APPLICATION OF THE ENSEMBLE KALMAN FILTER TO ATMOSPHERE-OCEAN COUPLED MODEL

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ABSTRACT

We report the first application of the ensemble Kalman filter (EnKF) to an intermediate coupled atmosphere-ocean model by [1], into which the sea surface height (SSH) anomaly observations by TOPEX/POSEIDON (T/P) altimetry are assimilated. Smoothed estimates of the 54,403 dimensional state are obtained from 1981 observational points with 2048 ensemble members. While data assimilated are SSH anomalies alone, an ensemble experiment of 2002/03 El Niño event based on the EnKF can predict consistent Niño 3 sea surface temperature (SST) anomalies about 5 months in advance.

1. INTRODUCTION

The El Niño-Southern Oscillation (ENSO) phenomenon is the strongest climate variation on seasonal to interannual timescales, and it depends essentially upon coupled interactions of the dynamics of ocean and atmosphere. ENSO is characterized by quasiperiodic interannual oscillation of tropical Pacific sea surface temperatures (SSTs) with a dominant period of approximately 4 years. Since ENSO affects not only global climate but ecosystems in and around the tropical Pacific and economies of several countries, successful prediction of ENSO is of great interest from scientific and social points of view.

With an atmosphere-ocean coupled model, Zebiak and Cane [1] (hereinafter referred to as ZC) have provided the first successful ENSO prediction. The ZC model is a nonlinear anomaly model of intermediate complexity, and reproduced an ENSO-like quasiperiodic variation. In the real ocean, however, even if a precursor of ENSO is detected, ENSO itself often does not follow the signature as the model predicts: the real ocean is more complex. On the basis of intermediate coupled models, data assimilation studies have been carried out for better prediction and reanalysis of ENSO events. Adopted assimilation schemes to the coupled models were the representer method [2], the adjoint method [3], a reduced-order Kalman filter (ROKF) [4], or the extended Kalman filter (EKF) [5]. However, the schemes listed above could not treat the ZC coupled model without modification or approximation. When the above two variational methods (the representer method and the adjoint method) were applied, the atmospheric component was modified or simplified in order to derive the adjoint equations. The two sequential methods (ROKF and EKF) require a linear approximation of the nonlinear model. It means that the advanced schemes may degrade the model's ability to reproduce nonlinear development.

In addition, the lack of system noise or Gaussian system noise employed by these works is another problem. If the model dynamics is assumed to be perfect (so-called "the strong constraint"), and the optimal initial and/or boundary conditions are estimated, assimilation experiments are carried out under the system noise-free condition [3, 4]. This condition assumes a system model without the noise,

$$x_t = f_t(x_{t-1}), \tag{1}$$

where x_t is a state vector and f_t describes the model dynamics. Apparently the assumption of perfect models is questionable for simple coupled models as the ZC model. On the other hand, the weak constraint was employed by [2] and [5]; they assumed Gaussian system noise added to the model equation as

$$x_t = f_t(x_{t-1}) + v_t \tag{2}$$

where v_t is system noise generated from the Gaussian distribution. This system model is more reasonable compared with the strong constraint, Eq. (1). The weak constraint model is, however, still questionable because the Gaussian system noise is added for all state variables. Since the model includes nonlinear transformation, Gaussian perturbation is converted to non-Gaussian perturbation, and some of the system variables then should be affected by non-Gaussian system noise. However, it is difficult to set alternative noise suitable for nonlinear systems and this is actually impossible for the ROKF and the EKF because the system noise is treated with covariance matrices.

The ensemble Kalman filter (EnKF) [6] can solve these problems. First, the EnKF can handle dynamic models without any modification or approximation since the EnKF requires neither linearized model nor adjoint model. Second, the EnKF can easily introduce a non-Gaussian system noise appropriate to nonlinear models. The EnKF treats the system noise by realizations generated from an assumed distribution. When we impose the realizations of the system noise on certain variables, their effects are conveyed to other variables according to the model equation. As a result, even if the imposed realizations are from Gaussian, received effects by the other variables can be non-Gaussian which can be considered as the system noise consistent with the nonlinear model. These two advantages of the EnKF produce more accurate prediction error. In addition, the EnKF can provide reasonable initial perturbation for ensemble forecasting.

In spite of these advantages, the EnKF has not been applied to coupled atmosphere-ocean models as far as the authors examined. One reason may be required computational load of the EnKF; mathematically, we need ensemble members larger than the number of data points. However, the recent development of computer technology gradually enables us to conduct EnKF experiments based on the coupled models. In the present paper we report the first application of the EnKF to a coupled atmosphere-ocean model.

