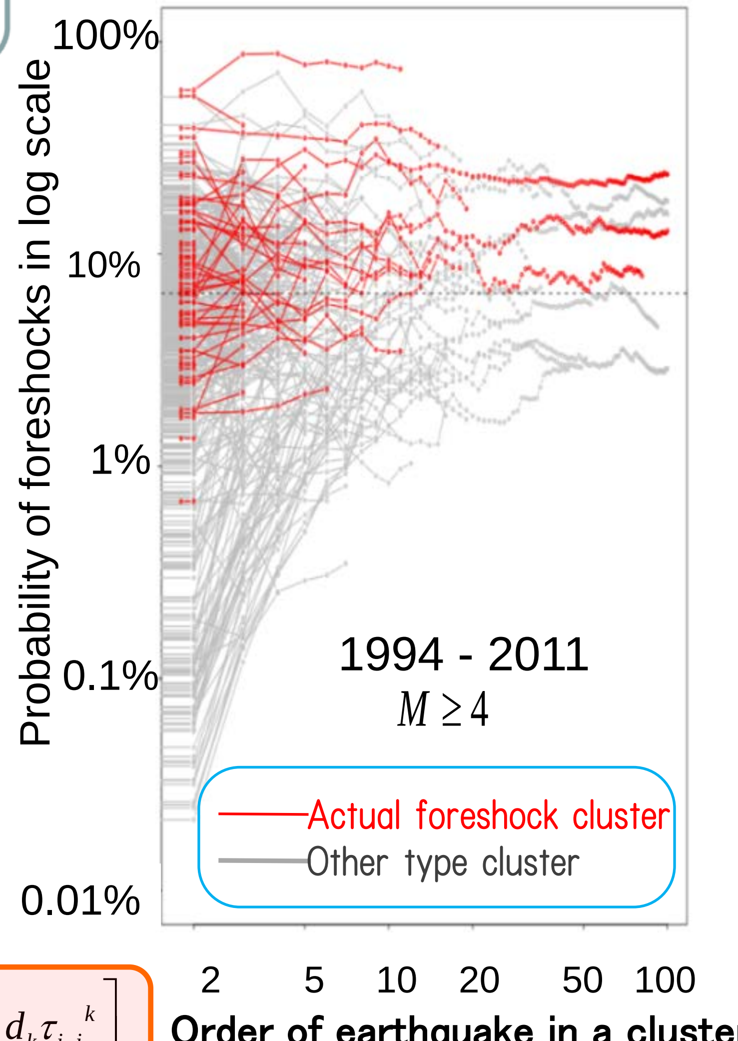






Transformed to unit cube
$(\tau_{i,j},\ \rho_{i,j},\ \gamma_{i,j})$

Forecasts and results



$$\ln\frac{1-p_c}{p_c} = \ln\frac{1-\mu(x_1,y_1)}{\mu(x_1,y_1)} + \frac{1}{\#\{i<j\}}\sum_{i<j}\left[a_1 + \sum_{k=1}^{3} b_k \gamma_{i,j}{}^k + \sum_{k=1}^{3} c_k \rho_{i,j}{}^k + \sum_{k=1}^{3} d_k \tau_{i,j}{}^k\right]$$