Fractal analysis of short time scale fluctuations of the interplanetary magnetic field

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Abstract. Fluctuations of the interplanetary magnetic field (IMF) having the time scale of less than 32 second and an amplitude of less than $2 \sim 3$ nT are investigated by using the fractal analysis, recently developed for analyzing a nonstationary and irregular time series. The results obtained by the fractal analysis indicate that the small-scale IMF disturbances have a self-affine structure within a wide frequency range from $1/32$ to $2 \sim 8$Hz and that their spectra behave a power law form. In short, the power spectrum of the IMF fluctuations follows a power law form with an index of $5/3$ in the wide frequency range. The fractal dimensions of disturbances correspond to the turbulence showing the Kolmogoroff power spectrum. This result means that an inertial range in the IMF turbulence can extend to high frequency of $2 \sim 8$ Hz, and also suggests that the dissipation range in the turbulence is possibly higher than $2 \sim 8$ Hz.

1. Introduction

The study of turbulent fluctuations observed in areas of space plasma physics has the potential yielding much insight into instabilities and wave particle interactions which influence the behavior of tenuous collisionless plasmas. In particular, the interplanetary magnetic filed (IMF) fluctuations within a low frequency range of about less than $10^{-2}$Hz has extensively investigated by using a magnetometer from spacecraft, because the IMF fluctuations satisfy the condition desirable for understanding the magnetohydrodynamic (MHD) turbulence. For example, they are found to satisfy the conditions of weak stationarity [Matthaeus and Goldstein, 1982b]. Hence, the power spectral analysis suited to weak stationary time series has been applied to the IMF fluctuation observed by the satellites [Matthaeus and Goldstein, 1982a; Viñas et al., 1984; Roberts and Goldstein, 1987]. Consequently, there are many reports that the power spectrum density of low frequency fluctuations in the interplanetary magnetic field is a Kolmogoroff spectrum for homogeneous, isotropic, and stationary turbulence: $P(f) \propto f^{-5/3}$ [Matthaeus and Goldstein, 1982a; Matthaeus et al., 1982; Burlaga and Goldstein, 1984; Roberts and Goldstein, 1987]. In particular, Matthaeus and Goldstein have extensively examined the low frequency IMF fluctuations at 1, 2.8, and 5 AU, by the spectral analysis with the Blackman-Tukey or FFT techniques, and presented clear results of the magnetic energy spectrum. [Matthaeus and Goldstein, 1982a, b]. Burlaga and Klein [1986] have shown that magnetic field fluctuations near 8.5 AU by Voyager 2 are self-affine over time scales from $\sim 20s$ to $\sim 3 \times 10^5s$.

It is well established from many observations that the large-scale IMF fluctuation in
the solar wind follows a power law spectrum, whereas the small-scale IMF fluctuation with a period of less than 20 second is not well understood in a framework of the turbulence because its power spectrum structure is not satisfactorily described by using conventional analysis procedures. In this paper, we will examine properties of turbulent IMF fluctuations having a short time scale (less than 32 second) by using the fractal analysis. The purpose of this paper is to indicate that the small-scale IMF disturbances have a self-affine structure and their power spectrum follows a power law spectrum. Moreover, we will show that the fractal dimension obtained by the present analysis corresponds to that of the Kolmogoroff spectrum.

2. Method

Power Spectral Analysis. The power spectrum analysis has been conventionally used as the useful and efficient method for analyzing the time series which show an irregular and complex behavior in a time domain. In particular, when the power spectrum follows a power law form; \( P(f) \propto f^{-\alpha} \), the exponent \( \alpha \) is an index for representing the irregularity of the time series. Although several methods for calculating the power spectrum are available [Kay and Marple, 1981], the finite Fourier transformation using a fast Fourier transform (FFT) algorithm is generally adopted, because the FFT is convenient without model selection which strongly affects the estimated power spectrum.

Despite of manageableness and usefulness of the FFT procedure, there are often cases where the characteristics obtained through the FFT are buried under noisy fluctuations superposed on the raw power spectrum. Such noisy fluctuations cause a power law index to be unstable, because the power law index generally given by a slope of a straight line through plots of \( \log f \) vs. \( \log P(f) \) is very sensitive to these noisy fluctuations.