2. ENKF AND ENKS

2.1. EnKF

The EnKF [6] can deal with a nonlinear system equation and a linear observation equation given by

$$x_t = f_t\left(x_{t-1}, v_t\right), \qquad (3)$$

$$y_t = H_t x_t + w_t, \qquad (4)$$

where x_t is a state vector at time t, f_t is a function describing model dynamics including the effects of system noise v_t, y_t is a time series, H_t is a matrix, and w_t is observation noise. The nonlinear system equation (3) includes the weak constraint system equation (2), and noises v_t and w_t can be non-Gaussian. In the EnKF, distributions of x_t, v_t, and w_t are approximated by many realizations (ensemble). With N ensemble members, the conditional distribution of the one-step-ahead predictor is approximated by an ensemble consisting of

$$x_{t|t-1}^{(n)} = f_t\left(x_{t-1|t-1}^{(n)}, v_t^{(n)}\right), \text{ for } n = 1, \cdots, N \qquad (5)$$

where $x_{t-1|t-1}^{(n)}$ $(n = 1, \cdots, N)$ is a member of an ensemble that approximates the previous filtered state distribution, and $v_t^{(n)}$ $(n = 1, \cdots, N)$ is a realization of v_t. An ensemble of the current filtered state $x_{t|t}^{(n)}$ $(n = 1, \cdots, N)$ is obtained with the approximated Kalman gain $\overline{K}_t$:

$$x_{t|t}^{(n)} = x_{t|t-1}^{(n)} + \overline{K}_t\left(y_t + w_t^{(n)} - H_t x_{t|t-1}^{(n)}\right), \quad \text{for } n = 1, \cdots, N \qquad (6)$$

$$\overline{K}_t = \overline{V}_{t|t-1} H_t^T \left(H_t \overline{V}_{t|t-1} H_t^T + \overline{R}_t\right)^{-1} \qquad (7)$$

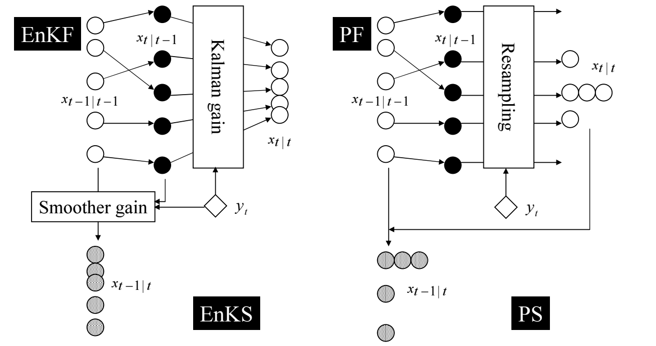

Fig. 1. EnKF, PF, EnKS and PS.

where $\overline{V}_{t|t-1}$ and $\overline{R}_t$ are covariance matrices of $x_{t|t-1}^{(n)}$ and $w_t^{(n)}$, respectively.

It may be instructive to compare the EnKF with the particle filter (PF [7, 8, 9]). As in the standard Kalman filter algorithm, both the EnKF and the PF consists of a one-step-ahead prediction step and a filtering step. The one-step-ahead prediction of the EnKF, Eq. (5), is a time transition of each realization of the state using a system noise realization, and is identical with that of the PF. What is different between the EnKF and the PF is their filtering steps. As in Eq. (6), the EnKF updates the realizations with the approximated Kalman gain (7) such that value of every realizations becomes closer to the observation. Upper parts of the left-hand panel in Figure 1 schematically shows the two steps of the EnKF. The PF, on the other hand, realizations of the filtered state are obtained through a resampling process from those of the predicted state with weights of their likelihood, $p\left(y_t|x_{t|t-1}^{(n)}\right)$ for $n = 1, \cdots, N$. That is, realizations which are located close to the observation will survive and also have their copies, while those far from the observation will be dismissed. It should be noted that value of realizations remains unchanged in the PF. In the upper parts of the right-hand panel in Figure 1, uppermost and lowermost realizations are dismissed, while central one obtains its two copies after the resampling.

2.2. EnKS

The ensemble Kalman smoother (EnKS [6]) is a smoothing algorithm for ensemble approaches, which stems from the fixed-lag smoothing algorithm. The smoothed distribution at time $t - j$ given $y_1, \cdots, y_t$ is approximated by an ensemble member computed as

$$x_{t-j|t}^{(n)} = x_{t-j|t-1}^{(n)} + \overline{K}_t(j)\left(y_t + w_t^{(n)} - H_t x_{t|t-1}^{(n)}\right), \quad \text{for } n = 1, \cdots, N \qquad (8)$$

$$\overline{K}_t(j) = \overline{V}_{t-j,t|t-1} H_t^T \left(H_t \overline{V}_{t|t-1} H_t^T + \overline{R}_t\right)^{-1}, \qquad (9)$$

where $\overline{V}_{t-j,t|t-1}$ is a cross-covariance matrix between $x_{t-j|t-1}^{(n)}$ and $x_{t|t-1}^{(n)}$. Index j denotes a time lag for smoothing and takes $1, \cdots, L$ where L is the fixed lag.

