Complexity Results for Calculating Power Indices of Weighted Majority Games

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Abstract

We discuss the problem for calculating power indices which satisfy some properties. We say a power index satisfies the *strict dummy property* when an element of the power index is equal to 0 if and only if the corresponding player is dummy. We say a power index satisfies the *strict symmetricity* when a pair of elements of the power index is equivalent if and only if the corresponding pair of players is symmetric. It is easy to show that (1) the Shapley-Shubik power index, the Banzhaf power index and the Deegan-Packel power index satisfy the strict dummy property and (2) the Shapley-Shubik power and the Banzhaf power index satisfy the strict symmetricity.

In this talk, we show the following theorem.

Theorem 1 *(1) Calculating a power index which satisfy strict dummy property is $\mathcal{NP}$-hard.*
(2) Calculating a power index satisfying strict symmetricity is $\mathcal{NP}$-hard.

References

[1] J.F.BANZHAF III, *Weighted Voting Doesn't Work*, Rutgers Law Review, 19 (1965), pp.317-343.

[2] J.DEEGAN AND E.W.PACKEL, *A New Index for Simple n-person Games*, Int. Journal of Game Theory, 7 (1978), pp.113-123, Physica-Verlag, Vienna.

[3] R.M.KARP, *Reducibility among combinatorial problems*, in R.E.Miller and J.W.Thatcher eds., COMPLEXITY OF COMPUTER COMPUTATIONS, Plenum Press, New York, 1972, pp.85-103.

[4] L.S.SHAPLEY, *A value for n-person games*, in *Contributions to the Theory of Games, Vol.II*, in H.W.Kuhn and A.W.Tucker eds., ANNALS OF MATHEMATICS STUDIES, 28, Princeton University Press, 1953, pp.307-317.

[5] L.S.SHAPLEY AND M.SHUBIK, *A method for evaluating the distribution of power in a committee system*, American Political Science Review, 48 (1954), pp.787-792.