Optimal Operation of Macroscopic Gas Transport Networks Over Time

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ZIB

MODAL GasLab – Zuse Institute Berlin

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Gas Physics and Transport à la Hoppmann

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- Gas is inserted at entries and withdrawn at exits

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- Gas can be stored in pipelines
- Gas flows from high pressure to low pressure
- Pressure loss while flowing through a pipe mainly due to friction
- Compressors can increase pressure
- Regulators can decrease pressure
- Valves can change network topology

The NAVI - Problem Description

Project Goal

- Short-term transient gas network operation of large-scale real-world networks
- "Navigation system" for dispatchers

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Problem

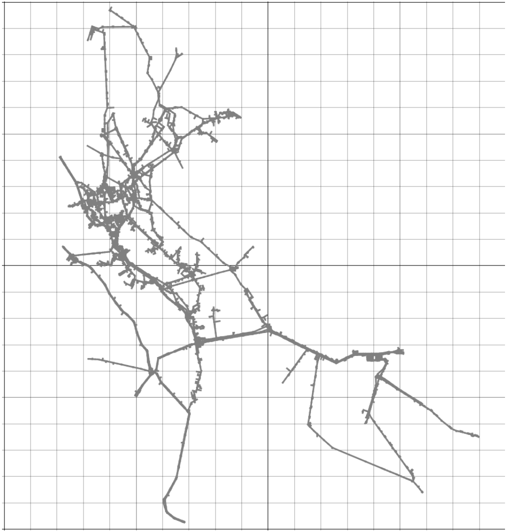
Given

- Network topology
- Initial network state
- Short-term supply/demand situation, e.g. 12–24 hours

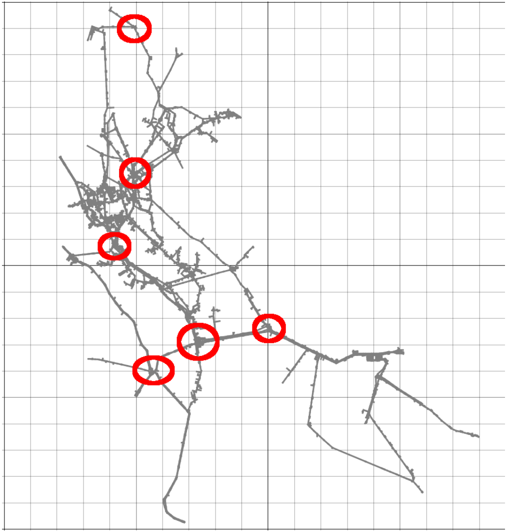
Goal

- Control each element s.t. the network is operated "best" (What does best mean?)

Sample Gas Grid



Sample Gas Grid - Navi Stations



Basic Algorithmic Framework

1. Simplify navi stations

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4. Result: For the boundaries of the navi stations
   - Pressure values for all timesteps
   - Flow values for all timesteps

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3. Solve transient operation problem using linearized gas flow equations (Netmodel-Algorithm)
4. Result: For the boundaries of the navi stations
   - Pressure values for all timesteps
   - Flow values for all timesteps
5. Solve transient operation problem for original navi stations

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Pipes

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Gasflow in a pipe (u, v) between timesteps t_i and t_{i+1} can be described by

$$
\begin{aligned}
\frac{p_{u,t_{i+1}} + p_{v,t_{i+1}}}{2} - \frac{p_{u,t_i} + p_{v,t_i}}{2} + \frac{R_s\, T\, z\, \Delta t}{L\, A} \left(q_{v,t_{i+1}} - q_{u,t_{i+1}}\right) &= 0 \\
\frac{\lambda\, R_s\, T\, z\, L}{4\, A^2\, D} \left(\frac{|q_{u,t_i}|\, q_{u,t_i}}{p_{u,t_i}} + \frac{|q_{v,t_i}|\, q_{v,t_i}}{p_{v,t_i}} \right) & \\
+ \frac{g\, s\, L}{2\, R_s\, T\, z} \left(p_{u,t_i} + p_{v,t_i}\right) + p_{v,t_i} - p_{u,t_i} &= 0
\end{aligned}
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$$\frac{\lambda\, R_s\, T\, z\, L}{4\, A^2\, D} \left(\frac{|q_{u,t_i}|\, q_{u,t_i}}{p_{u,t_i}} + \frac{|q_{v,t_i}|\, q_{v,t_i}}{p_{v,t_i}} \right)$$

$$+ \frac{g\, s\, L}{2\, R_s\, T\, z} \left(p_{u,t_i} + p_{v,t_i}\right) + p_{v,t_i} - p_{u,t_i} = 0$$