Two method are considered in order to treat this problem. One is to take a parametric modeling method of power spectrum estimation [Kitagawa and Gersch, 1985]. As mentioned above, it should be noticed that there exits a strong dependency of a power spectrum on a selected model [Kay and Marple, 1981]. The other one is to take an ensemble average of a power spectrum calculated by the FFT procedure. Unfortunately, the observational data in space physics are given by only one measurement with a single spacecraft, and thus we can only hope to take an average of the power spectrum over a long interval in which fluctuations are assumed to be statistically weak stationary. When it is unacceptable to assume stationarity for a long time interval, the power law index obtained with a small data number inevitably becomes to be unstable.

Fractal Analysis. The notion of "Fractals", which has been well understood, was introduced by Mandelbrot [1977] for describing the irregular and complex patterns. Mandelbrot [1977] also introduced a concept of the fractal set of points \((t, X(t))\) forming the graph of a function \(X\) defined on the unit interval. Along with his idea for describing the property of a time series, the methods to calculate the curve length of the time series have been proposed [Burlaga and Klein, 1986; Higuchi, 1988]. The procedure by Higuchi is an improvement of that by Burlaga and Klein, and there is difference in the definition of the curve length between them. Thus the index characterized by the curve length is \(-S\) for Burlaga and Klein's procedure, but \(-D\) for Higuchi's one, where \(S\) and \(D\) are and
fractal dimension of the time series, respectively [Burlaga and Klein, 1986; Higuchi, 1988]. Higuchi [1990] has also numerically investigated the relationship between the fractal and the power spectrum analyses.

The method of the fractal analysis for a time series can be referred to Higuchi [1988, 1990]. The fractal analysis is useful and effective for an irregular time series possessing a continuous power spectrum density such as a red noise type, where the red noise type indicates that its power spectrum exhibits appreciably more power at low frequencies than at high frequencies [Fougeres, 1985]. In particular, its performance in a stable estimation of the indices characterizing the time series is demonstrated for a nonstationary time series [Higuchi, 1988]. The notes on the description of the irregularity by using the fractal analysis were presented by several authors [Roberts and Goldstein, 1987; Brown, 1987; Feder, 1988; Higuchi, 1988, 1990; Bergamasco et al., 1990].

Analysis Procedure. In this study we apply the fractal analysis method to an examination of small-scale IMF fluctuations (less than 32 second) observed with ISEE1 in the upstream region of the Earth's foreshock. High time resolution data of 4Hz and 16Hz sampling were used for this purpose. In the first procedure, we divide the sequential data into subintervals with a time window of 3 min.. Thus far, each subinterval contains 720 (2880) data points for 1/4 second (1/16) magnetic field data. On each subinterval we calculate the length of a curve consisting of each component of the IMF. In this study we examine not only three components separately, but also a total component. Four curve lengths, \( < L_i(\tau) > \), are therefore obtained on each subinterval as a function of \( \tau \), where \( \tau \) is an "interval time" defined in a framework of the fractal analysis [Higuchi, 1988], and a subscript \( i \) represents the magnitude (T) and x, y, z components of the IMF, respectively. Roughly speaking, \( \tau \) corresponds to a period in the periodogram analysis, and so the unit is given by second. Since we adopt the GSE coordinates throughout this analysis, the subscript of \( i \) represents the \( i \) component in the GSE coordinate system.

\( < L_i(\tau) > \) is calculated against the interval time (time scale) from \( \Delta t \) to \( \tau_{max} \), where \( \Delta t \) is a sampling time. In determining \( \tau_{max} \), we should take into account the following two factors: that if \( \tau_{max} \) is too large, \( L_i(\tau) \) for large \( \tau \) shows to have a large variance around the mean value of \( L_i(\tau) \) (namely \( < L_i(\tau) > \)), on the other hand, if it is too small, the fractal dimension \( D \) determined by fitting a function to the \( \log \tau \) vs. \( \log < L_i(\tau) > \) plots comes to be unstable. The good \( \tau_{max} \) which balances the trade-off between above two factors is investigated by a numerical approach, and then is set to be 32 second in this study.

