Spatial-Temporal Statistics Between Successive Earthquakes on 2D Burridge-Knopoff Model

T. Hasumi

Department of Physics, Faculty of Science and Engineering, Waseda University, Shinjyuku-ku, Tokyo 169-8555, Japan

Corresponding Author: T. Hasumi, t-hasumi.1981@toki.waseda.jp

Abstract: We numerically investigate two-dimensional spring-block model for earthquakes called Burridge-Knopoff (BK) model and study the statistical properties of spatio-temporal interval between successive earthquakes. The cumulative distribution of spatio-temporal interval obeys the q-exponential distribution, namely in time interval (waiting time) case is $q_t > 1$, and spatial interval (distance between the hypocenters) case is $q_r < 1$, respectively. In the waiting time case only, the distribution depends on the frictional property. These results are consistent with the observation one, hence it is concluded that 2D BK model has a reality from a complex network and non-additive statistical mechanics.

1. Introduction

Earthquakes (EQs) are complex phenomena. The source mechanism of them has not been clear yet, however some empirical laws are known such as Gutenberg-Richter (GR) law and Omori law. Very recently, Abe and Suzuki (2003, 2005) have found the new statistical properties for EQ, namely spatio-temporal interval (hereafter waiting time and distance) between successive EQs with non-additive statistical mechanics proposed by Tsallis (1988). They have shown that the cumulative distribution of waiting time and distance have obeyed the $q$-exponential distribution with Southern California and Japan. More precisely, $q$-value of waiting time ($q_t$) and distance ($q_r$) are $q_t > 1$, $q_r < 1$, respectively.

In 1980s Bak et al. (1987, 1988) have proposed the Self-organized Criticality (SOC) in the non-equilibrium open system. Following their idea, EQ seemed to be SOC, hence many models have proposed and discuss the GR law. In particular, 2D spring-block model (originally proposed Burridge and Knopoff (1967)) have studied by Otsuka (1972) and Carlson (1991) to discuss the GR law. It is however that in this model, the waiting time and distance statistics have not been reported.

We report numerical investigation about 2D BK model to study the waiting time and distance statistics.

2. Model

The model is 2D spring-block model for earthquakes called BK model and shown in Fig. 1 (a). We assume that block’s slip is restricted $y$-direction. Following this, non-dimensional equation of motion of cite $(i,j)$ is

$$
\dot{U}_{i,j} = l_x^2 \left( U_{i+1,j} + U_{i-1,j} - 2U_{i,j} \right) + l_y^2 \left( U_{i,j+1} + U_{i,j-1} - 2U_{i,j} \right) - U_{i,j} - \phi \left( 2\alpha (v + \dot{U}_{i,j}) \right),
$$

(1)

where $U_{i,j}$ is scaled displacement. Another parameters are given by

$$
l_x^2 = k_c^x / k_p, \quad l_y^2 = k_c^y / k_p.
$$

(2)

The 4th term of right hand side is the nonlinear friction term, namely velocity weakening type
This system is characterized by four parameters: $l_x$, $l_y$, $\alpha$ and $v$. $l_x$ and $l_y$ are stiffness, $v$ is a scaled plate velocity, $\alpha$ is governing the friction function.

We numerically investigate eq. (1) the (100, 25) on the ($x$, $y$) plane under free boundary condition by 4th Runge-Kutta method. In simulation, $l_x^2 = 1$, $l_y^2 = 3$, $v = 0.01$ are fixed and $\alpha$ is changed.

### 3. Results and Discussion

In this model, we consider a slip of the block as EQ whose seismic moment $M_0$ and magnitude $M$ are defined by

$$
M_0 = \sum_{i,j}^{n} \delta U_{i,j}, \quad M = \log M_0, \quad (3)
$$

where $\delta U_{i,j}$ is the total displacement during a event and $n$ is the number of slipping block. The block which slips for the first time is regarded as a hypocenter. Hence, we define the distance and waiting time as the spatio-temporal interval between successive EQ-like events. In this research, 10$^5$ order of events are taken to discuss on the statistical properties, it is however, initial 10$^3$ order of events are neglected due to the randomness of initial configuration effect. Kumagai et al. (1999) have reported that the magnitude-frequency distribution depended on the friction parameter $\alpha$. More precisely, the case for $\alpha = 3.5$, those for 1.5 and 2.5 and for 4.5 and 5.5 corresponds to the critical state, subcritical state, supercritical state, respectively. Hence, we change the parameter $\alpha$ from 2.5 to 4.5 as a representative of three-types of critical state and discuss the waiting time and distance statistics.

Firstly, following the Abe and Suzuki (2005), we try to fit cumulative distribution of waiting time to $q$-exponential distribution that is defined by

$$
e_q(x) = [1 + (1 - q)x]^{1/(1-q)}, \quad \text{where} \quad [a]_+ = \max[0, a], \quad (4)
$$

if $q > 1$ this is equivalent to the Zipf-Mandelbrot power law. The inverse function called $q$-logarithmic function written by

$$
\ln_q(x) = \frac{1}{1-q} (x^{1-q} - 1) \quad (5)
$$

The cumulative distribution of waiting time in case for $\alpha = 3.5$ is shown in Fig. 2 (a), (b) with these fitting function. The simulation study says that $q_e$ is always more than 1 for any $\alpha$, it is concluded that the waiting time distribution obeys the Zipf-Mandelbrot power law. This is
consistent with the observation result (Abe and Suzuki, 2005). We also mention that this distribution depends on the frictional parameter $\alpha$.

Secondly, the cumulative distribution of distance is discussed as well as waiting time statistics. Fig. 2 (c), (d) is an example in case for $\alpha = 3.5$. In distance statistics, $0 < q_r < 1$ for any $\alpha$, so it is equivalent to the modified Zipf-Mandelbrot law. This result also satisfied the observation result (Abe and Suzuki, 2003).

![Figure 2](image)

**Figure 2** The spatio-temporal interval statistics between successive EQ-like events with $l_x^2 = 1, l_y^2 = 3, \alpha = 3.5$ : (a) and (b) are waiting time statistics, (c) and (d) are distance statistics. In each figure, dots are the simulation result and solid line is fitting function that is $q$-exponential function defined by eq. (4). In case for waiting time ($q, \tau_0 = (1.06, 4.50)$ and correlation coefficient $\rho = 0.989$, and for distance ($q, r_0 = (0.497, 50.1)$ and correlation coefficient $\rho = 0.9992$, respectively.

4. Conclusion

We report numerical investigation about the 2D BK model for EQs to discuss the waiting time and distance statistics. The cumulative distribution of waiting time and distance obey the $q$-exponential distribution in waiting time case for $q_t > 1$, in distance case for $q_r < 1$, respectively. These results are consistent with the observation one, hence it is concluded that 2D BK model has a reality from the complex network and the non-additive statistical mechanics.
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6. References