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Fixing absolute velocity:

$$\frac{\lambda\,L}{4\,A\,D}\left(|v_{u,0}|\,q_{u,t} + |v_{v,0}|\,q_{v,t}\right) + \frac{g\,s\,L}{2\,R_s\,T\,z}\left(p_{u,t_i} + p_{v,t_i}\right) + p_{v,t_i} - p_{u,t_i} = 0$$

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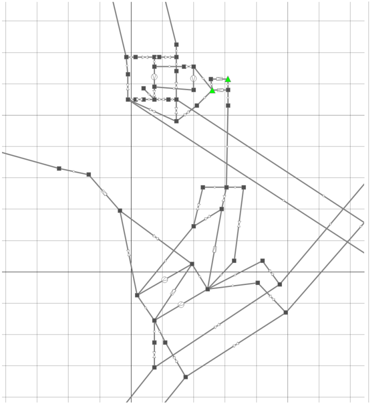
Simplifying Navi Stations

- Navi stations are bounded by fence nodes



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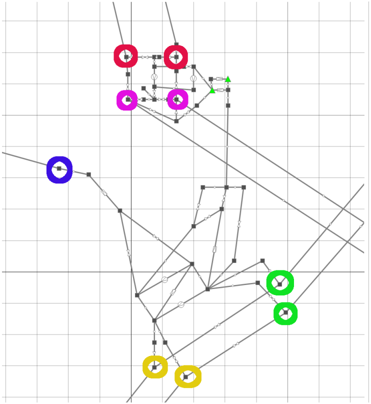

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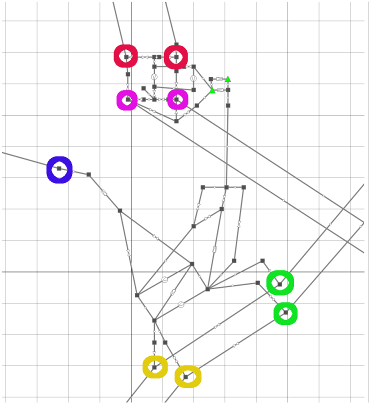

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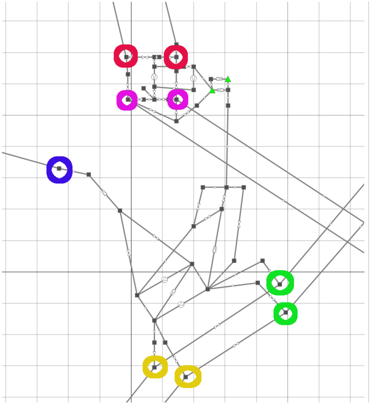

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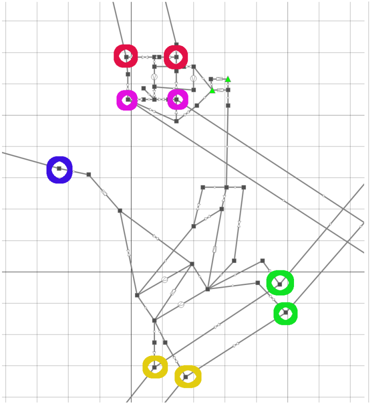

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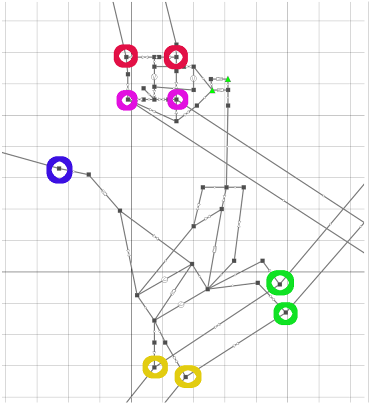

