Particle Filter Based Method for Maneuvering Target Tracking

Norikazu Ikoma*, Naoyuki Ichimura†, Tomoyuki Higuchi‡, Hiroshi Maeda*

* Kyushu Institute of Technology, Kita-Kyushu, Fukuoka 804-8550, Japan
† Electrotechnical Laboratory, Tsukuba, Ibaraki 305-8568, Japan
‡ The Institute of Statistical Mathematics, Minato-ku, Tokyo 106-8569, Japan

Abstract

The aim is to track a maneuvering target with abrupt change of its state(acceleration). The target is modeled by general state space representation that consists of system model for the target dynamics and observation model for the radar observation process with nonlinear formula. Using heavy-tailed distribution such as Cauchy distribution instead of Gaussian distribution for the system noise, tracking performance is improved. Particle filter is used for state estimation of such nonlinear non-Gaussian model. A simulational experiment shows the improvement by our method compared with the Gaussian model using extended Kalman filter.

Key Words: Particle filter, State space model, Target tracking

1 Introduction

The aim of this research is to track a maneuvering target such as ship, aircraft, and so on. The target is modeled by general state space representation, which consists of system model and observation model, and particle filter is used for estimation of the state. Dynamics of the target is described by system model with state vector in Cartesian coordinate. We assume to use a radar to detect the target position, so range and bearing(angle) to the target are observed. This measurement process is formulated by observation model in polar coordinate with independent measurement noises for each observation variable. Due to the difference of coordinate systems between state and observation, the observation model must be a nonlinear system. Consequently, we have to use some nonlinear method for the state estimation, e.g. extended Kalman filter, or some other kind of nonlinear filter.

There is a problem in tracking of maneuvering target that the target might have abrupt change of its state(acceleration) by sudden operation of acceleration pedal, break, or steering. In the conventional researches, Gaussian noise is used both for observation and system noises. The use of Gaussian system noise causes blunt estimation to such abrupt changes of the state. To overcome this problem, we propose a use of uni-modal heavy-tailed non-Gaussian distribution for system noise, in this paper. As the heavy-tailed distribution, Cauchy distribution is typical. This is interpreted that usual(continuous) movement is denoted by around the uni-mode and the abrupt change of the target is represented by the heavy-tail with low probability, which is relatively higher than Gaussian one.

A special care should be taken in dealing with a state estimation of this kind of non-Gaussian nonlinear model, because the distribution of the state can be multi-modal. Since the use of extended Kalman filter means an approximation of the multi-modal distribution by Gaussian uni-modal distribution, it causes a problem that uni-mode of Gaussian approximation might be placed at low probability area between modes of multi-modal distribution. Consequently, we have to use more exact approximation of non-Gaussian distribution in the state estimation, such as [2],[5], and [7]. Sequential Monte Carlo method[7] is computationally effective among them. This kind of technique is also called particle filter method, e.g., [3],[4], and [6].

In the following sections, we firstly define the model for maneuvering target tracking. Continuous time model is defined and discrete time model is derived from the continuous one. Heavy-tailed distribution is introduced into the discrete time model. Secondly we will explain the state estimation method by using particle filter. Finally, the efficiency of the method is shown through a simulational experiment of maneuvering target. The result is compared to the Gaussian model with extended Kalman filter.

2 Model

Dynamics of maneuvering target is described by continuous model firstly. By discretizing the continuous model, we obtain a discrete time model. For the discrete time model, we will introduce a heavy-tailed non-Gaussian distribution as the system noise.

2.1 Continuos time model

Let the dynamics of the target be written in differential equation

$$\dot{\mathbf{x}}(t) \quad = \quad \mathbf{F} \quad \mathbf{x}(t) \quad + \quad \mathbf{G} \quad \mathbf{u}(t), \tag{1}$$

where $t \in \Re$ shows continuous time index,

$$\mathbf{x}(t) = [r_x(t), r_y(t), s_x(t), s_y(t), a_x(t), a_y(t)]^T \tag{2}$$

is a state vector consists of position $\mathbf{r}(t) = [r_x(t), r_y(t)]^T$, velocity $\mathbf{s}(t) = [s_x(t), s_y(t)]^T$, and acceleration $\mathbf{a}(t) = [a_x(t), a_y(t)]^T$ vectors ($\mathbf{x}^T$ shows transpose of $\mathbf{x}$). $\mathbf{F}$ is a state transition matrix

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha \end{bmatrix}, \tag{3}$$

$\mathbf{G}$ is a matrix for addition of system noise

$$\mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4}$$

and

$$\mathbf{u}(t) = [u_x(t), u_y(t)]^T \tag{5}$$

is the system noise vector.

The system model eq.(1) is interpreted as follows; change of the target position is determined by velocity, change of the velocity is determined by acceleration, and the acceleration is driven by the input $\mathbf{u}(t)$. As the input $\mathbf{u}(t)$, it is proposed by [8] to use Gaussian white noise since the change of acceleration is unknown.

