A Model for Scheduling High-Cadence Telescope Observations

João Pedro Pedroso

INESCTEC and Faculty of Sciences, University of Porto

Joint work with:

- Shiro Ikeda, ISM, Tokyo
- Tomoki Morokuma, The University of Tokyo
- Shigeyuki Sako, The University of Tokyo

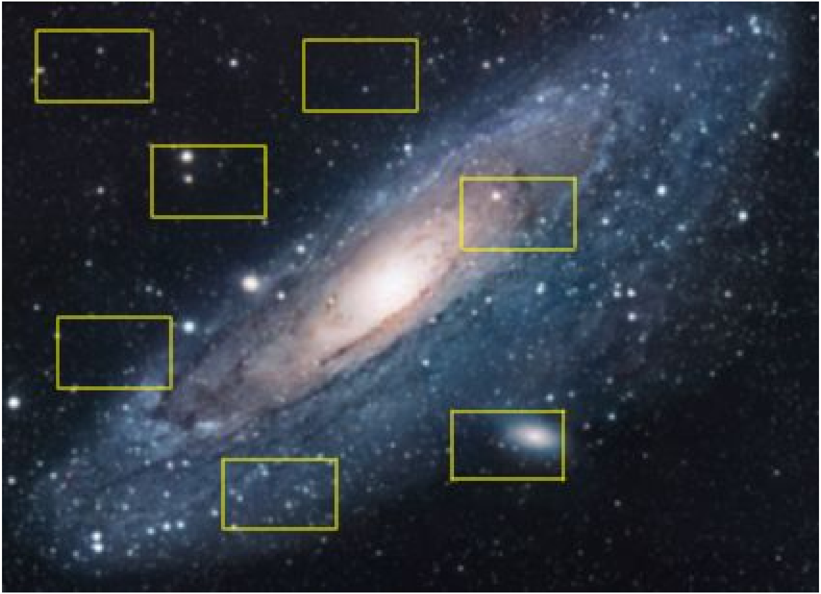
4th ISM-ZIB-IMI MODAL Workshop, ISM, Tokyo, 2019-03-29

The problem

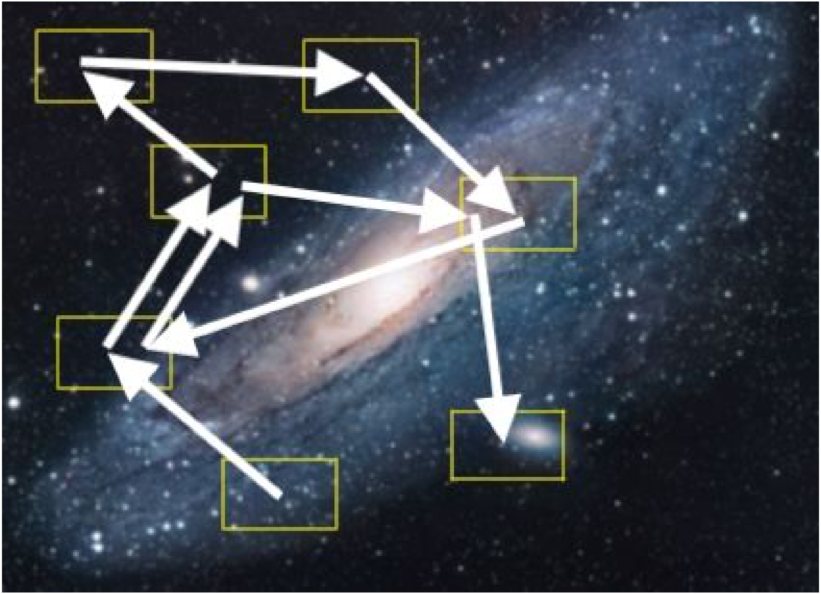
Situation (my understanding):

- Telescope used for detecting supernovae right after explosions
  - rapid increase in observed flux, requiring multiple observations during a night
- Strategy:
  - take successive images of a given zone
  - check for differences between them
- In this context:
  - try to observe the whole visible celestial sphere
  - repeat some time later
  - there must be a minimum delay between successive images
  - aim: maximize the number of observations made
  - in other words, minimize the time lost
    - telescope movements
    - waiting time

