3D Penrose Tiling and Related Topics

OGAWA Tohru

Institute of Applied Physics, University of Tsukuba,
Tsukuba, 305-8573 Japan
ogawa@bk.tsukuba.ac.jp

Main subject of the presentation is the story and some properties of 3D Penrose tiling. Some other related attempts are also introduced within the given duration. Subjects are itemized in below.

- Discovery of the Quasiperiodic long range order in some alloys.[1][2]
- Formulation of the Puzzle (Transformation model).[3]
- Symmetrical Transformation model vs. uniform Projection model.[4]
- Common feature of problems of quasiperiodicity and of chaos.[5]
- 2D Penrose tiling and *self-supporting* 3D rod construction.[6]
- Extension of similarity to a spherical surface and some fractal patterns.[7]

Unexpected discovery of quasiperiodic long range order of materials with icosahedral symmetry was achieved by D. Shechtman in 1984. Nobody expected the coexistence of Laue-spot type diffraction pattern and fivefold symmetry. The key concept of understanding the structure was Penrose tiling proposed 10 years before that. The motivation of R. Penrose was to find a set of tiles which forces nonperiodicity. It is a rare and wonderful fact that a mathematical fun predicted a new material structure in reality. However, the original Penrose tiling is for 2D and it is necessary to extend the idea to 3D. The author challenged and succeeded in it in 1985.

Formulation of the Puzzle in the first stage The golden rhombus is defined as the ratio of two diagonals is golden section ($\tau \equiv (1+\sqrt{5})/2$), whose acute angle is $\arctan 2$. There two types of golden rhombohedra; one is *Acute* in which three acute vertices meat at both ends of (three-fold) symmetry axes. and the other is *Obtuse* in which three obtuse vertices meat at both ends of (three-fold) symmetry axes. Find the 3D tiling with these two types of golden rhombohedra, *Acute* and *Obtuse* so that eight points at the vertex of some tiles construct a larger golden rhombohedra.

Transformation model The answer to the puzzle of the minimum size is as follows. The edge length is $\tau^3 = 2+\sqrt{5}$. All the faces of the two large rhombohedra have essentially same structure. In other words, they fit any others and the tiling can extend without limitation. The number of tiles for a large *Acute* rhombus are 55 *Acute* and 34 *Obtuse*. The number of tiles for a large *Obtuse* rhombus are 34 *Acute* and 21 *Obtuse*. All of the integers here are Fibonacci numbers, 8th to 10th terms of Fibonacci sequence. The detail of the tiling has some freedom which cannot be decided uniquely. The model is referred to as transformation model since it is based to the transformation from two sets of eight-point-relations to two sets of eight-point-relations. The transformation can be repeated endlessly. The freedom mentioned above can be used to construct some hierarchy of symmetry, for example, for T and T_h group.

Infinite quasiperiodic structure has the following difficulty, even in 1D case. One cannot say two given quasiperiodic arrangements are different, since finite times of failure of matching them are not enough to say difference. It is a similar feature to chaos where a infinitesimal difference may cause a big deference.

References

[1] D. Shechtman, I. Blech, D. Glatias, and J. W. Cahn; Phys. Rev Lett. **53** (1984) 1951.

[2] R. Penrose; J. Inst. Math and its Appl. **12** (1974), 266.

[3] T. Ogawa; J. Phys Soc. Jpn. **54** (1985), 3205.

[4] T. Ogawa; Mat. Sci. Forum; **22-24** (1987), 187.

[5] T. Ogawa; Nihon Kinzoku Gakkaihou (in Japanese) **29** (1990), 816.

[6] T. Ogawa, Y. Teshima, and Y. Watanabe; in *Katachi U Symmetry*, Eds. T. Ogawa, K. Miura, T. Masunari, and D. Nagy, p.219, (1996, Springer). (Also in Japanese, *Katachi no Bunkashi*, Vol.1, p.226 (1993)).
A. Hijikata and K. Fukuda; Nihon Fukugo Zairyo Gakkaishi, **18** (1992), 231 (in Japanese).
A. Hizume; Forma, **9** (1994), 259. (Also in Japanese, *Katachi no Bunkashi*, Vol.1, p.208 (1993)).

[7] T. Ogawa; Forma, **9** (1994), 273.