I. Model Assumptions and Basic Model Properties

(a) The magnitude of microseismic events in a sequence are distributed identically and independently identically distributed (i.i.d.) with the magnitude $M_i$ in the sequence with common distribution

$$P(M_i = m) = P(M = m) = \frac{1}{\Gamma(\alpha)c^\alpha m^{\alpha-1}} e^{-\frac{m}{c}}, \quad m = 0, 1, 2, ...$$

(Note: $\alpha > 0$ is a scaling exponent, commonly but not necessarily related to catalogue completeness. For mathematical convenience, in the rest of the paper we use logarithms to base $e$ rather than to base 10, hence $\beta$ rather than $\beta_{10}$.)

(b) The original aftershock structure is determined by the magnitude of the initiating event. In particular, the expected number $E(N(M)) = m \mu(M)$ of events of magnitude $M$ is given by $P(M = m) = \frac{1}{\Gamma(\alpha)c^\alpha m^{\alpha-1}} e^{-\frac{m}{c}}$.

(c) To catalogue completeness. For mathematical convenience, in the rest of the paper we use logarithms to base $e$ rather than to base 10, hence $\beta$ rather than $\beta_{10}$.)

(d) The expected number of events larger than the initiating event is given by

$$E(N(M)) = m \mu(M) = \frac{m}{\Gamma(\alpha)c^\alpha m^{\alpha-1}} e^{-\frac{m}{c}}$$

(e) $\alpha > 0$ is a scaling exponent, commonly but not necessarily related to catalogue completeness. For mathematical convenience, in the rest of the paper we use logarithms to base $e$ rather than to base 10, hence $\beta$ rather than $\beta_{10}$.)

(f) The left side can be interpreted as the probability that the initiating event is a foreshock. It is $\beta = 0$, in other words the fraction of the magnitude of the initiating event, $m_i$ + $m_a$ = $M$ is A. Results from the California data are shown in Figure 1(c) and compared with the proportion 0.6.

**Figure 1:** The population of all $M_i$'s, with $M_a = 3$, $M_b = 4$, and $M_c = 5$, are plotted in Figure 1(c) and compared with the proportion 0.6.

**Figure 2:** The frequency distribution of foreshock magnitude $m$ for 12 aftershock events

**Figure 3:** The frequency distribution of foreshock magnitude $m$ for 12 aftershock events.

**Figure 4:** A further note on Bäth's law

**Figure 5:** A further note on Bäth's law.

II. Expected value of the magnitude of the largest aftershock

(a) Let $M_{max}$ denote the magnitude of the largest event in the sequence (including the initiating event). The given, $M_i$ and the distribution $M_{max}$ is given by

$$P(M_{max} = m) = P(M = m) = \frac{1}{\Gamma(\alpha)c^\alpha m^{\alpha-1}} e^{-\frac{m}{c}}, \quad m = 0, 1, 2, ...$$

(b) This holds true if $\alpha = 1$, for which $N(M)$ is Poisson, and for $\alpha = 0$, for which $N(M)$ is exponential.

(c) The expected value of the magnitude of the largest event is given by

$$E(M_{max}) = \frac{1}{\beta}$$

(d) The expression for $E(M_{max})$ in terms of the parameters $\alpha$ and $c$ is

$$E(M_{max}) = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)c^\alpha}$$

(e) The expected value of the magnitude of the largest event is given by

$$E(M_{max}) = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)c^\alpha}$$

III. Distribution of the magnitude of the largest aftershock

(a) Let $G_\beta(A; X) = E(X)$ be the probability generating function of the random variable $X$ = $M_{max}$ of the largest event of magnitude $M$.

$$G_\beta(A; X) = E(X) = \frac{1}{\beta}$$

(b) Hence, if $\Delta = M_{max} - M_i$,

$$G_\beta(A; \Delta) = E(\Delta) = \frac{1}{\beta}$$

(c) From the above expression, we see that the probability generating function of $\Delta$ is

$$G_\beta(A; \Delta) = E(\Delta) = \frac{1}{\beta}$$

(d) The expected value of the magnitude of the largest event is given by

$$E(\Delta) = \frac{1}{\beta}$$

(e) The expected value of the magnitude of the largest event is given by

$$E(\Delta) = \frac{1}{\beta}$$

IV. Comparison with the California Data

Although the model is not on in our minds to compare empirical with theoretical data, we show in Figure 1(c) the empirical data and theoretical distribution $\beta_{10}$ is given by

$$\beta_{10} = \log_{10}\left(\frac{\alpha c}{\Gamma(\alpha)}\right)$$

We define the magnitude of the largest event as the magnitude of the largest earthquake in the sequence.

**Figure 3:** Three examples of the clipping fitting. Each magn is based on the sequence of the initiating event. The fitting of the model corresponds to the maximum likelihood estimate of model parameters in the context of maximum likelihood.

V. Discussion

Despite the simplicity of the model, there are good qualitative and even quantitative agreements between the predictions of the model and the empirical data. The most obvious agreement is in the observation of the magnitude of the largest event.

**Figure 4:** A further note on Bäth's law.

**Figure 5:** A further note on Bäth's law.

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