Background



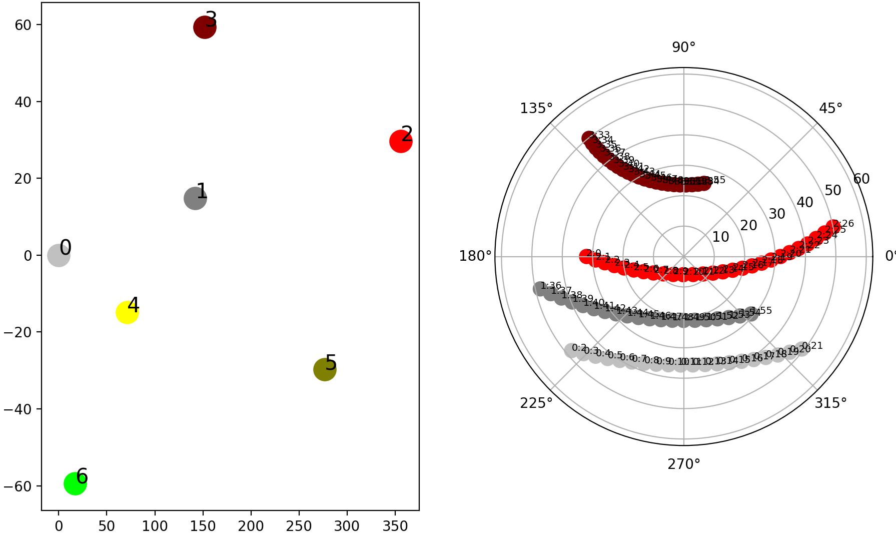
Background



The problem

- There is a set of positions to be observed in the sky
- Each of them can be observed on a given configuration of the telescope
- We want to minimize unproductive time
- Difficulty: sky "moves" during the night
  - setup between two telescope positions is time-dependent

Figure



60
40
20
0
−20
−40
−60
0
50
100
150
200
250
300
350
0
1
2
3
4
5
6
90°
45°
135°
180°
0°
225°
270°
315°
10
20
30
40
50
60

An optimization model

An optimization model

$$
\begin{aligned}
\text{maximize} \quad & \sum_{k \in K} z_k \\
\text{subject to} \quad & \sum_{i \in I} x_{it} \leq 1 && \text{for } t = 0, \dots, T \\
& x_{i,t-1} = \sum_{j \in I} w_{ijt} && \forall i \in I, t = 1, \dots, T \\
& x_{jt} = \sum_{i \in I : t - c_{ij} > 0} w_{ij,t-c_{ij}} && \forall j \in I, t = 1, \dots, T \\
& y_{k0} = 0 && \forall k \in K \\
& y_{kt} \leq \sum_{i \in I} a_{ikt} x_{it} && \forall k \in K, t = 1, \dots, T \\
& \sum_{t'=t}^{\min(T, t+d_k)} y_{kt'} \geq d_k (y_{kt} - y_{k,t-1}) && \forall k \in K, t = 1, \dots, T \\
& z_k \leq \sum_{t=1}^{T} y_{kt} && \forall k \in K \\
& \text{(all variables are binary)}
\end{aligned}
$$

Data

- $K \rightarrow$ set of positions to be observed in the sky
- $I \rightarrow$ set of positions in the telescope
- $T \rightarrow$ number or periods to consider (time discretization)
- $a_{ikt} \rightarrow$ connect telescope and sky's positions:
  - $a_{ikt} = 1$ if at period t telescope in position $i \in I$ observes sky's position $k \in K$
  - $a_{ikt} = 0$ otherwise
- $c_{ij} \rightarrow$ time necessary to move the telescope from position i to j
- $d_k \rightarrow$ time necessary to make observation at sky's position k