The second procedure is to determine a function form fitted to the \( \log \tau \) vs. \( \log < L_i(\tau) > \) plots in order to obtain the indices characterizing the time series [Higuchi, 1988]. When the \( \log \tau \) vs. \( \log < L_i(\tau) > \) plots lie very close to a straight line, the straight line is suitable for a fitted function. This represents that the power spectrum of such a time series is a power law from. In this case, we have a fractal dimension \( D_i \) describing property of the time series within a time scale of over \( \Delta t \sim \tau_{max} \). Although the straight line is satisfactorily fitted to the \( \log \tau \) vs. \( \log < L_i(\tau) > \) plots for the IMF fluctuations examined in this study, there are possibly cases where a continuous piece-wise linear function with a node is suitable; i.e., the curve of the \( \log \tau \) vs. \( \log < L_i(\tau) > \) plots has a break at which the slope of the line segment discontinuously changes. We call this curve
a two-segments curve.

In this study, we would like to know whether the IMF fluctuations show a power law spectrum in a time scale under consideration here. Now we compare a residual for fitting a straight line with that for a two-segments curve. By supposing that a fitted function with the smaller mean-square value of residuals is good approximation, a function with many parameters is always chosen, because residuals of a fitted function from the log $\tau$ vs. $\log < L_i(\tau) >$ plots reduce to be smaller with an increase in the number of parameters in a fitted function. Then, the residual for a two-segments curve ($S_2^2$) always becomes smaller than that for a straight line ($S_1^2$). A criterion that a function with the smallest residual is the best fit one, is inappropriate and thereby another criterion is necessary for determining a fitted function.

We examine the quantity of $R = S_1^2/S_2^2$ for a simulated data of which a power spectrum is a single power law form. Obviously, a straight line should be fitted to this time series. The numerical results indicate that $S_2^2$ is almost five times as large as $S_1^2$. Then, we select a straight line for a case of $R = S_1^2/S_2^2 \leq 5$. Finally, after determining the fitted function form, several indices characterizing the time series are obtained on each subinterval. In a case of fitting a straight line, $D_1$ can be defined. When a two-segments curve is chosen, values of $\tau_c$, $D_S$, and $D_L$ are given, where $\tau_c$ is a break in the log $\tau$ vs. $\log < L_i(\tau) >$ plots, and $D_S$ and $D_L$ are fractal dimensions within time scales of $\Delta t \sim \tau_c$ and of $\tau_c \sim \tau_{max}$, respectively.

3. Result

Data. We use the high resolution magnetic field data obtained by ISEE1 spacecraft during a period from November 1977 to August 1980. The fluxgate magnetometer on board ISEE1 has been described in detail elsewhere [Russell and Greenstadt, 1979]. We analyze 1/4 or 1/16 second data in this study. The intervals used in this study are selected to satisfy the following conditions. First, we search the interval during which the IMF observed by ISEE 3 satellite [Couzens and King, 1986] sustains a stable value in terms of the standard deviation. Namely, we picked up the intervals during which the IMF possesses a small value of the standard deviation. Next, since we would investigate the small-scale turbulence observed during the comparatively stationary intervals except for magnetic storms, the intervals showing the small value of the $K_p$ index (almost less than 3) are selected for the analyzed data set.

