The shapes of osculatory packings of disks

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We consider an Apollonian packing. Let T_0 be an initial curvilinear triangle determined by three mutually tangent disks. Packing a disk into T_0 solidly, we have three curvilinear triangles. From these we choose one at random and denote it by T_1. Continuing this process successively, we have a sequence of curvilinear triangles $\{T_n : n = 1, 2, 3, \ldots\}$.

First we investigate the shapes of these curvilinear triangles $\{T_n : n = 1, 2, 3, \ldots\}$. As is well-known, using the barycentric coordinate system, we can represent a triple of three real numbers (with their sum being equal to unity) by a point interior to an equilateral triangle M . With the aid of this device we can visualize the shapes of curvilinear triangles as a sequence of points in M. Then, as the limit set of the sequence of points, we obtain a similar fractal to the original Apollonian packing. In fact we can show that the Hausdorff dimension of this limit set is equal to that of the residual set of the original Apollonian packing.

Next we investigate an analogous problem for more general osculatory packings. For example, we consider the case that we pack several disks at a time into a curvilinear triangle, the case that we pack a disk into a curvilinear quadrange, and so on. Finally, leaving the deterministic situation, we consider random osculatory packings of disks. We present two models of random osculatory packings, which may have some statistical application.