The Snowball Effect: Statistical Evidence that Big Earthquakes are Rapid Cascades of Small Aftershocks

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Abstract: Subevents can be clearly seen in event seismograms, and subevent models are necessary to fit strong motion data. To what extent might earthquake subevents be made up of smaller subevents? And smaller subevents still? Vere Jones (1976) and Kagan and Knopoff (1981) demonstrated that an ultimate subevent model, in which the smallest, tiny component subevents are of equal size, could recreate the Gutenberg-Richter earthquake magnitude frequency distribution. In the Kagan and Knopoff (1981) model, mainshocks grow when tiny subevents rapidly trigger each other via a process identical to aftershock triggering. This model successfully reproduces realistic seismograms and realistic foreshock and aftershock sequences over longer time scales. Here I propose a model similar to Kagan and Knopoff (1981) except that equal faulting area rather than the original equal seismic moment characterizes the smallest subevents. This change is supported by physical and statistical considerations. My proposed model predicts that the number of aftershocks produced by an earthquake should scale with its faulting area and that no property of an aftershock sequence except for the total number of aftershocks should be influenced by mainshock size. These predictions are well supported by observations.

1. Introduction

There are significant differences in the rupture of homogeneous and heterogeneous materials. In a homogeneous glass or metal a small crack may often grow quickly and catastrophically. In a heterogeneous material, however, such as rock, strength and elastic moduli may vary and a crack that starts in one place may have difficulty in spreading through neighboring stronger material. The amount of growth difficulty will depend on the applied load. For a small load cracks will never propagate beyond the areas of lowest strength; for a high load unbounded growth will always occur. At the critical load growth is possible but jerky as the crack feels the small-scale heterogeneity. Dynamic elastic waves are emitted by this jerky motion, which may initiate slip in new areas. Scale invariant (power law) avalanche type slip occurs, and the crack tip is self affine (Ramanathan and Fisher, 1998).

Earthquakes clearly rupture through heterogeneous materials, and it is likely that they occur at critical stress loads. Tectonic stresses accumulate slowly, and slow stress accumulation means that stresses can stabilize fairly precisely at the level just large enough to make slip possible. The observed power law distribution of earthquake sizes provides further evidence for critical rupture dynamics.

Physically, the avalanche like behavior of critical point rupture is very difficult to model. Statistical stochastic models, may recreate the process quite easily and accurately. Kagan and Knopoff (1981) proposed a compelling stochastic age-dependent continuous state branching process model for earthquake propagation. In the Kagan and Knopoff (1981) model small seismic subevents of equal size (hereafter referred to as “unit earthquakes”) trigger each other rapidly, via the aftershock triggering process, to create larger earthquakes. The idea that aftershock triggering might occur over such a short time scale is supported by the fact that a minimum triggering time, while it must exist, has not yet been found (Peng et al., 2005; Kilb et al., 2004). Furthermore, Kagan and Knopoff (1981) find that allowing the
unit earthquakes to trigger each other with time delays that adhere to Omori’s law for aftershock decay creates realistic seismograms. When the activity level of the model is tuned to the critical point, where sustained growth just becomes possible (the point where each unit earthquake triggers, on average, one other) Gutenberg-Richter magnitude frequency statistics are also reproduced.

In the Kagan and Knopoff (1981) model unit earthquakes are defined as having equal seismic moment, but the model does not reproduce a $b$ value of 1.0 for the Gutenberg-Richter relationship. Alternatively, I propose that the unit earthquakes should be of equal area. Equal area subevents fit easily into a physical model. If we assume that points of high strength are randomly distributed in a heterogeneous medium, then this will produce a characteristic length scale over which we expect initiated slip to grow before it starts to slow down. The “equal area” model does produce a Gutenberg-Richter relationship with $b=1.0$.

Three testable predictions can be made from the equal-area unit earthquake stochastic model of earthquake growth:

1) The propagation of single earthquakes and the production of aftershocks are one and the same process. Therefore the triggering of aftershocks, like the growth of mainshocks, must be accomplished by dynamic stress changes.

2) All aftershock triggering is accomplished by unit earthquakes, whose characteristics (other than final slip) and triggering ability is always the same independent of final mainshock magnitude. Thus the characteristics of aftershock production by earthquakes of every magnitude should be the same except for the number of aftershocks, which should vary linearly as a function of faulting area (e.g. vary linearly with the number of unit earthquakes).

3) In the Kagan and Knopoff (1981) branching process, the activity of each branch is independent of the activity of all other branches. This means that each aftershock can be traced to a unique mainshock, and that subsequent earthquakes will not affect the timing of each aftershock.

In the following sections we will provide brief descriptions of tests of these predictions.

2. Test 1: Are aftershocks triggered by dynamic stress changes?

I have performed two tests with E. E. Brodsky (Felzer and Brodsky, 2005; Felzer and Brodsky, submitted), on whether aftershocks are triggered by static or dynamic stress changes. The first test was for stress shadows, or whether earthquake-induced stress changes are capable of slowing other earthquakes down. This is predicted to occur if earthquake timing is affected by static stress changes, but may not occur if dynamic stress changes are most important. We found no evidence for stress shadows in California, supporting the dynamic triggering hypothesis. A similar lack of stress shadowing has also been found by other authors, at least for the first several months following a mainshock (Parsons, 2002; Marsan, 2003; Mallman and Zoback, 2003).

