An asymmetric analogue of van der Veen conditions and the traveling salesman problem

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The Traveling Salesman Problem (TSP) is the problem of finding a shortest hamiltonian cycle in a complete weighted digraph. The TSP belongs to NP-hard class. So, we consider polynomially solvable cases (e.g. [2, 6]), that is, to find good conditions on the distances that assure an algorithm which finds an optimal tour in polynomial time. In this talk, we present the following theorem.

Theorem 1 *For the TSP which satisfies the following conditions an optimal tour can be computed in polynomial time.*
For all vertices $i, j, k, l \in \{1, 2, \cdots, n\}$ *with* $i < j < k < l$,

$$\begin{aligned} d(i,j) + d(k,l) &\leq d(i,l) + d(k,j) \\ d(j,i) + d(l,k) &\leq d(l,i) + d(j,k). \end{aligned}$$

This class is an asymmetric analogue of the conditions posed by van der Veen in 1994 [7]. Moreover, this class properly includes the class of Monge matrix (e.g. [1]), which is studied in several fields of computer science and combinatorics, and some polynomially solvable classes in the TSP [3, 5].

References

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