Toward urgent forecasting of aftershock hazard:
Simultaneous estimation of $b$-value of the Gutenberg-Richter’s law of the magnitude frequency and changing detection rates of aftershocks immediately after the mainshock

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Abstract: It is known that detection rate of aftershocks is extremely low during a period immediately after a large earthquake due to the contamination of arriving seismic waves. This has been made a considerable difficulty to attain the estimate of the empirical laws of aftershock decay and magnitude frequency immediately after the main shock. This paper presents an estimation method for predicting underlying occurrence rate of aftershocks of any magnitude range, based on the magnitude frequency model that combines Gutenberg-Richter’s law with the detection rate function. This procedure enables real-time probability forecasting of aftershocks immediately after the mainshock, when a majority of large aftershocks are likely to occur.

1. Introduction

The aftershock probability forecasting [Reasenberg and Jones, 1989] was proposed based on the joint intensity rate [Utsu, 1970]

$$\lambda(t,M) = m(M) n(t) = 10^{a+b(M_0-M)} / (t + c)^p \quad (a, b, c, p; \text{constant})$$

of aftershocks with magnitude $M$, at the time $t$ following a mainshock of magnitude $M_0$, where the parameters are independently estimated by the maximum likelihood method [Utsu, 1965; Aki, 1965; Ogata, 1983] for respective empirical law of the Omori-Utsu and the Gutenberg-Richter. Then, the probability of expecting one or more aftershocks is calculated in the interval of magnitude range and time range. The public forecast has been implemented by the responsible agencies in California and Japan. However, a considerable difficulty in the above procedure is that, due to the contamination of arriving seismic waves, detection rate of aftershocks is extremely low during a period immediately after the main shock, especially, during the first day, when the forecasting is most critical for public in the affected area. To avoid such a difficulty, they adopt a generic model with a set of the standard parameter values in California or Japan for the forecast of a probability during such a period.

In fact, the parameters, $a, b, c$ and $p$ are estimated by using completely detected aftershocks above a threshold magnitude where the aftershocks are almost all detected during the estimation period. However, detected earthquakes below the threshold magnitude do not take a minor portion in all detected earthquakes. Especially, such a threshold magnitude for the complete detection is very high during the early period after the mainshock. From a viewpoint of effective use of data, it is quite wasteful for the statistical analysis of seismic activity not to
utilize all detected aftershocks. This is particularly the case where we have to use data set which is inhomogeneous in the sense that the detection-rate at each magnitude varies depending on time or location. In particular, it is seen that the $a$-value in equation (1), accordingly $10^a$ called as aftershock productivity, takes values of considerably wide range, depending on aftershock sequence[Reasenberg and Jones, 1989], and therefore it is crucial to estimate the $a$-value suitably in the early stage immediately after the mainshock.

For an effective and practical estimation, Ogata and Katsura [1993] propose to utilize the statistical model formulated, taking the detection rate of earthquakes into account for simultaneous estimation of the $b$-value in equation (1) together with detection-rate (probability) of earthquakes of each magnitude-band from the data of all detected events. The detection-rate of earthquakes clearly depends on their magnitudes in such a way that large shocks are almost all detected but smaller ones are lower rate in detection. Some authors [see Ogata and Katsura, 1993, for the reference] studied the detection capability of a seismic station or network for earthquakes’ signal amplitudes that are observed over the seismic noise level. Consider a possible detection-rate function that is the cumulative function of the Normal distribution

$$q(M \mid \mu, \sigma) = \int_{-\infty}^{M} \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx \quad (\mu, \sigma, \text{constant}).$$