The second test performed for static vs. dynamic triggering was to inspect the rate of decay of aftershock density with distance from mainshocks in California. We found a constant, inverse power law decay rate from distances ranging from a fraction of a fault length (of M 5-6 earthquakes) to over a hundred fault lengths (of M 2-3 mainshocks). The single, continuous decay rate indicates that a single aftershock triggering mechanism is operating
over the entire distance range. Only dynamic stress changes are large enough to trigger at distances over several fault lengths. This suggests that dynamic stress changes must be the triggering agent at all distances (Figure 1).

![Figure 1](https://via.placeholder.com/150)

**Figure 1** Distance from the mainshock vs. aftershock linear density (aftershocks/km) for combined aftershock data of M 2-3(A) and M 3-4 (B) California mainshocks. All aftershocks are M>2 and occur in the first 5 minutes after their mainshock. The decay is well fit by $r^{-1.30 \pm 0.1}$ for the M 2-3 data and $r^{-1.37 \pm 0.07}$ for the M 3-4 data, where $r$ is the distance from the mainshock hypocenter.

### 3. Test 2: Aftershock scaling and mainshock magnitude

There are really two predictions that the model makes with respect to aftershock scaling with mainshock magnitude. The first is that the total number of aftershocks will scale linearly as mainshock faulting area, or as $10^{\alpha M}$ where $\alpha = 1$, since $10^M$ scales as faulting area. This scaling has been found by Yamanaka and Shimazaki (1990), Michael and Jones (1998), Felzer et al. (2004), and Helmstetter et al. (2005) among others. The second prediction is that no other property of aftershocks, besides number, will be affected by mainshock magnitude. Many authors have found that aftershock magnitude is independent of mainshock magnitude (Kagan 1991; Michael and Jones, 1998; Felzer et al. 2004), (Figure 2). Felzer and Brodsky (submitted) demonstrated that the spatial distribution of aftershocks is independent of mainshock magnitude. Helmstetter et al. (2005) and Gerstenberger et al. (2005) and others have found the temporal distribution of aftershocks to be independent of mainshock magnitude.

### 4. Test 3: Does each aftershock have a single parent?

The last prediction of the model is that aftershock triggering is a binary process, such that each aftershock is triggered by one and only one mainshock. This implies that aftershock triggering is effectively an “on” or “off” process; a fault can be either triggered or left alone by another earthquake, and once triggered cannot be turned off again before its aftershock occurs.

While this idea is counterintuitive with respect to many physical models (such as models of accelerating failure), it nonetheless finds support in the data. The density of aftershocks swiftly drops with distance from the mainshock fault. Yet Felzer (2005) found that the temporal distribution of aftershocks does not vary with mainshock distance. This indicates that stress change amplitude affects the total number of aftershock triggered, but not the timing of aftershocks that do occur – so the stress change must work as a simple “on” or “off” switch (Figure 3). While this finding does not prove that an aftershock cannot be multiply affected by different mainshocks, this “on or off” quality of aftershock triggering suggests that it is possible.
The average number of aftershocks/mainshock for different relative mainshock—aftershock magnitude differences. The data set is composed of 101,680 $M \geq 2.2$ California earthquakes (1975-2001). Mainshocks are used if they do not occur within the first 30 days of the aftershock sequence of a larger earthquake. Aftershocks occur within the first 2 days and 2 fault lengths of their mainshocks. The black line gives the relationship predicted if the aftershock magnitude distribution is simply the Gutenberg-Richter distribution (with $b=1$) regardless of mainshock magnitude, and if the total number of aftershocks produced varies linearly with mainshock faulting area.

We inspect the first 30 days of $M \geq 3$ aftershocks of the 1992 $M 7.3$ Landers, California earthquake. Groups of 200 earthquakes are taken from three different distance ranges from the mainshock fault. Stress change amplitudes and resulting seismicity rate changes affecting each group are different. The ratio of the average daily seismicity rate in the first 30 days of aftershocks vs. the average since 1975 is 4,546 for 0-1 km, 649 for 1-14.4 km, and 613 for 14.3 – 43.7 km, respectively (there is little difference between 1-14.4 km and 14.3 – 43.7 km because the Joshua Tree aftershock sequence created high prior seismicity rates in the 1-14.4 km zone). Despite the differences in stress amplitude and seismicity rate changes, the decay rates of the aftershocks in the three regions lie on top of each other – their temporal distribution is the same (Kolmogorov-Smirnoff test, 95% confidence). This indicates that stress change amplitude does not affect aftershock timing, suggesting that aftershock triggering is a binary “on” or “off” process.

5. Conclusion

I propose that earthquake propagation can be physically modeled as the propagation of a crack through a heterogeneous medium, with stress loading set to the critical point. I propose that the statistical characteristics of this model is well represented by the stochastic model of Kagan and Knopoff (1981) in which earthquake propagation is modeled as a branching process of rapid aftershock triggering by tiny, equal-sized subevents. I model the subevents, or unit earthquakes as having equal area, and make several predictions from the model. These include that aftershocks should be triggered by dynamic stress changes...
(because aftershock triggering and mainshock propagation are the same process), that aftershock production should scale with mainshock faulting area, that all properties of aftershock production, besides their total number, should be independent of mainshock magnitude, and that each aftershock should have a unique mainshock. Support for all of these predictions can be found statistically in aftershock catalog data.

6. Acknowledgement

I thank Emily Brodsky, Rachel Abercrombie, Göran Ekström, Jim Rice, Thorsten Becker, Lucy Jones, Alan Felzer, Mike Harrington, and many, many others for assistance with different components of this work.

6. References

Felzer, K.R., and Brodsky E. E. (2005 submitted), Decay rate of aftershocks with distance indicates triggering by dynamic stress change