Lower parts of Figure 1 compares the EnKS and the particle smoother (PS [8, 9]). In the EnKS (8), every realizations of the filtered state is updated by a future observation and the approximated smoother gain (9). In the PS, realizations of the smoothed state are obtained according to resampling at the future time step. That is, past realizations that generate the future resampled realizations will survive or rather duplicate, while those will dismiss if their descendant is not resampled.

3. SIMULATION MODEL AND DATA

The ZC model couples two linear shallow-water equations: a steady-state atmospheric model and a dynamic reduced-gravity ocean model. The atmospheric component is forced by heating that depends on SST and surface wind convergence. The SST evolves in time according to the thermodynamic equation. The ocean component is forced by surface wind stress calculated from surface wind through the bulk formula. The ZC model includes nonlinear equations for heating anomaly due to the surface wind convergence and for the wind stress. All variables are anomalies with respect to the prescribed monthly mean climatology. Dimension of the state of the ZC model amounts to 54,403.

We assimilated the sea surface hight (SSH) anomaly observations by TOPEX / Poseidon (T/P) altimetry from its cycle 1 to cycle 364 (from September 23, 1992 to August 11, 2002). The number of data points at each cycle is usually 1981, but it can decrease down to 1,720 due to partially missing data, and at cycle 118 the data are totally missing.

4. ASSIMILATION EXPERIMENT

4.1. System noise, Observation noise, and other parameters

As seen in most simulation models, the ZC model computes time evolution of the state x_t in the form of the system equation without system noise imposed:

$$x_t \quad = \quad f_t\left(x_{t-1}, v_t = 0\right). \tag{10}$$

If we add a process for imposing system noise somewhere in the computation, the ZC model immediately results in a system equation, Eq. (3). We make an EnKF experiment by adding a system noise to the thermocline depth anomaly,

$$h_{t-1} \quad \rightarrow \quad h_{t-1} + v_t, \tag{11}$$

which can be interpreted as an uncertainty in the prescribed monthly mean thermocline depth. We first generate Gaussian noise and then modified the realizations such that they satisfy a physical constraint of the ZC model: System noise v_t added to the Rossby component of the thermocline depth h_{t-1} should be orthogonal to the Kelvin waves.

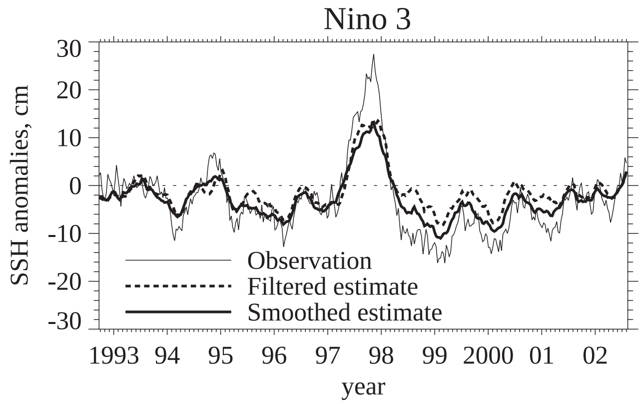

Fig. 2. Area-averaged SSH anomalies of the observation (thin), the filtered state (thick dashed), and the smoothed state (thick solid) in Niño 3 ($5°$N–$5°$S, $150°$–$90°$W).

We assume a Gaussian for observation noise: $w_t \sim N\left(0, R_t\right)$. To construct R_t, we at first treat the SSH anomaly data of each point as one dimensional time series and smooth it with the first-order trend model. Applying the same procedure to the other data points, we obtain the residuals for all points inside the basin. With the residuals we calculate a sample covariance matrix and assume it to be R_t.

With 2,048 ensemble members, we run the EnKF procedure from T/P cycle 1 to cycle 364. The number 2,048 is selected because it is larger than that of the maximum data points, 1,981. Model time step is set to be identical to the T/P sampling interval, 9.915625 days, and spin-up period is set to 90 years (3316 iterations), when the characteristic standing oscillation is reproduced.