Variables

- Main decision variables:
  - $x_{it} = 1$ if telescope is on position i at period t
  - $x_{it} = 0$ otherwise
- Telescope movement:
  - $w_{ijt} = 1$ if at period t telescope moves from position i to position j (possibly, $j = i$)
- Observed: (determined in terms of x)
  - $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- Positions observed: (determined in terms of y)
  - $z_k = 1$ if sky's position k has been observed

Constraints (#1)

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed

At each period, telescope is (at most) in one position

$$\sum_{i \in I} x_{it} \leq 1 \qquad \text{for } t = 0, \ldots, T$$

Constraints (#2)

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed

If the telescope was in position i at $t-1$, then at t it must move to some (possibly the same) position

$$x_{i,t-1} = \sum_{j \in I} w_{ijt} \qquad \forall i \in I, t = 1, \ldots, T$$

- if $x_{i,t-1} = 1$, then one of the w_{ijt} must be non-zero

Constraints (#3)

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed

For being in position j at period t, the telescope must have been in a position i (possibly the same) early enough to move to j

$$x_{jt} = \sum_{i \in I : t - c_{ij} > 0} w_{ij,t-c_{ij}} \qquad \forall j \in I, t = 1, \ldots, T$$

Constraints (#4)

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed

No observations can be made at $t = 0$

$$y_{k0} = 0 \qquad \forall k \in K$$

Constraints (#5)

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed
- $a_{ikt} \rightarrow 1$ if at period t telescope in position $i \in I$ observes sky's position $k \in K$

Observing sky's position k at period t is only possible if the telescope is in a position from which k can be observed

$$y_{kt} \leq \sum_{i \in I} a_{ikt} x_{it} \qquad \forall k \in K, t = 1, \ldots, T$$

Constraints (#6)

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed
- $d_k \rightarrow$ time necessary to make observation at sky's position k

If an observation at point k has started in period t, then the same position must be observed at least d_k successive periods

$$\sum_{t'=t}^{\min(T, t+d_k)} y_{kt'} \geq d_k (y_{kt} - y_{k,t-1}) \qquad \forall k \in K, t = 1, \ldots, T$$

- observing point k starts in period t iff $y_{k,t-1} = 0$ and $y_{kt} = 1$
- in that case, the right-hand side is positive
- otherwise, the constraint becomes redundant

Constraints (#7)

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed

A position is counted in the objective only if it was observed at some valid period

$$z_k \leq \sum_{t=1}^{T} y_{kt} \qquad \forall k \in K$$

Objective

- $x_{it} = 1$ if telescope is on position i at period t
- $w_{ijt} = 1$ if at period t telescope moves from position i to position j
- $y_{kt} = 1$ if sky's position k is observed at period t, 0 otherwise
- $z_k = 1$ if sky's position k has been observed

Objective: maximize the number of positions observed:

$$\text{maximize} \sum_{k \in K} z_k$$

Refinements: second-time observations

- What happens if all the positions can be observed?
- We should take into account second-time observations
  - also third-time, fourth-time, ...
- Additional variables:
  - $y'_{kt} = 1$ if position k is observed for the second time at some period t
  - $y'_{kt} = 0$ otherwise

Refinements: second-time observations

- A minimum number of periods (Δ) must elapse since the first observation
- In other words: y'_{ks} *must be zero for* Δ *periods after period* t *at which* y_{kt} *changed from 1 to 0*
- Additional constraints ($\forall k \in K, t = 1, \ldots, T$):

$$\begin{aligned} y'_{kt} &\leq 1 - (y_{k,t-1} - y_{kt}) \\ y'_{k,t+1} &\leq 1 - (y_{k,t-1} - y_{kt}) \\ &\ldots \\ y'_{k,t+\Delta} &\leq 1 - (y_{k,t-1} - y_{kt}) \end{aligned}$$

- A new variable z'_k is needed for counting the number of second-time observations (as with z_k)

Refinements: goal/hierarchical programming

- Previous model: a solution may have some nodes observed several times, and some other nodes not observed at all
- An improvement is solving successively:
  - find $Z = \max \sum_{k \in K} z_k$
  - find $Z' = \max \sum_{k \in K} z'_k$ subject to $\sum_{k \in K} z_k = Z$
  - ...
  - find $Z^{(n)} = \max \sum_{k \in K} z^{(n)}$ subject to $\sum_{k \in K} z^{(n-1)} = Z^{(n-1)}, \ldots, \sum_{k \in K} z' = Z'$
- This ensures a homogeneous number of observations to all sky positions

Issues

- The previous model is good, but...
- Is it acceptable in practice?

