

Oriented Graphs Generated by Random Points on a Circle

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Abstract

Extending the cascade model for food webs, we introduce a cyclic cascade model which is a random generation model of cyclic dominance relations. Put n species as n points Q_1, Q_2, \dots, Q_n on a circle. If the counterclockwise way from Q_i to Q_j on the circle is shorter than the clockwise way, we say Q_i dominates Q_j . Consider a tournament whose dominance relations are generated from the points on a circle by this rule. We show that when we take mutually independently distributed n points on the circle, the probability of getting a regular tournament of order $2r+1$ as the largest regular tournament is equal to $\binom{n}{2r+1}/2^{n-1}$. This probability distribution is for the number of existing species after a sufficiently long period, assuming a Lotka-Volterra cyclic cascade model.

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