

# On the Flip Operations of Clique-Acyclic Orientations of Graphs

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## Abstract

In this paper, we study the edge flip operations on some classes of clique-acyclic digraphs (that is, digraphs containing no directed triangle), especially we show that *if an undirected graph  $G$  is planar or its maximum degree  $\Delta(G) \leq 7$ , any two clique-acyclic orientations  $\pi$  and  $\pi'$  have a sequence of clique-acyclic orientations  $\pi = \pi_0, \pi_1, \dots, \pi_t = \pi'$  such that we obtain  $\pi_i$  by reversing the orientation of one single edge of  $\pi_{i-1}$  (then we call that  $\pi'$  is attainable from  $\pi$ ). The latter bound “ $\Delta(G) \leq 7$ ” is sharp. Actually, if  $\Delta(G) = 8$ , then there are exactly five examples of  $G$  which has an clique-acyclic orientation such that, if we flip any one edge of it, the resulting new orientation has a directed triangle. Last, we show that, *except for the above five examples of  $G$ , any two clique-acyclic orientations of  $G$  whose maximum degree  $\Delta(G) \leq 8$ , are attainable from each other.**

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