Non-constructible triangulations of balls

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A simplicial complex C is a finite set of simplices (called faces of C) in some Eucledian space R^n which satisfies: (i) if $\sigma \in C$ then all the faces of σ (including the empty set) is also in C, and (ii) if $\sigma, \tau \in C$ then $\sigma \cap \tau$ is a face of both σ and τ. It is called a triangulation of a manifold M with boundary if $\cup_{\sigma \in C} \sigma$ is homeomorphic to M.

Constructibility of simplicial complexes is defined recursively as follows.

(i) A simplex together with all its faces is defined to be constructible.

(ii) If two d-dimensional simplicial complexes C_1 and C_2 are constructible and $C_1 \cap C_2$ is a $(d-1)$-dimensional constructible complex, then $C = C_1 \cup C_2$ is defined to be constructible. (The dimension of a simplicial complex is the maximum dimension of its faces.)

This is a generalization of shellability of simplicial complexes, which appeared in [1]. If we restrict ourselves in the condition (ii) of the definition that C_2 must be a simplex together with its faces, we get a definition of shellable complexes.

The most mysterious fact around constructibility is that there are triangulations of balls which are not constructible if the dimension is ≥ 3, while all the constructible triangulations of manifolds are homeomorphic to balls or spheres. The same fact for shellability is well-known, see [3] for example. The existence of non-constructible triangulations of a 3-ball is shown in [2]. Stimulated by this fact, [2] discussed a way of testing constructibility of a given triangulation of a ball under the condition such that (i) the dimension of the ball is 3, and (ii) all the vertices of the triangulation is on the boundary of the ball. In this talk, we investivate such a testing algorithm for higher dimensions, and give a sufficient condition of non-constructibility as a partial result.

References

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