Abstract: Near real-time hazard assessment for large aftershocks post to a major earthquake is usually urgent for disaster relief and rescue efforts. For regions that are short of enough previous aftershock sequences for an appropriate empirical prior distribution, we suggest to implement a Bayesian analysis based on only one or few available previous aftershock sequences and limited current data in a short time after the mainshock. The prior information in this Bayesian analysis is obtained directly from the likelihood of previous aftershock sequence with a certain power. We illustrate the use of the power prior Bayesian analysis on the evaluation of the hazards of three aftershock sequences post to the Chi-Chi earthquake ($M_L=7.6$, 1999/9/21), the 331 event with $M_L=6.8$ on 2002/3/31 and the 1210 event with $M_L=6.4$ on 2004/12/10.

1. Introduction

Real-time information about the hazard of large aftershocks is often urgent for ongoing emergency services to reduce loss of life, especially, in a short time after a disastrous mainshock over a region near the mainshock epicenter. To fulfill this purpose, Reasenberg and Jones (1989) proposed to combine the modified Omori law (Utsu, 1961) and the frequency-magnitude relationship (Gutenberg and Richter, 1944) as a model, referred to as the RJ model hereafter, for describing the time-magnitude distribution of earthquakes after a mainshock in California.

Reasenberg and Jones (1989) considered the maximum likelihood estimated (MLE) RJ model. Due to the limited current data available within a short time after the mainshock, Reasenberg and Jones (1989) further suggested a Bayesian analysis of the RJ model with conjugate normal prior distribution. However, such a Bayesian analysis may not be appropriate as indicated in Rydelek (1990) and Chen et al. (2004) since the time and magnitude distributions of aftershocks are, in general, skewed. Therefore, Chen (2003) proposed using a Markov chain Monte Carlo (MCMC) algorithm for the Bayesian analysis for any prior distributions of the parameters in the RJ model. The MCMC Bayesian analysis of the RJ model is then expected to be better than the Bayesian analysis with conjugate normal prior if the pre-specified prior distributions are reasonable. To do so, the empirical prior distributions obtained from sufficiently large amount of previous aftershock sequences are needed.

In practice, however, there are many areas like Taiwan that have complex tectonic structures, but then do not have enough previous aftershock sequences in each tectonic region for the appropriate empirical prior distribution. Therefore, we propose a Bayesian analysis of the RJ model based on only one or few previous aftershock sequences. In the proposed Bayesian analysis, the prior information is obtained not for the establishment of the prior distribution of the parameters in the RJ model, but for the reference or contribution to the
current data through the likelihood of the available previous aftershock sequences with a certain power. Therefore, the proposed Bayesian analysis is termed as power prior Bayesian analysis hereafter (Chen et al., 2000). We demonstrate, herein, the use of the power prior Bayesian analysis on the evaluation of the hazards of three aftershock sequences and, for each of them, one historical aftershock sequence is employed for providing the prior information.

2. Power prior Bayesian analysis

Suppose that we observe the current data \( \{(t_{mi}, M_{mi}), i = 1,2,..., n_m\} \) after a major event with magnitude \( M_m \) and have a historical aftershock sequence \( \{(t_{hi}, M_{hi}), i = 1,2,..., n_h\} \) post to the mainshock with magnitude \( M_h \). Furthermore, suppose that both sequences of aftershocks can be well described by the RJ model:

\[
\lambda(t, M) = \alpha \exp\{-\beta(M - M_c)\}/(t + c)^{\rho} \quad \text{for} \quad M_c \leq M , \tag{1}
\]

where the parameters \( \alpha, \beta, c \) and \( p \) are nonnegative constants and \( M_c \) is the cut-off magnitude for both earthquake sequences. Then, following the Ogata (1983), we have the likelihood functions of the parameters given different sets of data

\[
L_u(\alpha, \beta, c, p) = L(\alpha, \beta, c, p | \{(t_{ui}, M_{ui}), i = 1,2,..., n_u\}) = \exp\{\sum_{i=1}^{n_u} \ln[\lambda(t_{ui}, M_{ui})] - \int_0^{T_u} \lambda(t_{ui}, M_c)dt\} \quad \text{for } u = m \text{ or } h . \tag{2}
\]

We consider herein the power prior information from a historical data set as \( \{L_a(\alpha, \beta, c, p)\}^a \), \( 0 \leq a \leq 1 \). Therefore, the posterior distribution of \( \alpha, \beta, c \) and \( p \) is proportional to

\[
\{L_a(\alpha, \beta, c, p)\}^a L_m(\alpha, \beta, c, p) . \tag{3}
\]

In this setting, we suggest to find the posterior modes of \( \alpha, \beta, c \) and \( p \), which correspond to the most probable values of \( \alpha, \beta, c \) and \( p \) given the current data by taking the reference from the historical data. In other words, we find the values of \( \alpha, \beta, c \) and \( p \) so that the equation in (3) reaches its maximum.

