Diagonal flips in pseudo-triangulations on closed surfaces

Seiya NEGAMI*

Negami has already shown that there is a natural number $N(F^2)$ for any closed surface F^2 such that two triangulations on F^2 with n vertices can be transformed into each other, up to equivalence, by a sequence of diagonal flips if $n \geq N(F^2)$. Any diagonal flip in the sequence has to keep the simpleness of triangulations. However, we shall consider diagonal flips in pseudo-triangulations to give an upper bound for $N(F^2)$. A pseudo-triangulation G on a closed surface F^2 is a triangular embedding of a graph G on F^2 which may have multiple edges or self-loops. Define the *diagonal crossing number* $\mathrm{cr}_\nabla(G_1, G_2)$ of two labeled graphs G_1 and G_2 on F^2 with the same vertex set $V(G_1) = V(G_2)$ as the minimum number of crossings on their edges when we put them together on F^2. Then we have the following theorem:

Theorem 1. *Let G_1 and G_2 be two labeled pseudo-triangulations on a closed surface F^2 with the same number of vertices. Then they can be transformed into each other, up to equivalence, by a sequence of diagonal flips of length at most $\mathrm{cr}_\nabla(G_1, G_2)$.*

As an application of this theorem, we can prove the following theorem as we expect.

Theorem 2. *There is a linear upper bound for $N(F^2)$ with respect to the genus g of the surface F^2; $N(F^2) = O(g)$.*

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*Department of Mathematics, Faculty of Education and Human Sciences, Yokohama National University, 79-2 Tokiwadai, Hodogaya-Ku, Yokohama 240-8501, Japan. Email: negami@ms.ed.ynu.ac.jp