Estimating stress heterogeneity from aftershock rate
Agnès Helmstetter and Bruce E. Shaw
Lamont-Doherty Earth Observatory, Columbia University, New York

Corresponding author: agnes.helmstetter@ujf-grenoble.fr

Abstract: We estimate the rate of aftershocks triggered by a heterogeneous stress change, using the rate-and-state model of Dieterich [1994]. We show that an exponential stress distribution $P(\tau) \sim \exp(-\tau/\tau_0)$ gives an Omori law decay of aftershocks with time $\sim 1/t^p$, with an exponent $p = 1 - A\sigma_n/\tau_0$, where $A$ is a parameter of the rate-and-state friction law, and $\sigma_n$ the normal stress. Omori exponent $p$ thus decreases if the stress "heterogeneity" $\tau_0$ decreases. We also invert the stress distribution $P(\tau)$ from the seismicity rate $R(t)$, assuming that the stress does not change with time. We apply this method to a synthetic stress map, using the (modified) scale invariant "$k^2$" slip model [Herrero and Bernard, 1994]. We generate synthetic aftershock catalogs from this stress change. The seismicity rate on the rupture area shows a huge increase at short times, even if the stress decreases on average. Aftershocks are clustered in the regions of low slip, but the spatial distribution is more diffuse than for a simple slip dislocation. This stochastic slip model gives a Gaussian stress distribution, but nevertheless produces an aftershock rate which is very close to Omori’s law, with an effective $p \leq 1$, which increases slowly with time. The inversion of the full stress distribution $P(\tau)$ is badly constrained for negative stress values, and for very large positive values, if the time interval of the catalog is limited. However, constraining $P(\tau)$ to be a Gaussian distribution allows a good estimation of $P(\tau)$ even with a limited number of events and catalog duration. We show that stress shadows are very difficult to observe in a heterogeneous stress context.

1 Introduction

Much progress has been made in describing earthquake behavior, in particular the Omori law decay of aftershocks with time, based on the predictions of rate-and-state friction law [Dieterich, 1994]. However, this model does not explain the abundance of aftershocks on the rupture surface, unless introducing a strong heterogeneity of the slip and stress on the fault. A simple model of an earthquake source, with a uniform stress change on the rupture area, predicts a decrease of the seismicity rate on the rupture area. In order to explain the increase of the seismicity rate on the fault, we need to introduce stress heterogeneity in the rate-and-state model, which can be due to the variability of slip on the fault [Helmstetter and Shaw, 2005; Marsan, 2005], or to fault roughness [Dieterich, 2005]. Here we estimate the seismicity rate triggered by a heterogeneous stress change. We also develop a method to invert for the stress distribution on the fault from the seismicity rate, assuming that stress changes instantaneously after the mainshock.

2 Relation between stress distribution and seismicity rate

Dieterich [1994] estimated the seismicity rate $R(t, \tau)$ triggered by a single stress step $\tau$, assuming a population of faults governed by rate-and-state friction

$$R(t, \tau) = \frac{R_r}{\left(e^{-\tau/\dot{\tau}_r} - 1\right)} e^{-t/\dot{\tau}_r + 1},$$

(1)

where $t_a = A\sigma_n/\dot{\tau}_r$ is the duration of the aftershock sequence, and $\dot{\tau}_r$ the tectonic loading stressing rate. For each positive stress value, the seismicity rate is constant for $t \ll t_a e^{-\tau/\dot{\tau}_r}$, and then decreases with time as $R(t) \sim 1/t$ for $t \ll t_a$. For a negative stress change, the seismicity rate decreases after the mainshock. In both cases, the seismicity rate recovers its reference value $R = R_r$ for $t \gg t_a$.

For a heterogeneous stress field $\tau(\vec{r})$, with a distribution (probability density function) $P(\tau)$, the seismicity rate integrated over space is

$$R(t) = \int R(t, \tau(\vec{r})) d\vec{r} = \int_{-\infty}^{\infty} R(t, \tau) P(\tau) d\tau.$$

(2)
The rate-and-state model gives an Omori law decay of aftershocks with time \( R(t) \sim 1/t^p \), for \( t \ll t_a \), if the stress distribution follows an exponential distribution \( P(\tau) \sim e^{-\tau/\tau_0} \). The Omori exponent \( p = 1 - A\sigma_n/\tau_0 \) increases toward 1 as the stress heterogeneity \( \tau_0 \) increases. The rate-and-state model with a uniform stress step (1) cannot explain an Omori law decay with \( p > 1 \). The only solution in order to obtain \( p > 1 \) in the rate-and-state model, as observed for many aftershock sequences, is to have a variation of stress with time, which may be due to postseismic slip or viscous relaxation.

We can invert for the complete stress distribution \( P(\tau) \) from the temporal evolution of the seismicity rate using (2), provided we observe \( R(t) \) over a wide enough time interval. In practice, the estimation of \( P(\tau) \) for large \( \tau \) is limited by the minimum time \( t_{\min} \) at which we can reliably estimate the seismicity rate. The largest stress we can resolve is of the order of \( \tau_{\max} = -A\sigma_n \log(t_{\min}/t_a) \). For negative stress, we are limited by the maximum time \( t_{\max} \) after the mainshock, and by our assumptions that secondary aftershocks are negligible, and that the stress does not change with time (e.g., neglecting post-seismic relaxation).