Issues

- The previous model is good, but...
- Is it acceptable in practice?
- For the telescope at Kiso observatory:
  - sky positions: $> 300 \rightarrow \sim 100000$ arc variables
  - time discretization:
    - each image: $\sim$ 48 seconds
    - each movement: from a few seconds to $\sim$ 1 minute
- If we discretize to 1 second: > 4000 million variables...

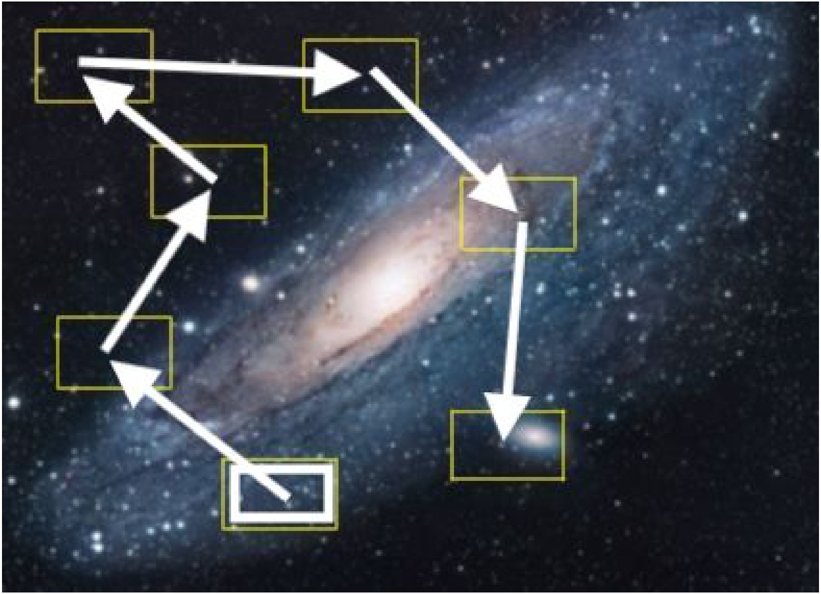
Practical approach # 1

Practical approach # 1

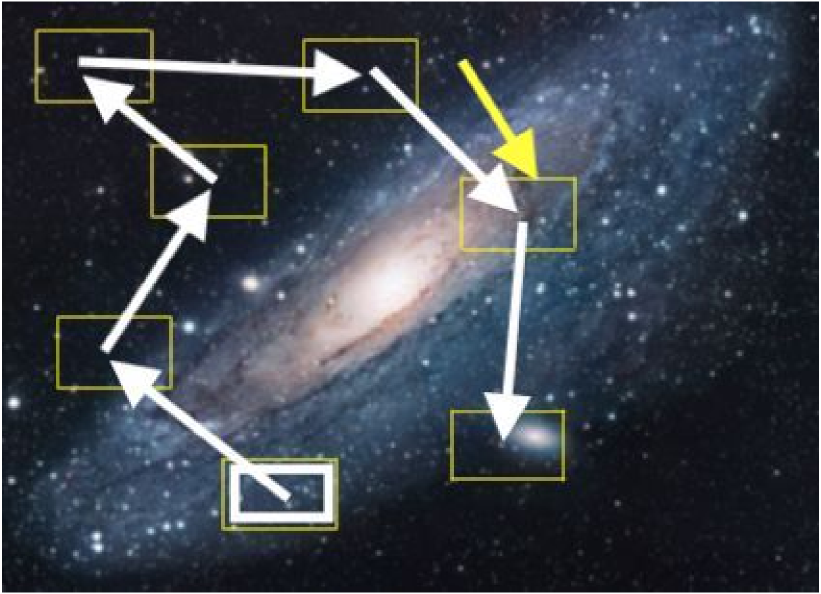
For dealing with the practical problem:

- approximate dynamics of the movement between two celestial positions:
  - consider movement times at present
  - obtain the "optimal" sequence
    - variant of the traveling salesman problem
  - check if there was a significant error
    - if so, recompute the movement times, solve another TSP

Background



Background



Algorithm

1. for the current time, get list of available sky positions
2. estimate the cost of moving between each pair of them, at the current time, in the telescope
3. schedule them using a TSP model
4. for each observation in this "optimal" sequence:
    4.1 "simulate" it, advancing the simulation clock and calculating the exact delay for movement
    4.2 if the discrepancy between this delay and the corresponsind movement time considered in the TSP is less than, say, 1 second:
        - commit this observation
        - go to the next observation in the TSP solution
    4.3 else:
        - discard the current observation and break this cycle
5. update time and repeat from step 1, while sky conditions allow

Practical approach # 2

Practical approach # 2

- Motivation: as we cannot afford much detail on future data, concentrate on the next movement
- Very simple idea: use a *nearest-neighbor approach*
- Well known heuristic method for the TSP

Algorithm: nearest-neighbor

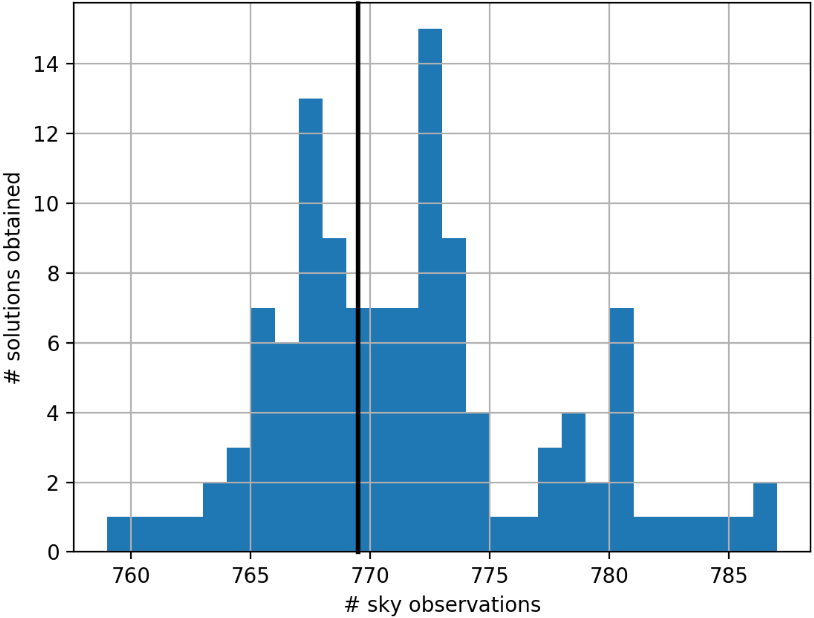
Contruction procedure:

- set the initial distance as zero to all visible points
- repeat:
  - move to closest point
  - update time with movement + exposure durations
  - update set of "visitable" points
    - visible and with minimum delay from previous observation
  - determine distance from current point to all visitable

These solution constructions can be repeated:

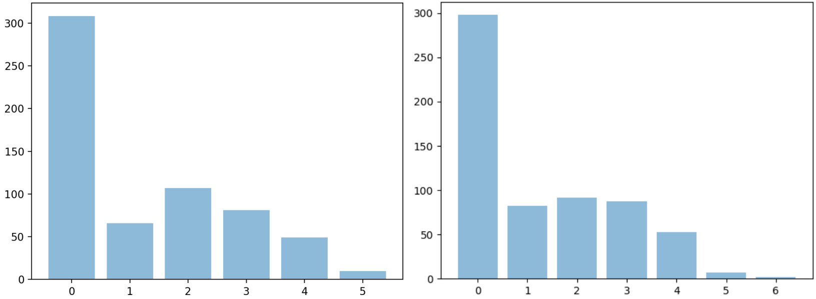
- choose all different starting points
- for each of them, construct a solution starting from thene
- at the end, choose the best

