The connectivities of leaf graphs of sets of points in the plane

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Let U be a set of n points in the plane. If no three points are collinear, then we say that U is in *general position*. We can draw a graph of which vertices are points in U and edges are straight line segments between two points in U. If any two edges in the graph do not intersect except at endpoints, then the graph is called a *non-selfintersecting* graph on U.

Y. Ikebe et al. [3] showed that any rooted tree with n vertices can be embedded as a non-selfintersecting tree on a given set U, the root being mapped to an arbitrary specified point of U. Furthermore A. Kaneko and M. Kano extended the theorem for a forest with two rooted trees in [4]. A non-selfintersecting tree with n vertices on U is called an *affine spanning tree* on U. E. Campo and V. Glicia [2] showed that the tree graph on U is connected. A tree graph on U is defined in such a way that the vertex set is the set of all the affine spanning trees on U and two affine spanning trees $\mathbf{t}_1$ and $\mathbf{t}_2$ are said to be adjacent if there exist edges $e_i \in E(\mathbf{t}_i)$ such that $\mathbf{t}_1 - e_1 = \mathbf{t}_2 - e_2$.

In this paper, we study a spanning subgraph of the tree graph on U, called a *leaf graph.* An adjacency relation of a leaf graph is defined so that two affine spanning trees $\mathbf{t}_1$ and $\mathbf{t}_2$ are said to be adjacent if there exists $u \in U$ such that $\mathbf{t}_1 - u = \mathbf{t}_2 - u$ and this graph is connected. The authors studied the connectivity of the leaf graph of a 2-connected graph with minimum degree k in [5]. We shall prove the following theorem in this paper.

Theorem 1 *Let U be the set of points in the plane in general position. Then the leaf graph on U is 2-connected.*

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