On graphs of girth 5

by

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Abstract

We consider graphs Γ satisfying the following properties: (a) the valency of Γ is $(q^n - 1)/(q - 1)$ where q is a prime power; (b) the shortest cycle in Γ has length 5; (b) there is an automorphism group G of Γ which acts transitively on the set of 2-arcs in Γ; (c) if x is a vertex of Γ, $G(x)$ is the stabilizer of x in G and $\Gamma(x)$ is the set of neighbours of x in Γ then the action induced by $G(x)$ on $\Gamma(x)$ contains a doubly transitive normal subgroup isomorphic to $PSL_n(q)$. There exist eight exceptional graphs with these properties associated with the sporadic simple groups Mat_{22}, Mat_{23}, Co_2, J_4 and BM and their non-split extensions. In addition there is an infinite family of such graphs with $q = 4$. The identification problem of the infinite series is reduced to the determination problem of the minimal submodules of the $GF(2)$-permutational module of $PSL_n(4)$ acting of the set of 2-dimensional subspaces in its natural $GF(4)$-module.