Observations. We demonstrate in Figure 1 a typical example of IMF variations analyzed in this study. The magnetic field data were observed during 12:00~14:00UT on Nov. 19, 1977. The magnetic field data shown in Figure 1 are 10 second averages in order to explicitly exhibit no fluctuation having a coherent structure with a characteristic time scale. In short, we would like to present that the analyzed fluctuations include neither upstream waves [Russell and Hoppe, 1983] nor the foreshock disturbances [Greenstadt and Baum, 1986]. No large fluctuation is observed for any component during 12:00~13:00UT; $\Delta B \leq 5$ nT. We give in Figure 2 the unsmoothed plots of the IMF in order to make a small disturbance visible. The time resolution is 4 Hz in this interval. It is evident that there is no quasi-sinusoidal (quasi-monochromatic) waves with a characteristic time scale.
The IMF during 12:00–13:00 UT on Nov. 19, 1977 is examined by using the fractal analysis. The log $\tau$ vs. log $< L_i(\tau) >$ plots obtained along the aforementioned analysis procedure hold a similar value on each subinterval throughout 1 hour; the IMF variations show a same self-affine structure during 1 hour. Then, we examine what time scale the self-affine structure extends from 1/4 second. We demonstrate in Figure 3 their $\tau$ vs. $< L_y(\tau) >$ plots in a log–log scale. As previously mentioned, $\tau_{max}$ is normally set to be 32 second, but we temporally set $\tau_{max}$ to be 1000 second in Figure 3. According to an extension of $\tau_{max}$, the time window of the subinterval naturally is longer (1 hour) than the normally used value (3 min.). There is no sharp bend in Figure 3, and hence a straight line is satisfactorily suited for this case. The self-affine structure holds in a wide time scale across the proton gyro-period (~11 second). Strictly speaking, it is seen that there is a round corner around $\tau = 2$ second. However, this corner is negligible according to the previously introduced criterion. The value of a minus slope of the straight line successfully fitted to the log $\tau$ vs. log $< L_y(\tau) >$ plots, 1.64, represents the fractal dimension, $D_1$.

According to the aforementioned criterion, the straight line is fitted to the log $\tau$ vs. log $< L_i(\tau) >$ plots for cases with $R \leq 5$, and the fluctuations can be considered to hold a self-affine structure accordingly. We here examine the values of $R$ in order to specifically show that the self-affine structure is intrinsic to the IMF fluctuations. Figure 4 shows a histogram of $R$ for the $y$ component of fluctuations of the IMF, together with that of the fluctuations observed in the magnetosheath by ISEE1. The analysis procedure, mentioned in the previous section, is also applied to an analysis of the magnetic field fluctuations observed in the magnetosheath. The white and black bars denote distributions for the magnetosheath and for the IMF, respectively. As for the magnetosheath, no event with $R_y$ of less than 5 is observed, whereas the cases with $R_y \leq 6$ are dominant for the IMF (64%). This indicates that the log $\tau$ vs. log $< L_i(\tau) >$ plots for the IMF fluctuations have no sharp bend, and then a straight line appropriately fits. On the contrary, a two-segments curve is a good approximation to that for the magnetosheath.

These results suggest that the IMF fluctuations have a self-affine structure from 1/4 (1/16) to 32 second, and then the characteristic time scale is poorly or not defined. As for the fluctuations observed in the magnetosheath, there is always the characteristic time scale. The fluctuations in the magnetosheath have a characteristic time scale around the local proton gyroperiod [Higuchi et al., 1987]. Thus, an interpretation from Figure 4 obviously indicates that an existence of the characteristic time scale is intrinsic to the fluctuations in the magnetosheath. The detail analysis of the turbulent fluctuations observed in the magnetosheath will be extensively discussed in Higuchi and Kokubun [1990].

Although the IMF fluctuations show high occurrence in a range of $R \leq 6$, the cases with $R \geq 6$ are fairly significant. These IMF fluctuations with a large value of $R$ sometimes show a wave form similar to that of the up-stream waves associated with the diffuse ion population [Hoppe and Russell, 1981]. Of course, we remove the up-stream waves consisting of nearly monochromatic wave packets in this statistics, because we would like to investigate the IMF fluctuations without a coherent structure. The IMF fluctuations sometimes look like the turbulent fluctuations observed in the magnetosheath, and thereby have a large value of $R$. However, the magnitude of these IMF fluctuations is significantly
different from that normally observed in the magnetosheath.

The fractal dimension, $D_1$, is averaged for the events with the small value of $R_i$ less than 6 for the IMF fluctuations. We show in Table 1 the mean value and standard deviation (S.D.) of $D_1$ for individual components. The mean value for each component is nearly $5/3$, except for the total components. This suggests that there is possibly anisotropy between the transversal and compressional components. Together with statistical quantities (mean and S.D.) of $D_1$ for the IMF, we present those values of $D_1$ for the fluctuations in the magnetosheath with $R_i$ less than 6. The mean value of the fluctuations in the magnetosheath is very different from that of the IMF for any component. Moreover, even if we take account of convergence of the S.D. for $D_1$ according to an increase in the number of the events used in averaging, it is noteworthy that the S.D. of $D_1$ for the fluctuations in the magnetosheath is much larger than that for the IMF.

