Abstract: We perform analytical and numerical studies of aftershock sequences following abrupt steps of strain in a rheologically-layered model of the lithosphere. The model consists of a weak sedimentary layer, over a seismogenic zone governed by a visco-elastic damage rheology, underlain by a visco-elastic upper mantle. The damage rheology accounts for fundamental irreversible aspects of brittle rock deformation and is constrained by laboratory data of fracture and friction experiments. A 1-D version of the visco-elastic damage rheology leads to an exponential analytical solution for aftershock rates. The corresponding solution for a 3-D volume is expected to be sum of exponentials. The exponential solution depends primarily on a material parameter $R$ given by the ratio of timescale for damage increase to timescale for gradual inelastic deformation, and to a lesser extent on the initial damage and a threshold strain-state for material degradation. The parameter $R$ is also inversely proportional to the degree of seismic coupling across the fault. Simplifying the governing equations leads to a solution following the modified Omori power law decay with an analytical exponent $p = 1$. In addition, the results associated with the general exponential expression can be fitted for various values of $R$ with the modified Omori law. The same holds for the decay rates of aftershocks simulated numerically using the 3-D layered lithospheric model. The results indicate that low $R$-values (e.g., $R \leq 1$) corresponding to cold brittle material produce long Omori-type aftershock sequences with high event productivity, while high $R$-values (e.g., $R \geq 5$) corresponding to hot viscous material produce short diffuse response with low event productivity. The frequency-size statistics of aftershocks simulated in 3-D cases with low $R$-values follow the Gutenberg-Richter power law relation, while events simulated for high $R$-values are concentrated in a narrow magnitude range. Increasing thickness of the weak sedimentary cover produces results that are similar to those associated with higher $R$-values. Increasing the assumed geothermal gradient reduces the depth extent of the simulated earthquakes. The magnitude of the largest simulated aftershocks is compatible with the Båth law for a range of dynamic damage-weakening parameter. The results provide a physical basis for interpreting the main observed features of aftershock sequences in terms of basic structural and material properties.

1. Introduction

Large earthquakes typically produce an increasing number of earthquakes referred to as aftershocks, with decay rates that can be described (e.g., Utsu et al., 1995) by the modified Omori law

$$\frac{\Delta N}{\Delta t} = K(c + t)^{-p},$$

where $N$ is the cumulative number of events with magnitude larger than a prescribed cutoff, $t$ is the time after the mainshock, and $K$, $c$, and $p$ are empirical constants. However, aftershock decay rates can also be fitted (e.g., Gross and Kisslinger; 1994; Narteau et al., 2002) with exponential and other functions. The occurrence and properties of aftershocks exhibit a number of spatio-temporal variations that are unlikely to be associated with statistical fluctuations. The depth extent of aftershocks is correlated with the geothermal gradient (Magistrale, 2002) and the early parts of aftershock sequences may extend deeper than the regular ongoing seismicity (Rolandone et al., 2004). Cold continental regions with low heat flow have high aftershock productively and long event sequences associated, while hot continental regions and oceanic lithosphere have low aftershock productively and short event sequences with fast decay (e.g., Kisslinger and Jones, 1991; Utsu et al., 1995; Utsu, 2002; McGuire et al., 2005). Geothermal areas and volcanic regions with high fluid (magma, water) activity and high heat flow often have swarms of events without a clear separation between mainshocks and aftershocks (Mogi, 1967). In this work we attempt to establish a physical basis for aftershocks behavior using a lithospheric model with a seismogenic crust governed by a continuum damage rheology that accounts for key observed features of irreversible rock deformation (Lyakhovsky et al., 1997; Hamiel et al., 2004).

2. Results

A detailed background on the employed damage rheology and full derivation of the results are given by Ben-Zion and Lyakhovsky (2006). Here we outline the main analytical results for a 1-D version
of the model, and show illustrative examples associated with the analytical results and 3-D numerical simulations. For a 1-D version of the model, associated with uniform damage evolution, the stress-strain relation is

\[ \sigma = 2\mu_0(1-\alpha)\varepsilon, \]  

where \( \mu_0(1-\alpha) \) is the effective elastic modulus with \( \mu_0 \) being the initial rigidity of the undamaged solid.

The 1-D version of the kinetic equation for damage evolution is

\[ \dot{\alpha} = C_d(\varepsilon^2 - \varepsilon_0^2), \]  

where \( \varepsilon_0 \) is a threshold value separating states of strain associated with material degradation and healing and \( C_d \) is a rate constant for material degradation. For positive damage evolution (\( \varepsilon > \varepsilon_0 \)), the damage-related inelastic strain rate before the final macroscopic failure is given by

\[ e = C_v \dot{\alpha} \sigma, \]  

where \( C_v \cdot d\alpha/dt \) is the inverse of an effective damage-related viscosity.

