Polyhedral M-convex and L-convex Functions
— Two Classes of Combinatorial Convexity over Real Space —

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Abstract

In the area of combinatorial optimization, there exist many well-solved problems, and many researchers have considered "discrete convexity" or "convexity over integer lattice", aiming to identify the well-behaved structure in combinatorial optimization. Recently, two classes of discrete convexity, called M-convexity and L-convexity, are proposed by Murota in 1996, which provide us with a unified framework for well-solved combinatorial optimization problems. M/L-convex functions enjoy various nice properties as discrete convexity: they can be extended to ordinary convex functions, and a Fenchel-type duality and a (discrete) separation theorem hold for them.

In this presentation, we extend the concept of M/L-convexity to polyhedral convex functions.

One motivation comes from the results described in Rockafellar's book published in 1984, where it is shown that two convex functions arise from the cost functions of the minimum cost flow problem and the minimum cost tension problem. Though both functions are classified in the same category from the viewpoint of convex analysis, they have different combinatorial structures. The concept of polyhedral M-convex (L-convex, resp.) function is a generalization of the cost function of the minimum cost flow (tension, resp.) problem. By investigating polyhedral M/L-convex functions, we clarify the combinatorial structure of those cost functions.

Another motivation is an extension of the concept of matroid to a polyhedron called polymatroid and submodular polyhedron, for which the same greedy algorithm still works. This fact shows that the validity of greedy algorithms does not necessarily depend on the integrality or discreteness. The polyhedral extension of M/L-convex functions enables us to capture the greedily solvable structure which is independent of integrality.

Our interest is also in the polyhedral structure of M/L-convex functions, and polyhedral characterization of M/L-convex sets and functions are shown.

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