The relationship between the earthquake rate change and the gradient of stress change

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Abstract: It has been widely discussed that the seismicity rate change before and after a mainshock is related to the mainshock induced stress change. To see if it is true for the complex fault rupture of the 1999 Chi-Chi, Taiwan, earthquake, we use a universal Kriging method to explore how well the depth-dependent Coulomb stress change explains the seismicity rate change. We also make such an exploration by correlating the rate change with the magnitude of the gradient of the stress change. The results demonstrate that the gradient of the stress change is preferred to the stress change itself for describing the seismicity rate change.

1. Introduction

One problem of interest regarding to stress transfer leading to changes in the probability of earthquake occurrence (Steacy et al., 2005) is how the mainshock induced Coulomb stress change is related to the seismicity rate change. Toda et al. (1998) found that the rate change is related to the stress change induced by the 1995 Kobe, Japan, earthquake. However, how well the stress change explains the spatial variation of the rate change remains unknown. Ma et al. (2005) calculated the depth-dependent Coulomb stress change induced by the 1999 Chi-Chi, Taiwan, earthquake based on strike-slip fault models. They found that, even for the complex fault rupture of the Chi-Chi earthquake, the spatial distribution of the seismicity rate change is still consistent to the calculated Coulomb stress change. However, again, they did not investigate how well these two changes are correlated in space.

To investigate the contribution of the stress change induced by the Chi-Chi earthquake to the explanation of the associated seismicity rate change, we employ a universal Kriging method (Cressie, 1993) to correlate the stress change and seismicity rate change. Note that the stress changes calculated from heterogeneous slip fault models (Ma et al., 2005) are not only depth-dependent, but also quite different in stability over the study region. Therefore, we also measure the associated magnitude of the gradient of the stress change and then consider the spatial model for relating the gradient magnitude and seismicity rate change.

2. Spatial statistical model

In this study, we consider the non-overlapping grids in the study region. In each grid, let \( R_A \) and \( R_B \) be the rate of earthquakes after the Chi-Chi earthquake and the associated background earthquake rate, respectively. Then, the seismicity rate change in the grid is defined to be \( R_B/R_A \) and the response of interest is, in fact, \( Z(\cdot) = \log(R_B/R_A) \). For modeling
the spatial variation of the response $Z(\cdot)$, we consider a spatial process \( \{S(s): s \in D\} \), where
\[
S(s) = \mu(s) + \delta(s), \quad s \in D,
\]
where \( \mu(\cdot) \) is the overall trend related to the covariate, denoted by \( X(\cdot) \), in the way of \( \mu(s) = \beta_0 + \beta_1 X(s) \) and \( \delta(\cdot) \) is a zero-mean Gaussian process with a certain spatial-dependent covariance function. Note that, in general, the correlation of \( Z(\cdot) \) in two different locations becomes weaker when the distance of the two locations, \( h \), gets further. Therefore, we employ the covariance function suggested in Matern (1986) in the following:
\[
C_s(h) = \text{cov}(\delta(s + h), \delta(s)) = \begin{cases} \sigma^2 \frac{(a^2 || h || / 2)^\nu 2\kappa_\nu(a^2 || h ||)}{\Gamma(\nu)}; & \text{if } h \neq 0, \\ \sigma^2_\delta; & \text{if } h = 0, \end{cases}
\]
where \( \kappa_\nu(\cdot) \) is the second kind of modified Bessel function with order \( \nu > 0 \), \( \nu \) is a smoothness parameter, \( a > 0 \) is a scaling parameter, and \( \sigma^2_\delta = \text{var}(\delta(s)) \).