Since we are interested in the relaxation process in a region following the occurrence of a large event, we assume a constant total strain boundary condition. This implies that the rate of elastic strain relaxation is equal to the viscous strain rate, i.e., \( 2\cdot\dot{\varepsilon} = -e \) (the factor 2 stems from the common definitions of the strain and strain-rate tensors). With this set of equations and boundary conditions, the result for damage evolution is

\[ \frac{d\alpha}{dt} = C_d \cdot \{ \varepsilon_s^2 \exp[R(1-\alpha)^2 - R(1-\alpha_s)^2] - \varepsilon_0^2 \}, \]  

where \( \alpha_s \) and \( \varepsilon_s \) are the initial levels of damage and strain, respectively, and \( R = \mu_0 \cdot C_v \) is a material parameter. As demonstrated below analytically and numerically, the parameter \( R \) plays a dominant role in the aftershock behavior. Ben-Zion and Lyakhovsky (2006) show that \( R \) characterizes the ratio between a brittle damage timescale and a viscous relaxation timescale, and that \( R \) is inversely proportional to the degree of seismic coupling across the fault.

To convert the evolution of the damage state variable \( \alpha \) to evolution of aftershocks number \( N \), we assume that \( \alpha \) increases linearly with aftershocks number, \( \alpha = \alpha_s + \phi N \). We thus get

\[ \phi \frac{dN}{dt} = C_d \cdot \{ \varepsilon_s^2 \exp[R(1-\alpha_s-\phi N)^2 - R(1-\alpha_s)^2] - \varepsilon_0^2 \}. \]  

The corresponding solution for a 3-D volume with many interacting elements is expected to be associated with a sum of exponentials similar to (6). Figure 1 illustrates results based on equation (6) for several values of \( R \) and the simplifying limit values \( \alpha_s = 0 \) and \( \varepsilon_0 = 0 \). Small \( R \)-values, corresponding to highly brittle cases with little viscous relaxation, produce long aftershock sequences with slow decay and high aftershock productivity. In contrast, high \( R \)-values, corresponding to relatively viscous cases, produce short aftershock sequences with fast decay and low aftershock productivity. While the aftershock rates are generated using the exponential equation (6), the results can be fitted well by the modified Omori power law relation \( K(c+t)^p \).

The obtained good fits to the results with the modified Omori law suggest that the leading term of equation (6) is associated with a power law. This can be derived explicitly by making 2 simplifications. Assuming that \( \phi N \) is sufficiently small so that the \( (\phi N)^2 \) term in (6) can be neglected, gives

\[ \phi \frac{dN}{dt} = C_d \cdot \{ \varepsilon_s^2 \exp[-2\phi NR(1-\alpha_s)] - \varepsilon_0^2 \}. \]  

The obtained good fits to the results with the modified Omori law suggest that the leading term of equation (6) is associated with a power law. This can be derived explicitly by making 2 simplifications.
If we assume further that the initial strain $\varepsilon_s$ induced by the mainshock is large enough so that $\varepsilon_0^2 \ll \varepsilon_s^2 \exp[-2\phi R (1-\alpha_s)]$, we get a decay rate following the modified Omori law

$$\frac{dN}{dt} = \frac{\dot{N}_0}{2\phi R (1-\alpha_s) N_0 t + 1} = \frac{\dot{N}_0}{2\phi R (1-\alpha_s) N_0} \cdot \frac{1}{t + 1/2\phi R (1-\alpha_s) N_0},$$

where $\dot{N}_0 = C_d \varepsilon_s^2 / \phi$ is the initial aftershocks rate. Equation (8a) provides a mapping between parameters of the modified Omori law and parameters of our damage rheology model (in a 1-D analysis of uniform deformation),

$$k = \frac{1}{2\phi R (1-\alpha_s)}, \quad c = \frac{1}{2\phi R (1-\alpha_s) \dot{N}_0} = \frac{k}{\dot{N}_0}, \quad p = 1.$$  

Numerical simulations in a 3-D lithospheric structure confirm that the material parameter $R$ is the major factor controlling the decay rate, duration and productivity of aftershocks, as well their frequency-size statistics. The simulations also show that a sedimentary layer thicker than a few km produces weaker aftershock sequences with a smaller number of events and faster decay, similar to what is produced by high $R$-values. Increasing the assumed temperature gradient leads to a smaller number of events, but the aftershock decay rate and duration remain largely unaffected (if the $R$-value is kept the same). However, increasing temperature gradient reduces the overall depth extent of the brittle seismogenic zone and the simulated aftershocks. In addition, the maximum hypocenter depth decreases with time from the mainshock due to a transient deepening of the brittle-ductile transition depth produced by the high strain rates generated by the mainshock (Figure 2). These results are compatible with observed correlations between the depth of seismicity and temperature gradient (Magistrale, 2002), and temporal evolution of the depth of aftershocks following the 1992 Landers CA earthquake (Rolandone et al., 2004).

References


Events rate for several values of $R = \frac{\text{Timescale of fracturing}}{\text{Timescale of stress relaxation}}$

Small $R$:
- Expect long active aftershock sequences

Large $R$:
- Expect short diffuse sequences

Modified Omori law with $p = 1$

Fig. 1. Analytical results based on equation (6) for aftershock decay rates in a 1-D version of the damage model for different $R$-values. The dotted line is least-squares fit to the exponential analytical solution with the modified Omori power law.

Fig. 2. Depth of aftershock hypocenters in a 3-D model realization with $R = 0.1$, sedimentary cover of 1 km, and temperature gradient of 20 °C/km. Increasing thermal gradient and/or $R$ lead to a thinner seismogenic zone. The maximum event depth decreases with time from the mainshock.