Quadrangulations on closed surfaces covered by vertices of degree 2, 3 and 4

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A *quadrangulation* on a closed surface F^2 is a simple graph which has been embedded in F^2 so that each face is quadrilateral. A quadrangulation G is said to be *k-covered* if for any $e = xy \in E(G)$, $\deg(x) = k$ or $\deg(y) = k$. In this talk, we first show that for any closed surface F^2 and any natural number $k \geq 2$, there exist k-covered quadrangulations on F^2, and secondly characterize the k-covered quadrangulations on closed surfaces with $k = 2, 3, 4$.

Theorem 1. *A quadrangulation G is 2-covered if and only if G is a planar quadrangulation isomorphic to $K_{2,n}$ for some $n \geq 2$.*

Let G be an embedding on a closed surface F^2. Put a vertex v on a face f of G and join v with all vertices lying on the boundary walk of f. Apply this operation to each face of G and delete all edges of G. Then the resulting bipartite graph on F^2 is called the *radial graph* $R(G)$ of G.

Theorem 2. *A quadrangulation G is 3-covered if and only if either G is isomorphic to the radial graph $R(K)$ for some triangular embedding K (possibly with multiple edges) on a closed surface, or G is isomorphic to one of the exceptions on the sphere and the projective plane.*

For the 4-covered quadrangulations on closed surfaces, we give a constructible characterization.

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