On the Flip Operations of Clique-Acyclic Orientations of Graphs

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Abstract

In this paper, we study the edge flip operations on some classes of clique-acyclic digraphs (that is, digraphs containing no directed triangle), especially we show that *if an undirected graph G is planar or its maximum degree $\Delta(G) \leqq 7$, any two clique-acyclic orientations π and π' have a sequence of clique-acyclic orientations $\pi = \pi_0, \pi_1, \ldots, \pi_t = \pi'$ such that we obtain π_i by reversing the orientation of one single edge of π_{i-1}* (then we call that *π' is attainable from π*). The latter bound "$\Delta(G) \leqq 7$" is sharp. Actually, if $\Delta(G) = 8$, then there are exactly five examples of G which has an clique-acyclic orientation such that, if we flip any one edge of it, the resulting new orientation has a directed triangle. Last, we show that, *except for the above five examples of G, any two clique-acyclic orientations of G whose maximum degree $\Delta(G) \leqq 8$, are attainable from each other.*

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