The looseness of triangulations on closed surfaces

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A triangulation G on the closed surface F^2 is called *tight* if there is a face of G which has three distinct colored vertices on its boundary for any surjective color assignment $f : V(G) \to \{1, 2, 3\}$. (Such a face is called a *heterochromatic* face.) For example, Arocha, Bracho and Neumann-Lara have discussed on triangular embeddings of the complete graph K_n into surfaces and shown a certain method to constract a series of tight triangulations with complete graphs and a series of untight ones. This concept is extended by Negami and Midorikawa. A triangulation G is called *k-loosely tight* if there is a heterochromatic face for any surjective color assignment $f : V(G) \to \{1, 2, 3, \ldots, k+3\}$. The *looseness* of a triangulation G is defined as the minimum value of k such that G is k-loosely tight, and is denoted by $\xi(G)$. The looseness $\xi(G)$ depends on the embedding of G in general. However, we shall show that $\xi(G)$ is determined only by the combinatorial structure of G in some special cases. We denote the independence number and the diameter of G by $\alpha(G)$ and $\mathrm{dia}(G)$, respectively.

Theorem 1. *Let G be a triangulation on the sphere, the projective plane, the torus or the Klein bottle. Then G is 1-loosely tight if and only if $\alpha(G) \leq 2$ and $\mathrm{dia}(G) \leq 2$.*

Theorem 2. *If G_1 and G_2 are two triangulations on the projective plane that are isomorphic to each other as graphs, then $\xi(G_1) = \xi(G_2)$.*

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