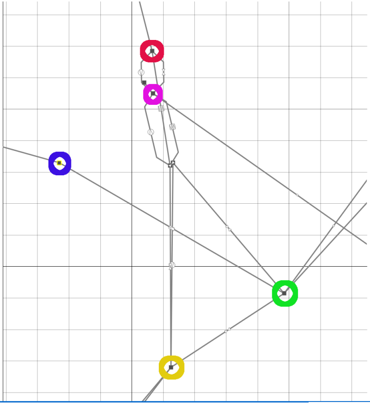

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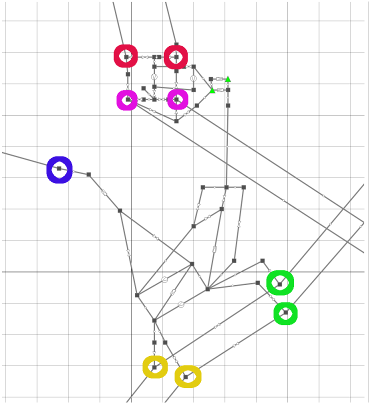

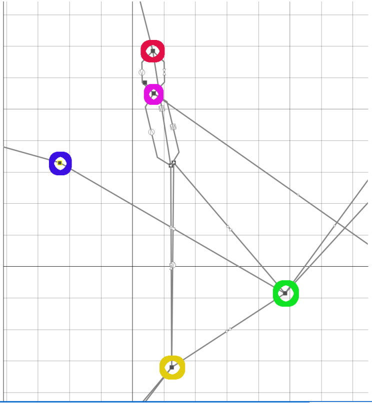

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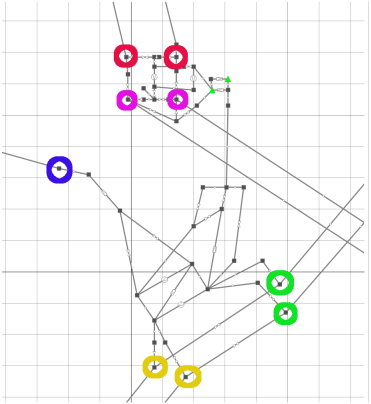

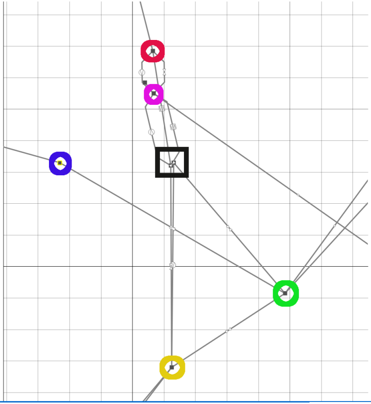

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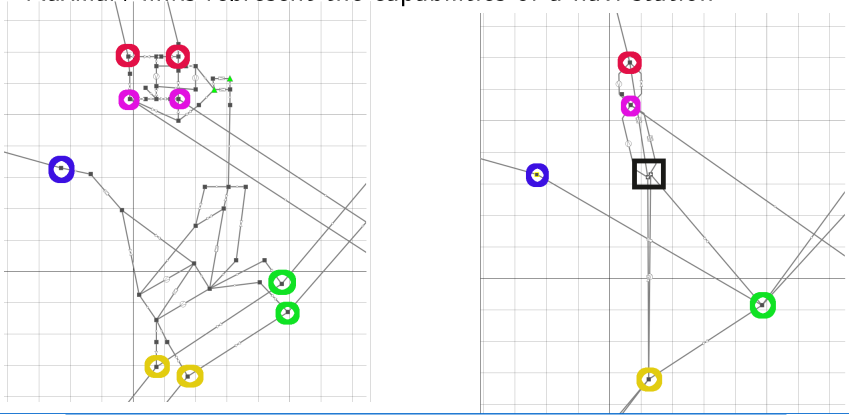
- Auxiliary links represent the capabilities of a navi station