2.2 Discretization

Solution of differential equation (1) is

$$\mathbf{x}(t) = \mathbf{A}(t - t_0)\mathbf{x}(t_0) + \int_{t_0}^{t} \mathbf{A}(t-\tau)\mathbf{G}\mathbf{w}(\tau)d\tau, \tag{6}$$

where

$$\mathbf{A}(t) \equiv e^{\mathbf{F}t} = \mathbf{I} + \mathbf{F}t + \frac{1}{2!}\mathbf{F}^2t^2 + \frac{1}{3!}\mathbf{F}^3t^3 + \cdots . \tag{7}$$

Let Δt is sampling time of discretization, and we will use discrete time points $t = t_0 + k\Delta t$ for $k = 0, 1, 2, \cdots$ in the following discussion. For these time points, we have

$$\begin{aligned} \mathbf{x}(t_{k+1}) \quad = \quad & \mathbf{A}(\Delta t)\mathbf{x}(t_k) \\ & + \int_{t_k}^{t_{k+1}} \mathbf{A}(t_{k+1} - \tau)\mathbf{G}\mathbf{w}(\tau)d\tau. \end{aligned} \tag{8}$$

By assuming zero-th order hold to system noise, i.e., $\mathbf{w}_k \equiv \mathbf{w}(t_k)$, we have the discretized formula

$$\mathbf{x}_{k+1} = \mathbf{A}(\Delta t)\mathbf{x}_k + \mathbf{B}(\Delta t)\mathbf{w}_k \tag{9}$$

where

$$\begin{aligned} \mathbf{B}(t) \quad & \equiv \quad \int_0^t \mathbf{A}(\tau)\mathbf{G}d\tau \\ & = \quad \mathbf{G}t + \tfrac{1}{2!}\mathbf{F}\mathbf{G}t^2 + \tfrac{1}{3!}\mathbf{F}^2\mathbf{G}t^3 + \cdots . \end{aligned} \tag{10}$$

2.3 Discrete time model

By applying the discretization shown in the previous subsection to the continuous model (1), we have the discretized model

$$\mathbf{x}_k \quad = \quad \mathbf{A} \quad \mathbf{x}_{k-1} \quad + \quad \mathbf{B} \quad \mathbf{v}_k, \tag{11}$$

where

$$\mathbf{x}_k = [r_x(k), r_y(k), s_x(k), s_y(k), a_x(k), a_y(k)]^T \tag{12}$$

is state vector,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 & a_1 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & a_1 \\ 0 & 0 & 1 & 0 & a_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 0 & e^{-\alpha\Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\alpha\Delta t} \end{bmatrix} \tag{13}$$

is state transition matrix,

$$\mathbf{B} = \begin{bmatrix} b_1 & 0 \\ 0 & b_1 \\ b_2 & 0 \\ 0 & b_2 \\ b_3 & 0 \\ 0 & b_3 \end{bmatrix} \tag{14}$$

is a matrix for addition of system noise, and

$$\mathbf{v}_k = [v_x(k), v_y(k)]^T \tag{15}$$

is system noise vector. The items of eq. (13) and (14) are as follows;

$$b_1 \quad = \quad \frac{1}{\alpha}\left(\frac{(\Delta t)^2}{2} - a_1\right), \tag{16}$$

$$a_1 = b_2 \quad = \quad \frac{1}{\alpha}\left(\Delta t - a_2\right), \tag{17}$$

$$a_2 = b_3 \quad = \quad \frac{1}{\alpha}\left(1 - e^{-\alpha\Delta t}\right). \tag{18}$$

As a result of discretization of Gaussian white system noise $\mathbf{u}(t)$ in continuous time model, we have system noise vector

$$\mathbf{v}_k \sim N(\mathbf{0}, \mathbf{Q}), \quad \mathbf{Q} = \begin{bmatrix} {\tau_x}^2 & 0 \\ 0 & {\tau_y}^2 \end{bmatrix}. \tag{19}$$

As shown above, it becomes Gaussian white and independent in each element.

2.4 Observation model

Measurement process by radar of the target position is denoted by observation model

$$\mathbf{y}_k \quad = \quad \mathbf{h}(\mathbf{x}_k) \quad + \quad \mathbf{w}_k, \tag{20}$$

where

$$\mathbf{y}_k = [y_\theta(k), y_g(k)]^T \tag{21}$$

is observation vector consists of bearing $y_\theta(k)$ and range $y_g(k)$,

$$\mathbf{h}(\mathbf{x}_k) = \begin{bmatrix} \tan^{-1}\left\{\dfrac{r_x(k)}{r_y(k)}\right\} \\ \\ \sqrt{r_x(k)^2 + r_y(k)^2} \end{bmatrix} \tag{22}$$

denotes nonlinear measurement process by radar characteristic, and

$$\mathbf{w}_k = [w_\theta(k), w_g(k)]^T \tag{23}$$

is observation noise vector with its distribution

$$\begin{bmatrix} w_\theta(k) \\ w_g(k) \end{bmatrix} \sim N(\mathbf{0}, \mathbf{R}), \quad \mathbf{R} = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_g^2 \end{bmatrix}. \tag{24}$$

Note that it is possible to use transformation on observation series from polar coordinate to Cartesian coordinate. However, then, we have to use non-Gaussian distribution for the observation noise caused by the translation. Due to the direct formulation of measurement process by nonlinear equation (22), we can use simple distribution (24). So it is rather straightforward to use the above defined nonlinear model than using transformation on observation series.