3. Data analysis

For illustrative purpose, we consider three current aftershock sequences post to the major events occurred in central (\( M=7.6, 1999/9/21, \) event 921), northeastern (\( M=6.8, 2002/3/31, \) event 331) and southeastern (\( M=6.4, 2004/12/10, \) event 1210) of Taiwan. The historical sequence as a reference for the aftershocks of the 921 event corresponds to the event with \( M_L=6.2 \) on 1998/7/17 is selected. The aftershock sequence post to the event with \( M_L=6.5 \) on 1994/6/5 is chosen as the historical data for the aftershocks of the 331 event. Finally, for the aftershock sequences post to the 1210 event, we take the historical sequence after the event with \( M_L=7.1 \) on 1996/9/5. The related epicenters are shown in Figure 1.

Note that the background colors, showing the spatial variation of the minimum magnitude for the complete earthquake catalog (Wiemer and Wyss, 2000), indicate an appropriate overall
cut-off magnitude of $M_c = 3.0$. Therefore, for simplicity, we demonstrate herein the application of the power prior Bayesian analysis of the modified Omori’s model (Utsu, 1961) for assessing the hazard of $M \geq 3.0$ aftershocks. The power under study is $a = 0.1, 0.5$ and $1$. We investigate how well the occurrence of current aftershocks can be forecasted by using the modified Omori’s law based on power prior Bayesian analysis. To do so, we computed the number of aftershocks during a week $N$ days after the current major event based on the estimated Omori’s law obtained from both the current $N$-day data post to the major event and the associated historical data. We also study if the power prior Bayesian analysis is preferred to the MLE-based analysis for the forecasting, where the forecasted number of current aftershocks is calculated based on the Omori’s law with MLE obtained from the first $N$-day current data only. The results are then presented in Figure 2.

Figure 1: The epicenters of the current and historical major earthquakes. The rectangular regions indicate the areas of the epicenters of the current aftershocks under study.

Figure 2: The forecasted number of aftershocks in next 7 days based on the first 2-day (upper three) and 10-day (lower three) current data post to the 921 event (left), 331 event (middle) and 1210 event (right). The black dots are the observed number, the black line is the forecasted one from MLE, the red, green and blue lines are the forecasted ones from the power prior Bayesian analysis with $a = 0.1, 0.5$ and $1.0$. 
4. Conclusions and Discussions

Figure 2 shows that, when the magnitude of previous mainshock is much smaller than that of the current one, like the case with 921 event, the power prior Bayesian analysis tends to give a lower aftershock hazard for using a larger power of the prior information. On the other hand, if the magnitude of previous mainshock is much greater than that of the current one, like the case with 1210 event, then the power prior Bayesian analysis produces a higher aftershock hazard for using a larger prior power. However, when the magnitudes of the current and historical major events are about the same, like the case with 331 event, the power prior Bayesian analysis gives a good forecast for the aftershock hazard in the future 2 days with a small prior power about 0.1, but, yields a better forecast for the aftershock hazard in the future days with a larger prior power. Figure 3 further indicates that, comparing to the power prior Bayesian analysis, the MLE-based Omori’s model estimated from 10-day current data gives a competitive or even better forecast for the aftershock hazard in next week.

Note that, in the power prior Bayesian analysis, the related parameters in the RJ model remain the same for both the historical and current aftershock sequences. Therefore, in the use of the power prior Bayesian analysis, the selection of a historical aftershock sequence that has a similar behavior in the time-decaying and the frequency-magnitude distribution to the current one is important. In fact, the power prior Bayesian analysis can be easily extended to the situation with more than one historical aftershock sequences.

In general, the Bayesian analysis with reasonable prior information preserves some benefit for the near-time assessment of the aftershock hazard. However, in our study, about 10 days after the current mainshock when the available current data are getting more, the MLE-based analysis becomes more competitive. For simplicity, the MLE-based analysis is then suggested.

5. Acknowledgement

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6. References

Chen, Y. I. (2003), Bayesian analysis of aftershock hazard, contributed paper in AGU Fall Meeting.