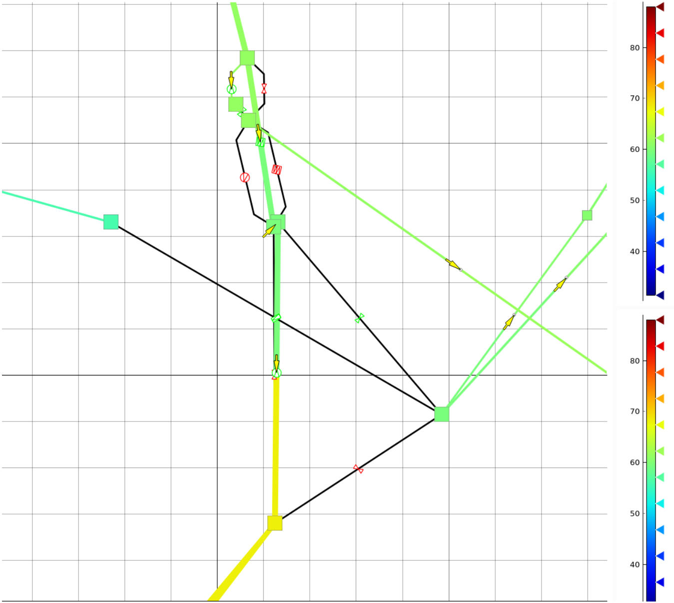


Flow Directions and Simple States

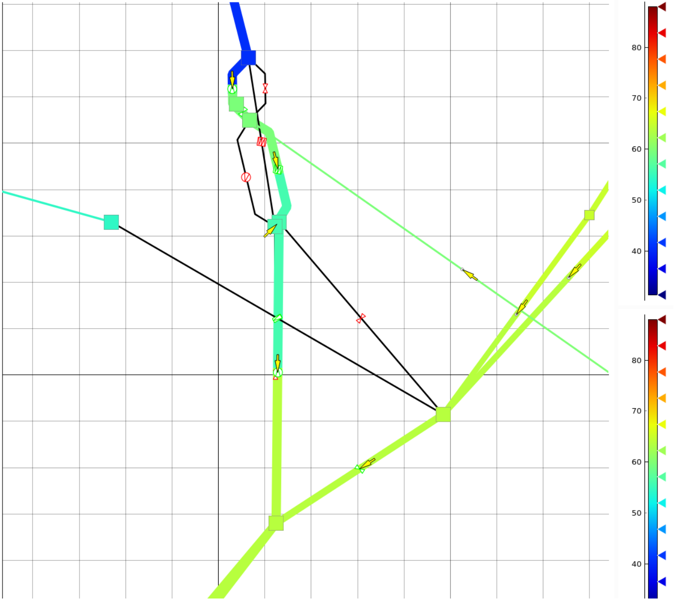
For each navi station (V, A) we are given

- Flow directions $\mathcal{F} \subseteq \mathcal{P}(V) \times \mathcal{P}(V)$ with $f = (f^+, f^-) \in \mathcal{F}$
- Simple states $\mathcal{S} \subseteq \mathcal{P}(\mathcal{F}) \times \mathcal{P}(A) \times \mathcal{P}(A)$ with $s = (s_f, s_a^{on}, s_a^{off}) \in \mathcal{S}$

Example I



Example II



Flow Directions and Simple States

For each navi station (V, A) we are given

- Flow directions $\mathcal{F} \subseteq \mathcal{P}(V) \times \mathcal{P}(V_i)$ (example: (f^+, f^-))
- Simple states $\mathcal{S} \subseteq \mathcal{P}(\mathcal{F}) \times \mathcal{P}(A) \times \mathcal{P}(A)$ (example: $(s_f, s_a^{on}, s_a^{off})$)
- $x_{f,t} \in \{0,1\}$ for flow direction $f \in \mathcal{F}$ and time $t \in T$
- $x_{s,t} \in \{0,1\}$ for simple state $s \in \mathcal{S}$ and time $t \in T$
- $x_{a,t} \in \{0,1\}$ for auxiliary arc $a \in A$ and time $t \in T$

