Contributions of Professor Tokuji Utsu to Statistical Seismology and Recent Developments

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Abstract and Introduction: *Late Professor Tokuji Utsu made so many contributions to seismology especially in the research of seismicity. It is out of my ability to introduce many of his important contributions; his extensive works have been most frequently cited in seismological community in Japan. I can only talk about works that are related to statistical seismology and that have been important for my personal works. Especially, I have been influenced very much by the series of his extensive studies on aftershocks and earthquake statistics* [*Utsu,* 1969, 1970, 1971 *and* 1972].

Omori-Utsu formula. One of his prominent works is the empirical formula for the decaying rate of aftershocks which has been called the modified Omori formula, $\lambda(t) = K / (t + c)^{-p}$, ($K, c, p$; constant), where t is the elapsed time from the mainshock, usually in days. This formula is modestly named by *Utsu* [1961]. However, this formula is quite substantial extension of the original Omori formula [*Omori*, 1894] for the case of $p = 1$. Although *Utsu* [1957] first used the form $\lambda(t) = Kt^{-p}$, he has become confident this form including the scale coefficient c (see below for the reason). On this occasion, I would like to call it as the “Omori-Utsu formula” after Marsan and Nalbant [2005].

Utsu [1961, 1969] analyzed more than 100 aftershock sequences in Japan and world by using the doubly logarithmic diagram of occurrence rate against the elapsed time from the mainshock, and he showed that the characterizing parameter p-value was different from place to place, varying from 0.9 to 1.8, to be a geophysical quantity. In fact, p-value is shown to be correlated with the crustal temperature and related to the stress relaxation of the main rupture [*Mogi* 1967; *Kisslinger and Jones*, 1991; *Creamer and Kisslinger*]. The p-values may also correlate with the degree of heterogeneity of the mainshock’s fault zone.

The p-value is usually independent of threshold magnitude [*Utsu*, 1962], which is equivalent to the fact that b-value of the Gutenberg and Richter’s (G-R) magnitude frequency law is constant throughout the aftershock activity.

The p-value and other parameters of the Omori-Utsu formula is nowadays estimated by the maximum likelihood procedure directly from the series of occurrence times assuming the non-stationary Poisson process model with the intensity function $\lambda(t)$ [*Ogata*, 1983]. The duration of aftershock activity can last more than one century [*Utsu*, 1969; *Utsu et al.*, 1995] until it will merge into the normal activity of the region, the time of which is independent of magnitude threshold of the sequence [*Ogata and Shimazaki*, 1984].

Further studies on empirical laws of aftershocks. *Utsu* [1970] discovered that the secondary aftershock sequence following after a large aftershock also obeyed the Omori-Utsu law. *Utsu* [1970] provided the standard formula of aftershock occurrence rate, the size of which is proportional to the exponential to magnitude of the mainshock, which is in fact the numerical basis of aftershock probability forecasting [*Reasenberg and Jones*, 1989] in California and Japan. These works by Utsu was integrated into the ETAS point-process model [*Ogata*, 1986, 1988, 1989].

His study on the aftershock area against the magnitude of the mainshock led well known as the Utsu-Seki law [*Utsu and Seki*, 1954]. This is probably the first seismic scaling law related to the moment magnitude. One of the interesting discovered facts is that the scaling law for the inland earthquakes is clearly discriminated from those of offshore [*Utsu*, 1969]. His scaling law is a basis of modeling the spatial aftershock distribution of the space-time ETAS models [*Ogata*, 1998, 2003; *Ogata et al.*, 2004]. A comprehensive review on the studies of the decaying law of aftershock activity and their development is given by *Utsu et al*. [1995].

Remarks on the *c*-value of the Omori-Utsu formula. True *c*-values are difficult to estimate accurately unless very careful observation is started immediately after the main shock. When data are taken from ordinary earthquake catalogs, the estimated *c*-value may partially reflect the effect of incomplete detection of small aftershocks shortly after the main shock.

There is an opinion that the *c* value is essentially 0 and all reported positive *c* values result from the above-mentioned incompleteness. If $c = 0$, $\lambda(t)$ above diverges at $t = 0$. *Kagan and Knopoff* [1981] explained this difficulty by considering that the main shock is a multiple occurrence of numerous sub-events occurring in a very short time interval, and thus the stochastic seismicity model by *Kagan and Knopoff* [1987] uses this transfer function after certain dead times following the earthquakes, which is actually different from the Omori-Utsu transfer function of the ETAS model, though the both are extended from the Hawkes' self-exciting point-process model [*Hawkes and Adamopoulos*, 1973].