Histogram: # observations with nearest-neighbor



Analysis

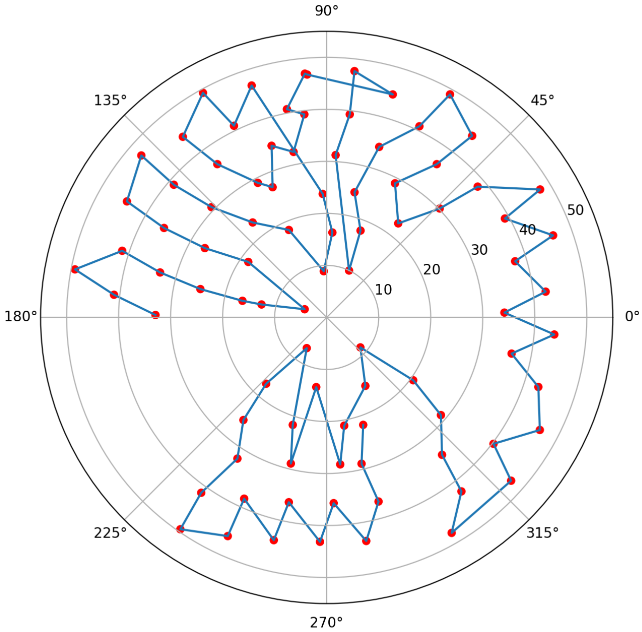
TSP-based and Nearest-Neighbor heuristics



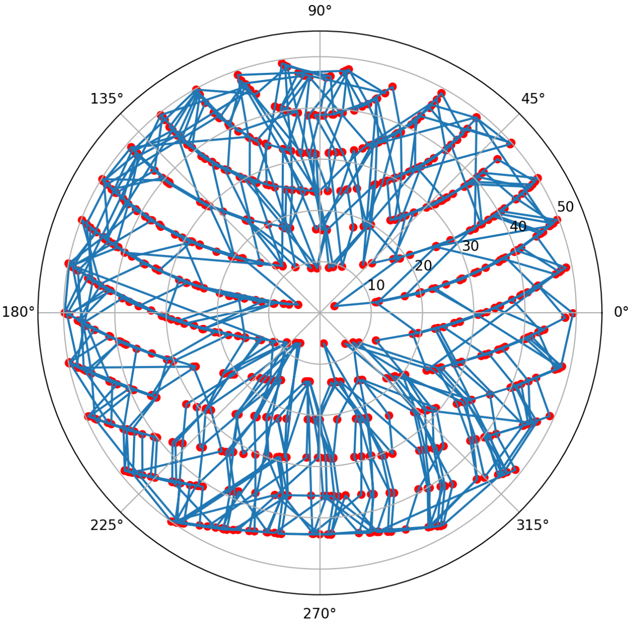
Comparison:

- Different points observed
  - TSP-model heuristic: 313 / 621 points
  - (Best) nearest-neighbor heuristic: 323 / 621 points
- Total points observed (including multiple visits)
  - TSP-model heuristic: 769 points
  - (Best) nearest-neighbor heuristic: 787 points

Initial part of the solution



Full solution



Further issues

- Real time data:
  - weather conditions: clouds may obstruct observation
  - use whole sky image analysis to select observable points
  - also, forecast future positions
  - → further advantage to nearest-neighbor...
- Bounds:
  - can we use optimization model to compute bounds?
  - → determine *minimum time* between any two sky positions
- "Expected image interest":
  - can we somehow estimate how much new information a new image will bring about?
  - objective: maximize "total interest" of images collected
  - possibly, some advantage for a mathematical model here

In summary

- First attempt to model/solve telescope scheduling
- Ongoing work, no definitive results yet
- Methods:
  1. Telescope scheduling as a mathematical optimization problem
  2. Heuristic methods:
     - based on the TSP model
     - nearest-neighbor
- Future work:
  - online version (image processing)
  - extend to different objectives