If neither the straight line nor the two-segments curve is suitable for fitting to the log $\tau$ vs. log $<(P_i(\tau) >$ plots, the value of $R$ is smaller than that for the case where a two-segments curve is successfully fitted. However, the value of $D_1$ defined from a slope of poorly fitted straight line widely scatters around the mean of $D_1$. The results presented in Table 1 suggest that the small value of $R$ for the fluctuations observed in the magnetosheath does not arise from their self-affine structure, but from the log $\tau$ vs. log $<(P_i(\tau) >$ plots which poorly fit both to a straight line and to a two-segments curve. The fluctuations in the magnetosheath often change their characteristics such as the magnitude of disturbances and the power spectrum form during a short time interval. It is suggested from the results by Higuchi and Kokubun [1990] that the poor fitness both of a straight line and of a two-segments curve arises from nonstationarity of the fluctuations during the subinterval (3 min.).

5. Discussion

We demonstrated in Figure 3 that there is no characteristic time scale in the log $\tau$ vs. log $<(P_i(\tau) >$ plots for the IMF fluctuations within the time scale of 1/4 to 1000 second. This result represents that the IMF fluctuations show self-affine structure within the time scale of 1/4~1000 second. This self-affine structure leads a small value of $R$. The magnetic field data shown in Figure 2 are a typical short time scale IMF fluctuations with a small value of $R$ less than 5. The distribution of $R_i$ illustrated in Figure 4 for the IMF fluctuations represents that most of the IMF fluctuations have no characteristic time scale within the time scale of 1/4 (or 1/16) $\sim$ 32 second. The smallest time scale depends on the sampling time at times of observations.

The self-affine structure without a characteristic time scale represents that a power spectrum follows a single power law form. If a time series is self-affine within a time scale of $\Delta\tau \sim \tau_{max}$, a power law spectrum holds in a frequency range of $1/\tau_{max} \sim 1/2\Delta\tau$. The reason why the highest frequency is given by $1/2\Delta\tau$ is that a power spectrum can be defined only in a frequency range below the Nyquist frequency. As mentioned above, the IMF fluctuations exhibit a self-affine behavior from $1/4$ ($1/16$) to 32 second. Hence they have a power law spectrum from 1/32 to 2Hz (or 8Hz).

The results of the power spectrum analysis for the IMF fluctuations have been presented [Siscoe et al. 1968; Goldstein et. al, 1984; Burlaga and Goldstein, 1984; Burlaga and Klein, 1986; Roberts and Goldstein, 1987]. Among these results, the fluctuations
having the power law spectrum with the index of $\sim 5/3$ are recognized as the Kolmogoroff turbulence [Kolmogoroff, 1941]. The power law index can be estimated to be $5/3$ from the fractal dimension of $D_1 = 5/3$ for the IMF fluctuations examined in this study [Berry, 1979; Higuchi, 1988]. This value of the power law index is very interesting, because it is the same as that reported in the lower frequency range of the magnetic field [Goldstein et. al, 1984; Burlaga and Goldstein, 1984; Burlaga and Klein, 1986; Roberts and Goldstein, 1987].

Obviously, the Kolmogoroff spectrum is given in the wavelength space, and not in the frequency space. The previous reports assumed the validity of the Taylor "froze-in-flow" hypothesis and the frequency and wave number can be interchangeably used; $P(f)$ can be considered as the representative quantity of $P(k)$. The frequency range within which the power spectrum behaves as $P(f) \propto f^{-5/3}$ is called an "inertial range" of turbulence. The inertial range is bounded at longer wavelengths by the correlation length which is a measure of the scale of the energy containing structures. At the small wavelength boundary of the inertial range, the spectrum is exponentially damped by dissipative processes. The observed power law spectrum with a power law index of $5/3$ have suggested that the magnetic field fluctuation in this inertial range is the homogeneous, isotropic, and stationary turbulence.