### 3 Application of the method to a stochastic slip model

We have tested the rate-and-state model on a realistic synthetic slip pattern. Herrero and Bernard [1994] proposed a kinematic, self-similar model of earthquakes. They assumed that the slip distribution at small scales, compared to the rupture length \( L \), does not depend on \( L \). This led to a slip power-spectrum for high wave-number equal to \( u(k) = C\sigma_0 L^2k^{-2}/\mu \), where \( \sigma_0 \) is the stress drop (typically 3 MPa), \( \mu \) is the rigidity, and \( C \) is a shape factor close to 1. We have modified the \( k^2 \) slip model in order to have a finite standard deviation of the stress distribution. We used a slip power-spectrum given by \( u(k) = C\sigma_0 L^3(kL + 1)^{-\eta}/\mu \), with \( \eta = 2.3 \). We have then computed the stress change on the fault from this synthetic slip model, for a fault of 50 x 50 km, with a resolution \( dx = 0.1 \) km, and a stress drop \( \sigma_0 = 3 \) MPa. The stress field has large variations, from about -90 to 90 MPa, due to slip variability (see Fig 1).

We have then estimated the seismicity rate predicted by the rate-and-state (2) on the fault from the stress change, assuming \( A\sigma_n = 1 \) MPa (see Fig 2). While the stress on average decreases on the fault, the seismicity rate shows a huge increase after the mainshock. It then decays with time approximately according to Omori law, with an apparent exponent \( p = 0.93 \). At large times \( t \approx t_a \), the seismicity rate decreases below its reference rate due to the negative stress values. While the pure Omori law with \( p < 1 \) occurs for the exponential distribution of shear stress changes, we find that a Gaussian stress distribution (which the \( k^2 \) model and many other models give), also gives realistic looking \( p \)-values over wide ranges of time scales.

**Figure 1:** Slip generated with the modified \( k^2 \) model (a), shear stress change (b) and (c) map of aftershocks generated with the rate-and-state model, using the stress change shown in (b) and \( A\sigma_n = 1 \) MPa.
We have generated synthetic earthquake catalogs according to the rate-and-state model (1), using the stress change shown in Fig 1b, without including earthquake interaction. We found that, for a realistic catalog (less than 6 orders of magnitude in time), the limited time interval available does not allow the inversion of the complete stress distribution \( P(\tau) \) from the seismicity rate \( R(t) \) (see Fig 2b). However, we can estimate \( P(\tau) \) with a good accuracy if we constrain \( P(\tau) \) to be a Gaussian distribution, as predicted by the \( k^2 \) slip model, and if we know the stress drop and \( A\sigma_n \). We can measure the standard deviation \( \tau^* \) of the stress distribution and the aftershock duration \( t_a \) from the observed aftershock rate.

4 Off-fault aftershocks

We can estimate the stress change and seismicity rate off of the fault plane for the synthetic \( k^2 \) slip model shown in Fig 1. The power-spectrum of the shear stress change off of the fault, at a distance \( y \) smaller than the rupture length \( L \), is given by \( \tau(k,y) \sim e^{-ky} \tau(k,0) \). The standard deviation of the stress distribution decreases very fast with the distance to the fault, which produces a strong drop of the seismicity rate off of the fault (see Fig 3). The average stress decreases much slower with \( y \). For \( y/L > 0.1 \), the stress field is very homogeneous and mostly negative (the standard deviation is smaller than the absolute mean stress). Therefore, the seismicity rate for \( y/L > 0.1 \) is smaller than the reference rate at all times \( t < t_a \).

In practice, it is difficult to analyze the rate of off-fault aftershocks, since very few earthquakes occur off of the fault plane, because the stress changes are smaller, and because the reference seismicity rate off the fault is much smaller that on the fault. Also, secondary aftershocks triggered by off-fault events will perturb the stress field and seismicity rate with additional stress heterogeneity.

5 Conclusion

We have shown how the rate-and-state seismicity model, with an heterogeneous stress field, can explain the spatial distribution of aftershocks on or close to the rupture area, and an Omori law decay with exponent \( p \leq 1 \).
Figure 3: (a) Seismicity rate for different values of the distance to the fault $y/L$ decreasing from $y/L = 0$ (top) to $y/L = 0.2$ (bottom), using the slip model shown in Figure 1a, and assuming $A\sigma_n = 1$ MPa. (b) Standard deviation (solid line) and absolute value of the mean (dashed line, the average stress change is always negative) of the stress distribution as a function of the distance to the fault.

Deviations from $p = 1$ are mapped onto measures of stress change heterogeneity on the fault. We have shown that modest catalogue lengths allow an accurate inversion for the standard deviation of the stress distribution and for the aftershock duration $t_a$. The stress drop is however not constrained if the catalog is too short ($t_{max} < t_a$). This model cannot explain why some aftershock sequences have $p > 1$. A possible explanation involves stress relaxation with time due to post-seismic deformation. Inverting the stress distribution from aftershock rate requires to know $A\sigma_n$. We used $A\sigma_n = 1$ MPa, based on laboratory experiments, which give $A = 0.01$ [Dieterich, 1994], and assuming $A\sigma_n = 100$ MPa (lithostatic pressure at 5 km). Dieterich [1994] suggests $A\sigma_n = 0.1$ MPa from the relation between aftershocks duration, tectonic loading rate, and recurrence time. We have neglected in our study the effect of secondary aftershocks. Ziv and Rubin [2003] found, using numerical and analytical study of the rate-and-state model, that secondary aftershocks increase the reference rate $R_r$ but do not change $p$ or $\tau^*$. We have also assumed that $A\sigma_n$ is uniform. But fluctuations of $A\sigma_n$ do not change the seismicity rate if $A\sigma_n$ is less heterogeneous than the coseismic shear stress change $\tau$.

References