Note that the related covariate under study is either the mean stress change or the associated gradient’s magnitude. Denote the mean stress change in grid \((i, j)\) by \( SC_{i,j} \), the magnitude of the gradient of the related stress change is then given by
\[
MG_{i,j} = \sqrt{(SC_{i,j+1} - SC_{i,j-1})^2 + (SC_{i+1,j} - SC_{i-1,j})^2},
\]
which reflects the heterogeneity of the stress change in space. For modeling the spatial variation of the seismicity rate change, we consider the following two models:
\[
\log(R_B/R_A) = \alpha_0 + \alpha_1 \text{sgn}(SC_{ij}) \log(|SC_{ij}|) + \delta_{ij} + \epsilon_{ij}, \quad (2)
\]
where \( \text{sgn}(a) = +1 \) for \( a > 0 \), \( 0 \) for \( a = 0 \), and \(-1\) otherwise, and
\[
\log(R_B/R_A) = \beta_0 + \beta_1 \log(MG_{ij}) + \delta_{ij} + \epsilon_{ij}, \quad (3)
\]
where the errors \( \epsilon_{ij} \)'s are independent normal white noises with variance \( \sigma^2_\epsilon \). Since we employ the universal Kriging method (Cressie, 1993) and find the maximum likelihood estimates (MLE) of the parameters in the two spatial models:

3. Data analysis and results
To correlate the seismicity rate change to the stress change induced by the 1999, \( M=7.6 \), Chi-Chi, Taiwan, earthquake, we consider 900 (=30x30) grids over the region \((119.5^\circ E, 122.5^\circ E)\times(22.5^\circ N, 25.5^\circ N)\) and the area of each grid is \(0.1^\circ x0.1^\circ\). The \( M \geq 3.0 \) earthquakes 50 months before the Chi-Chi earthquake provide the background and the \( M \geq 3.0 \) earthquakes 50 months after the Chi-Chi earthquake are under study. Since the hyper center of the Chi-Chi earthquake is at 8 km, the seismicity rate changes, the mean stress changes and the associated magnitudes of gradients for the layer with depth 3-9km are computed and presented in Figure 1.
Applying the universal Kriging method, we have the fitted model (2) as given by
\[ \mu = 0.088 - 0.004 \text{sgn}(SC)\log(|SC|) \] and \[ (\nu, a, \sigma^2_\nu, \sigma^2_\epsilon) = (0.227, 0.579, 0.453, 0.001), \] while the fitted model (3) is obtained as
\[ \mu = 0.511 + 0.607 \log(MG) \] and \[ (\nu, a, \sigma^2_\nu, \sigma^2_\epsilon) = (1.628, 0.044, 0.290, 0.000). \]
Since the fitted model (4) showing a negative correlation between the stress change and seismicity rate change is contradiction to the general knowledge, we only present, in Figure 2, the overall trend and the unexplained spatial variation in the fitted model (5).

![Figure 1](image1.png)

**Figure 1.** The seismicity rate change (left), the mean stress change (middle) and the magnitude of the associated gradient (right) for the Chi-Chi earthquake.

![Figure 2](image2.png)

**Figure 2.** The fitted overall trend (left) and spatial variation (right) for the model with the magnitude of the gradient of the stress change as the covariate.

4. Conclusions and Discussions

The fitted model (4) implies that a grid with a larger positive stress change has a more serious decrease in the seismicity rate post to the Chi-Chi earthquake. Meanwhile, a grid with a smaller negative stress change or stress shadow has a more increase in the
seismicity rate after the Chi-Chi earthquake. This result is somehow against our
knowledge about the relationship between the stress change and the seismicity rate change.
On the other hand, the fitted model (5) shows that a grid located in the area with more
heterogeneous stress change in space tends to have a larger seismicity rate post to the
Chi-Chi earthquake.

Figure 2 indicates that the overall trend that counts on the stability of the stress
trend contributes about 50% of the variation of seismicity rate change. Therefore, the
result sheds light on the study of earthquake probability changes by taking into account of
the homogeneity of the stress change in space. Note that, we also find, though not shown
here, the unexplained spatial variation is irrelevant to b value (Gutenberg and Richter,
1944) change or the p value (Utsu, 1961; Ogata, 1983). Therefore, it remains a problem
in a future study to explore something else that might be related to the seismicity rate
change.

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6. References
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