The Kolmogoroff spectrum was found both in the Voyager 1 and in the ISEE3 within the inertial range of $10^{-6} \sim 10^{-4}$Hz [Roberts and Goldstein, 1987]. Burlaga and Klein [1986] has shown by using the magnetic field data obtained by Voyager 2 that the inertial range in the turbulence of the IMF can extend over more than 4 decades in frequency from $3 \times 10^{-6}$ to $5 \times 10^{-2}$Hz. Moreover, our result indicates that the inertial range extends toward higher frequency up to at least 2Hz. In short, the power spectrum of the IMF fluctuations follows a single power law form with the power law index of $5/3$ within a frequency range of $10^{-6} \sim 2$Hz. Consequently, this result suggests that the dissipation range in the turbulence of the IMF is higher than 2Hz. However, it must be noteworthy that the validity of the Taylor froze-in-flow does not hold in the frequency range around 2Hz. Moreover, although the IMF fluctuations with a spectrum characteristic of Kolmogoroff turbulence in the low frequency were obtained in the region far from the earth (4.1~5.2 AU), the IMF analyzed in this study were obtained around the earth. However, the result suggesting that the inertial range reaches the high frequency up to at least 2Hz over the proton-gyrofrequency is very interesting for an interpretation of the turbulence in the IMF fluctuations.

Acknowledgments

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References


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Figure Captions

Fig. 1  Plots of the averaged magnetic field (IMF) are shown in the GSE coordinates during 12:00~14:00UT on Nov.19, 1977.

Fig. 2  The interplanetary magnetic field (IMF) are shown in the GSE coordinates during 12:30~12:35UT on Nov.19, 1977. It is noteworthy that the vertical value scale is changed from that used in Figure 1 in order to demonstrate the disturbances of the IMF in detail.

Fig. 3  $\langle L(\tau) \rangle$, of the magnetic field for the $y$ component (in GSE coordinates) during 12:00~13:00 UT on Nov.19, 1977 is shown against the time scale, $\tau$, on doubly logarithm scale. The maximum value of the time scale, $\tau_{\text{max}}$, in this figure is set to be 1000 second, because we would like to clearly demonstrate that there is no $\tau_c$ around the proton gyro-period (~11 second). It is evident that a self-affine structure of the IMF disturbances holds for the time scale range of 1/4~1000 second.

Fig. 4  The distribution of $R_y$. The white and black bars indicate $R_y$ for the fluctuations in the magnetosheath and for the IMF, respectively. The IMF disturbances with the small $R_y$ mean that there is no sharp bend in the $\log \tau$ vs. $\log \langle L_y(\tau) \rangle$ plots. Most of fluctuations of the IMF have the value of $R_y$ less than 6. The cases of the IMF with $R_y$ larger than 6 represent disturbances having the characteristic time scale similar to the up-stream waves.
TABLE 1. Fractal dimension $D_1$ for events with $R = S_1^2/S_2^2 \leq 6.0$

<table>
<thead>
<tr>
<th>comp.</th>
<th>IMF</th>
<th>Sheath</th>
</tr>
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<tr>
<td>x</td>
<td>1.70±0.22</td>
<td>1.54±0.51</td>
</tr>
<tr>
<td>y</td>
<td>1.70±0.18</td>
<td>1.54±0.96</td>
</tr>
<tr>
<td>z</td>
<td>1.68±0.23</td>
<td>1.52±0.81</td>
</tr>
<tr>
<td>T</td>
<td>1.76±0.20</td>
<td>1.53±0.80</td>
</tr>
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</table>
Fig. 3
Normalized Occurrence

Fig. 4
Abstract. Fluctuations of the interplanetary magnetic field (IMF) having the time scale of less than 32 second and an amplitude of less than $2 \sim 3$ nT are investigated by using the fractal analysis, recently developed for analyzing a nonstationary and irregular time series. The results obtained by the fractal analysis indicate that the small-scale IMF disturbances have a self-affine structure within a wide frequency range from $1/32$ to $2 \sim 8$Hz and that their spectra behave a power law form. In short, the power spectrum of the IMF fluctuations follows a power law form with an index of $5/3$ in the wide frequency range. The fractal dimensions of disturbances correspond to the turbulence showing the Kolmogoroff power spectrum. This result means that an inertial range in the IMF turbulence can extend to high frequency of $2 \sim 8$ Hz, and also suggests that the dissipation range in the turbulence is possibly higher than $2 \sim 8$ Hz.