4.2. Result

The observation, the filtered state, and the smoothed state are compared in Figure 2, which shows time evolution of the area-averaged SSH anomalies in the Niño 3 SST regions. Thin, thick dashed, and thick solid lines represent the observation, the filtered estimate, and the smoothed estimate, respectively. Here we let the fixed lag $L = t - 1$. All states before time t then will be updated by y_t, and resultant states smoothed by y_{364} are equivalent to those obtained by the fixed interval smoother. Time delay in the filtered estimates was adjusted in the smoothed estimates. The retrieved negative anomalies were clearly seen in Niño 3 1995 and during 1998–2000.

Figure 3 shows a prediction experiment of 2002/03 ENSO event. Predicted states based on the data until October, 2001, showed a gradual increase of the Niño 3 anomalies until May, 2002, and a rapid increase that may correspond to the 2002/03 ENSO event. However, the estimated peak amplitude amounted to over $3°$C, which was overestimated compared with the observation ($\sim 1.5°$C). If we started the prediction in later months,

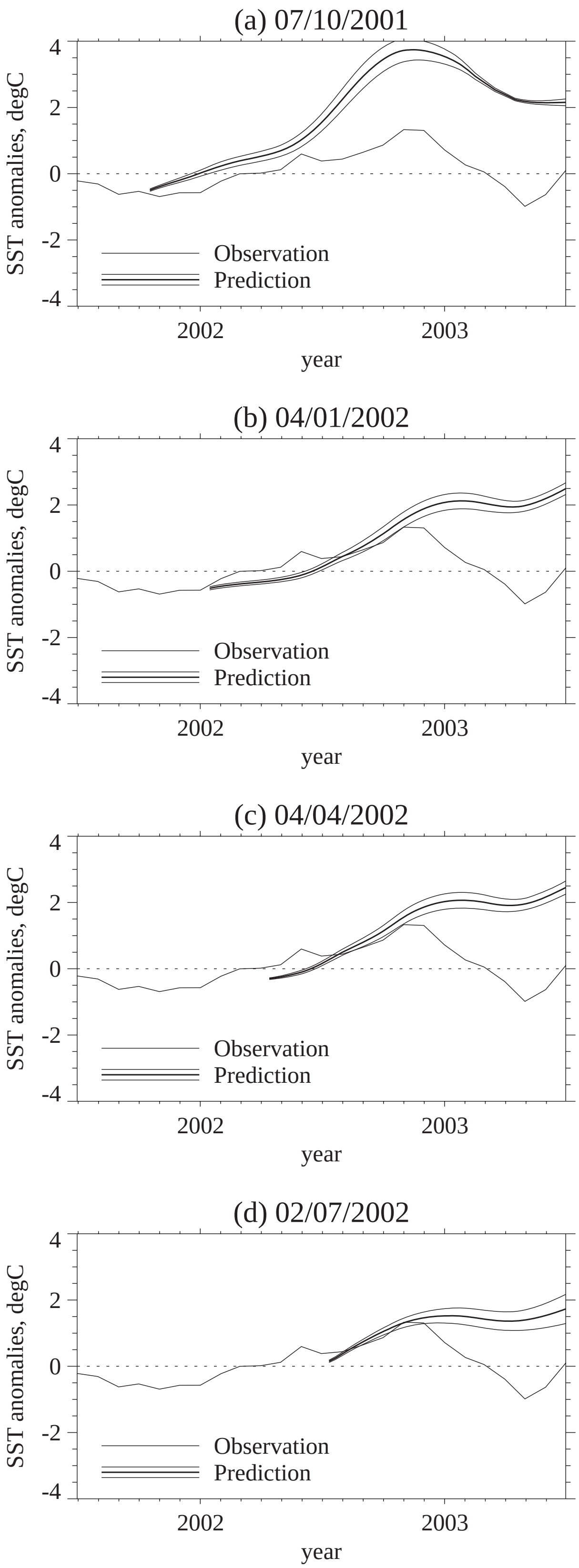

Fig. 3. SST anomalies observed in Niño 3 (5°N–5°S, 90°–150°W, black lines), accompanied with predicted ones (red lines) starting from the filtered state on (a) October 7, 2001, (b) January 4, 2002, (c) April 4, 2002, and (d) July 2, 2003.

predicted Niño 3 indices became more reasonable to the observation. Actually, after three or six months, predicted states also demonstrated increasing trends of the Niño indices but the peak amplitudes became smaller. Finally, prediction from the filtered state on July, 2002, gave a much accurate state distribution. An observed positive excursion related to the ENSO event was predicted within the deviations. However, while observed SST anomalies decrease after December, 2002, the prediction gave rather continued warm period. We infer that the prediction gives reasonable Niño 3 index within future five months.

5. REFERENCES

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