$$
\begin{aligned}
\sum\nolimits_{f \in \mathcal{F}} x_{f,t} &= 1 && \forall t \in T \\
\sum\nolimits_{f \in s_f} x_{f,t} &\geq x_{s,t} && \forall s \in \mathcal{S}, \forall t \in T \\
\sum\nolimits_{s \in \mathcal{S}} x_{s,t} &= 1 && \forall t \in T \\
x_{s,t} &\leq x_{a,t} && \forall s \in S, \forall a \in s_a^{on}, \forall t \in T \\
1 - x_{s,t} &\geq x_{a,t} && \forall s \in S, \forall a \in s_a^{off}, \forall t \in T
\end{aligned}
$$

... flow direction constraints ...

Shortcuts

For a shortcut $a = (u, v)$ and each $t \in T$:

Not Active ($x_{a,t} = 0$):

- Decoupled pressure values
- No flow allowed

Active ($x_{a,t} = 1$):

- Coupled pressure values
- Bidirectional flow up to an amount of $\overline{q}_a$ (Big-M).

$$
\begin{aligned}
p_{u,t} - p_{v,t} &\leq (1 - x_{a,t})(\overline{p}_v - \underline{p}_u) \\
p_{u,t} - p_{v,t} &\geq (1 - x_{a,t})(\underline{p}_v - \overline{p}_u) \\
q^{\rightarrow}_{a,t} &\leq x_{a,t}\overline{q}_a \\
q^{\leftarrow}_{a,t} &\leq x_{a,t}\overline{q}_a.
\end{aligned}
$$

Regulating Arcs

For a regulating arc $a = (u, v)$ and each $t \in T$:

Not Active ($x_{a,t} = 0$):

- Decoupled pressure values
- No flow allowed

Active ($x_{a,t} = 1$):

- Pressure at u not smaller than pressure at v
- Unidirectional flow up to an amount of $\overline{q}_a$ (Big-M).

$$p_{u,t} - p_{v,t} \geq (1 - x_{a,t})(\underline{p}_v - \overline{p}_u)$$
$$q_{\overrightarrow{a,t}} \leq x_{a,t}\overline{q}_a.$$

Compressing Arcs

Not Active ($x_{a,t} = 0$):

- No machine assigned
- Decoupled pressure values
- No flow allowed

Active ($x_{a,t} = 1$):

- Assign machines to compressing arc
- Pressure at v not smaller than pressure at u
- Pressure at v at most r_a times greater than $p_{u,0}$
- Flow limited by sum of max flows of assigned machines
- Respect approximated power bound equation

Compressing Arcs - MIP Model

For each machine $i \in M$ and for each timestep $t \in T$ we have

$$\sum_{a \in A: i \in M_a} y^i_{a,t} \leq 1$$

$$y^i_{a,t} \leq x_{a,t}$$

For each compressing arc a and for each timestep $t \in T$ we have

$$q^{\rightarrow}_{a,t} \leq \sum_{i \in M_a} F^i y^i_{a,t}$$

$$r_{a,t} = 1 + \sum_{i \in M_a} (1 - R^i) y^i_{a,t}$$

$$\pi_{a,t} \leq \sum_{i \in M_a} P^i y^i_{a,t}$$

$$p_{u,t} - p_{v,t} \leq (1 - x_{a,t})(\overline{p}_v - \underline{p}_u)$$

$$r_a p_{u,0} - p_{v,t} \geq (1 - x_{a,t})(p_{u,0} - \overline{p}_{v,t})$$

$$\alpha_1 p_{u,t} + \alpha_2 p_{v,t} + \alpha_3 q^{\rightarrow}_{a,t} + \alpha_4 \pi_{a,t} \leq \beta x_{a,t} + (1 - x_{a,t})(\alpha_1 \underline{p}_u + \alpha_2 \overline{p}_v + \alpha_3 \overline{q}_a)$$

$$\alpha_1 p_{u,t} + \alpha_2 p_{v,t} + \alpha_3 q^{\rightarrow}_{a,t} + \alpha_4 \pi_{a,t} \geq \beta x_{a,t} + (1 - x_{a,t})(\alpha_1 \overline{p}_u + \alpha_2 \underline{p}_v + \alpha_4 \overline{\pi}_a)$$

