Statistical Models Based on the Gutenberg-Richter $a$ and $b$ values for Estimating Probabilities of Moderate Earthquakes in Kanto, Japan

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Abstract: We attempt to construct models estimating probabilities of moderate size earthquakes in Kanto, central Japan, with recurrent times of target earthquakes considered in reverse relation to the hazard. In estimating the recurrent times, the $a$ and $b$ values of the Gutenberg-Richter relation for an area of interest are estimated and the relation is extrapolated to a magnitude range of targets. We consider two alternative cases; the case of the $b$ value varying from point to point and the case of the $b$ value remaining constant over the whole space under study. Retrospective analysis based on the catalogue of the NIED Kanto-Tokai network for the period from 1990 to 1999 indicates that the model with constant $b$ performs better than that with varying $b$. This implies that the spatial variation in $b$ will not work effectively for estimating earthquake probability. However, a comparison between the $b$ value distributions during the background period and over a set of time-space points conditioned by target earthquakes suggests that some information could be obtained from the $b$ value, since the Kullback-Leibler information statistic between the two distributions indicates a significant value. In order to incorporate variations in $b$ into the model with $b$ remaining constant, a hazard function with a single parameter, $b$, is considered, in which a normal distribution is adopted and optimized to the data. The $b$ value is distributed independently from the distribution of the $a$ value. Taking these two values as the parameters for earthquake precursors, a formula for earthquake probabilities based on multi-disciplinary parameters could be applied. Thus, we obtain a model of earthquake probability using both $a$ and $b$ values which performs better than either model of the Gutenberg-Richter relation, with constant or variable $b$.

1. Introduction

Utsu (1977, 1982), Aki (1981) and others have formulated an earthquake probability calculation based on multiple precursory phenomena. These calculations assume that different precursory phenomena occur independently of each other. In such a case, the probability expected from detecting multiple precursory phenomena is given by the product of the probability gains for respective observations and the probability calculated from secular seismicity. Therefore, the probability becomes large when multiple precursory phenomena are detected simultaneously. Only a small number of reports have been published based on this formula, perhaps due to a common understanding that no reliable precursor has been observed except for foreshocks and others. This formulation, in which phenomena are treated as two categorical values in the original form, could be reformulated in light of recent point-process modeling to apply to cases of a quantitative observed value. As an example, the $a$ and $b$ values of the Gutenberg-Richter relation could be treated as such parameters and are indeed being used for building probability models for moderate earthquakes in Kanto, central Japan.
2. Data and Models

Hazard functions for a moderate-size earthquake (M≥5.0) occurring within the study volume (200 x 200 x 90 km³) for the period from January 1990 to December 1999 were obtained using the Gutenberg-Richter relation. The $a$ and $b$ values at a given space-time point, which measure the recurrent intervals of targets, are calculated based on earthquakes within 20 km of the point occurring over a period of 3650 days prior to the assessment. The assessment was made at 2km-spaced points and at 10-day intervals. We classified the points of assessment into two categories, points of target events and points of background. A point of target event was the grid point of assessment nearest to a target in space and just before it in time. The other grid points are classified as background. We thus obtained two distributions, the background distribution and conditional distribution, for each of the $a$ and $b$ parameters. Figures 1a and 1b illustrate the background distribution with a blue dashed line and the conditional distribution with a red solid line for $a$ and $b$, respectively. In Fig. 1a, the difference between the two distributions is quite obvious. On the other hand, in Fig. 1b, the difference between the two distributions is not extremely large but is still significant.

![Diagram](image)

**Figure 1a** Cumulative distributions of the $a$ value for background points (blue dashed line) and for points conditioned by target earthquakes (red solid line).
A-model: Assuming a single constant $b$ value over the study volume, only the spatial variation in $a$ is taken into account when estimating the recurrent interval of a target event. The hazard function for this model is given as:

$$\lambda_a(s; M \geq 5.0) = N(s; M \geq 2.0) \cdot 10^{-\Delta_m \cdot b} \cdot r_v,$$

(1)

where the first term of the right-hand side is the number of earthquakes in a sample space and time volume, the common logarithm of which defines the $a$ value. The second term is a constant in this case; the last term is a normalizing factor between the sample volume and the unit volume for the hazard function.

B-model: The same as the A-model but with the $b$ variation. The hazard for this case is given as:

$$\lambda_b(s; M \geq 5.0) = N(s; M \geq 2.0) \cdot 10^{-\Delta_m \cdot b(s)} \cdot r_v.$$

(2)

AxB-model: Taking the two distributions of $b$ in Fig. 1b into consideration, a hazard function is assumed as in the following form:

$$\lambda_{axb}(s; M \geq 5.0) = N(s; M \geq 2.0) \cdot 10^{-c_1 \cdot b + c_2 \cdot b^2} \cdot r_v,$$

(3)

where $c_1$ and $c_2$ are optimized to data for the learning period (January 1990 to December 1999).
3. Performance

To estimate the performance of the above-mentioned models, the difference in the log likelihood between a proposed model and the stationary Poisson model, which is called information gain (IG value; Darley and Vere-Jones, 2003), is calculated using data from both the learning period and a test period (January 2000 to September 2005). Table 1 lists the IG values for the A-, B-, and AxB-model for both periods. The IG values for the A-model are larger than those for the B-model for both periods. This suggests that the spatial variation in the b value does not work effectively for estimating probabilities. The IG values for the AxB-model, however, are larger than those for either the A- or B-model for both periods. Thus, the b parameter of the Gutenberg-Richter relation is built more effectively in this model than in the model that uses the Gutenberg-Richter relation directly, with the b value constant or varying in space.

<table>
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<th>Period</th>
<th>N</th>
<th>A-model</th>
<th>B-model</th>
<th>AxB-model</th>
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<td>38.07</td>
<td>36.49</td>
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<td>Test</td>
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<td>33.05</td>
<td>31.61</td>
<td>37.23</td>
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Table 1. IG values for the A-, B-, and AxB-model for the learning and test periods.

4. References