In fact, positive *c* values have been obtained for a number of adequately observed and selected datasets. Moreover, consider a case where the aftershock activity obeys the law $\lambda(t) = Kt^{-p}$, but the observational data fit the relation $\lambda(t) = K(t+c)^{-p}$ owing to the above mentioned incompleteness. In this case, $100(1\text{-}2^{-p})$ percent of aftershocks is considered to be missing at $t=c$ (50% if p =1 and 60% if $p = 1.3$). Such a large percentage of missing events seems unlikely for many of the aftershock sequences to which relatively large *c* values have been estimated.

Magnitude distributions. The *b*-value of the G-R magnitude frequency was graphically estimated by measuring the slope of the fitted straight line to the log-frequency plot against magnitude. Assuming the exponential distribution of the G-R law, *Utsu* [1965] derived the estimate of *b*-value by the method of moment, which is nothing but the MLE [*Aki*, 1965]. The compensation must be taken for the MLE when we use discrete (rounded) magnitude values [*Utsu*, 1970]. *Utsu* [1967] provided the testing procedure for different *b*-values from two regions. For example, it is seen that *b*-values of the mainshocks is different from that of aftershocks in and near Japan [*Utsu*, 1969]. To measure a deviation from the G-R law, Utsu [1988] proposed the η-estimate based on the second order moment of the empirical magnitude distribution.

Discrimination of foreshocks from earthquake swarm is difficult problem, because no easily recognizable differences in statistical or properties are found between them. *Utsu* [1978] searched for the sequential pattern of three largest values of magnitude in a cluster, which provides the highest probability of being foreshocks, using the JMA hypocenter data. *Utsu* [1988] showed that the η-value in foreshocks tends to be smaller than that of aftershocks in the same sequence. *Ogata, Utsu and Katsura* [1995, 1996] investigated data sets of earthquake clusters to discriminate features of foreshocks from earthquakes of other cluster types in a statistical sense, and found several features of some predictive value, including the fact that foreshocks were more closely-spaced in time than either swarms or aftershocks, the fact that foreshocks were closer together in space than other types of events, and that foreshocks' sizes were more likely to increase chronologically than the other types. By modeling such discriminating features *Ogata, Utsu and Katsura* [1996] developed probability forecasts of an earthquake cluster being of foreshock type.

Multi-element prediction formula. The occurrence probability of earthquake is enhanced when we assess the probability using the composite anomalies of independent record. *Utsu* [1977, 1978b] calculated such probability and retrospectively applied to the reported several anomalies of long-, intermediate- and short-term preceding Izu-Oshima Kinkai Earthquake of 1978, showing the high probability. *Aki* [1981] provide a simple, approximated version of this formula assuming that the probability due to each anomaly is small. *Ogata, Utsu and Katsura* [1996] make use of a logit model as the extended version of this formula to assess the probability of foreshocks based on the location of the first event and inter-event features (time, distance and magnitude) of a cluster.

Causal relation of seismic activity between two regions. Relatively clear correlation is seen in subduction zones. *Utsu* [1975] found such a significant correlation between shallow and intermediate-depth earthquakes. Since earthquakes have a tendency to clustering, the correlation coefficient between two regions easily exceeds the significance level expected for the Poissonian occurrence, even if there is no correlation. This paper was my first occasion to know Professor Utsu's work, and I was motivated to apply a point-process model to his compiled dataset to show the one way causal relationship [*Ogata*, 1982; *Ogata and Katsura*, 1986; also see LINLIN program of IASPEI Software by *Utsu and Ogata*, 1997].

Utsu catalog of large earthquakes in Japan 1885-1980. A homogeneous catalog of earthquakes of M>=6 in and near Japan is compiled by *Utsu* [1979, 1982, 1985] based on both instrumental and macroseismic data in the determination of focal parameters. Based on the Utsu catalog, *Ogata* [1986, 1988] compared the goodness-of-fit of the ETAS with other models to the earthquakes from a certain active offshore region to show the best fit, and revealed some relative quiescence before some great earthquakes.

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