Combined Arcs

Not Active ($x_{a,t} = 0$):

- No machine assigned
- Decoupled pressure values
- No flow allowed

Active ($x_{a,t} = 1$):

- Assign machines to compressing arc (if compressing)
- Pressure at v at most r_a times greater than $p_{u,0}$
- Flow limited by sum of max flows of assigned machines
- Respect power bound approximation equation (if compressing)

Introduce binary variables $x^r_{a,t}, x^c_{a,t} \in \{0, 1\}$ indicating whether the arc is regulating or compressing.

$$x^r_{a,t} + x^c_{a,t} = x_{a,t}$$

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Connecting In- and Outside and Objective function

Flow conservation holds at all nodes in the network

$$\sum \text{ingoing flow} - \sum \text{outgoing flow} = b_{v,t}$$

where $b_{v,t} = 0$ for inner nodes, $b_{v,t} \geq 0$ for entries, and $b_{v,t} \leq 0$ for exits.

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The (current) objective of Netmodel-MIP is to minimize the number of

1. flow direction changes,
2. simple state changes,
3. and auxiliary link switches.

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The (current) objective of Netmodel-MIP is to minimize the number of

1. flow direction changes,
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3. and auxiliary link switches.

Currently, we discuss to additionally penalize

- compressor/combined links being active
- and the power used for compression.

3-Stage Approach

To avoid infeasibility, we have a 3-Stage Approach

1. Initial MIP

3-Stage Approach

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1. Initial MIP
2. Stage 1 infeasible $\Rightarrow$ add (expensive) slack on supplies/demands
3. Stage 2 infeasible $\Rightarrow$ add (highly expensive) slack on pressure bounds

In theory the last MIP always admits a feasible solution.

Netmodel-Algorithm

Solve MIP
if MIP is infeasible **then**
    Add slack on supply/demands and resolve
    if MIP is infeasible **then**
        Add slack on pressure bounds and resolve
$\text{sol}_0 \leftarrow$ solution of MIP

Netmodel-Algorithm

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if MIP is infeasible **then**
    Add slack on supply/demands and resolve
    if MIP is infeasible **then**
        Add slack on pressure bounds and resolve
$\text{sol}_0 \leftarrow$ solution of MIP

for i in $1 \dots k$ **do**
    Determine average velocities using last $\min\{i, j\}$ solutions
    Update momentum equations and solve MIP
    if MIP is infeasible **then**
        Add slack on supply/demands and resolve
        if MIP is infeasible **then**
            Add slack on pressure bounds and resolve
            $\text{sol}_i \leftarrow$ solution of MIP
Return pressure and flow values of fence group nodes in sol_k

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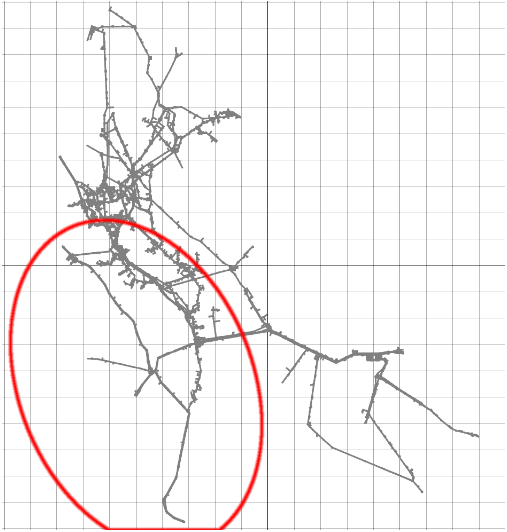
Subnetwork of Prototype



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Thank you for your attention!