The parameter $\mu$ represents the magnitude where earthquakes are detected in the rate of 50%, and $\sigma$ represents a range of magnitudes where earthquakes are more or less partially detected. The shape of the product function $m(M)q(M \mid \mu, \sigma)$ agrees very well with histograms of magnitude frequency for homogeneously detected earthquakes in a seismic area and a period from the Hypocenter Catalog of Japan Meteorological Agency as well as Harvard catalog of
global seismicity. Figure 1 illustrates the schematic diagrams of the present model. Therefore, if we fit the magnitude data of all detected earthquakes to the normalized version of $m(M)$ $q(M | \mu, \sigma)$ as the probability density by the maximum likelihood procedure, we can simultaneously estimate the parameters, $b$, $\mu$, and $\sigma$, where 'ln' represents the natural logarithm. This function of magnitude with the coefficients of the maximum likelihood estimates ($\hat{b}, \hat{\mu}, \hat{\sigma}$) shown in Figure 1 demonstrates very well fit to the empirical frequency of all the detected aftershocks of the 2003 Miyagi-Ken-Oki earthquake of M7.0, northern Japan, during the period of 350 days from 20th day since the main shock, where homogeneous detection of events of every magnitude band is expected.

Moreover, these parameters are extended to be a function of time, $b(t)$, $\mu(t)$, and $\sigma(t)$, to estimate these assuming a prior distribution for the constraint of a smooth changes. Given the estimation of the changing parameters, $\hat{b}(t)$, $\hat{\mu}(t)$ and $\hat{\sigma}(t)$, we have the following intensity rate function

$$\lambda(t, M) = 10^{a + \hat{b}(t)(M - M_0)}(t + c)^{-p}q[M | \hat{\mu}(t), \hat{\sigma}(t)]$$

Here, as mentioned above, estimation of the productivity coefficient, $a$, is crucial in the early stage of the aftershock activity. This intensity rate leads to the occurrence rate of the all detected aftershocks. Here, for a real-time probability forecasting of future period, we may well assume that $\hat{b}(t)$ and $\hat{\sigma}(t)$ are constants during the period for estimation from the mainshock to extrapolate it for the forecast. As regards the $\mu(t)$ we consider to use a certain parametric form. Then the equation (3) is used to get the maximum likelihood estimates of the parameters, $c$, $p$, and $K=10^{a+\hat{b}M_0}/\hat{b}/\ln10$ of the intensity in (1) for the underlying aftershock activity including undetected aftershocks, by fitting to all the detected aftershocks from the beginning of the period.

Eventually, we obtain the estimates $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{p}$ to evaluate the joint intensity (1) of the underlying aftershocks of any magnitude. Table 1 and Figure 2 show each set of updated estimates using the data of detected aftershocks during all observed magnitude range of the early periods of the aftershock activity of the 2005 Miyagi-Ken-Oki earthquake. Figure 3 shows estimated decay curve of underlying aftershocks of $M>0$, due to (1) with the coefficients $\hat{K}=10^{a+\hat{b}M_0}/\hat{b}/\ln10$, $c$ and $p$ estimated by applying (3) to the events during respective period from the mainshock up until the updated times. From the figure we see that the rate of the underlying aftershocks predict well up to now by the observations of the detected aftershocks up until 1/4 day (6hours) from the mainshock.
On the other hand, the expected magnitude frequency of the detected aftershocks in a time interval \([S, T]\) is given from (7) by

\[
A_{S,T}(M) = \int_{S}^{T} \lambda(t, M) dt \\approx \int_{S}^{T} \frac{\hat{K}}{(s + \hat{c})^p} ds \cdot f(M | \hat{b}, \hat{\mu}(t), \hat{\sigma}), \quad S \leq t \leq T, \tag{4}
\]

where \(f(M | b, \mu, \sigma)\) is the probability density normalized from \(m(M) q(M | \mu, \sigma)\). From this we can predict the magnitude frequency of aftershocks during some period ahead the updated time of the observation. Figure 4 shows such predicted \(\Lambda(M)\) curves predicted at \(t=S\) and \(t=T\) for each panel of the figure, and actual frequency in the considered period \([S,T]\). This figure demonstrates the feasibility and practical utility of such forecasts in the early stage for case of aftershock activity of the 2005 Miyagi-Ken-Oki earthquake.

References


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