2.5 Heavy-tailed system noise

For the aim to tracking the maneuvering target with abrupt change of its acceleration, we use heavy-tailed non-Gaussian distribution instead of Gaussian one for system noise eq.(19) as follows;

$$\mathbf{v}_k \sim C(\mathbf{0}, \mathbf{Q}_c), \quad \mathbf{Q}_c = \begin{bmatrix} {q_x}^2 & 0 \\ 0 & {q_y}^2 \end{bmatrix}, \tag{25}$$

where, $\mathbf{C}$ denotes heavy-tailed distribution with central position $\mathbf{0}$ and dispersion $\mathbf{Q}_c$. Cauchy distribution is typical as the heavy-tailed distribution, as shown in **Figure 1**. We will explain more in scalar case for convenience. Probability density function Cauchy distribution is given by

$$p_c(v; q) = \frac{q}{\pi(v^2 + q^2)}, \tag{26}$$

where central position is 0 and dispersion is controlled by q. It has relatively high probability for large $|v|$ values compared with Gaussian distribution.

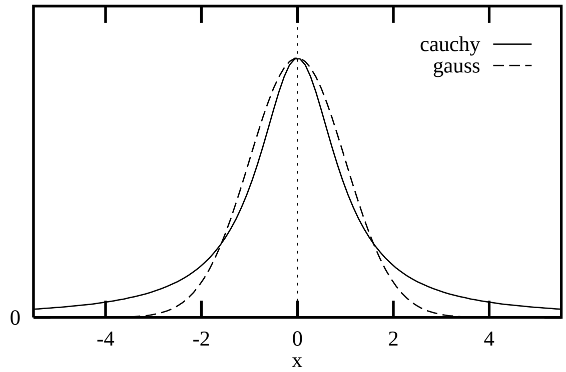

Figure 1. Probability density function of Cauchy distribution(solid line) and Gauss distribution(dashed line)

3 Particle filter

For state estimation of non-Gaussian nonlinear model as described above, non-Gaussian nonlinear filtering method is required. The key of the filtering method is how to approximate the non-Gaussian distribution of the state. There are some conventional researches, e.g., Gaussian sum approximation [2], numerical representation of non-Gaussian distribution [5], and approximation by particle called Sequential Monte Carlo method[7].

The use of particles to approximate non-Gaussian distribution has computational advantage compared with other approximation. Gaussian-sum approximation [2] has combinatorial problem in the computation, and numerical approximation of distribution [5] has exponential order of computation. Contrary to them, approximation by particles has computational cost of the order of the number of particles.

There are several methods in SMC, such as bootstrap filter[3], conditional density propagation (CONDENSATION) [4], and Monte Carlo filter [6]. We have employed Monte Carlo filter (MCF) among them, and will be is explained in the following section.

3.1 General state space representation

MCF can estimate the state for general class of state space model[6]. We use a definition of general state space representation, which is a subset of the class, and is defined as follows;

$$\mathbf{x}_k = \mathbf{g}(\mathbf{x}_{k-1}, \mathbf{v}_k), \qquad \mathbf{v}_k \sim q(\,\cdot\,; \mathbf{Q}) \tag{27}$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{w}_k), \qquad \mathbf{w}_k \sim r(\,\cdot\,; \mathbf{R}) \tag{28}$$

where $\mathbf{g}(\mathbf{x},\mathbf{v})$ and $\mathbf{h}(\mathbf{x},\mathbf{w})$ are nonlinear functions, and $q(\ \cdot\ ;\mathbf{Q})$ and $r(\ \cdot\ ;\mathbf{R})$ are non-Gaussian distributions. The target tracking model defined at the previous section is a special case of this representation.

Let the observation series is denoted by

$$\mathbf{Y}_N = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_N\} \tag{29}$$

The problem of state estimation is divided into three types of sub-problems depending on the time relationship between the state and the observations. They are called prediction, filtering, and smoothing, in general. Let us fix the problem to estimate the current state $\mathbf{x}_k$, then the three sub-problems become the estimation of the following probability density function(pdf)s; $p(\mathbf{x}_k|Y_{k-1})$ is for one-step-ahead prediction, $p(\mathbf{x}_k|Y_k)$ is for filtering, and $p(\mathbf{x}_k|Y_{k+L})$ is for smoothing with fixed lag L.

3.2 State approximation by particles

The key idea of MCF is an approximation of non-Gaussian distribution by many number of its realizations. These realizations are called "particles". Filtering procedures are done by using these particles instead of distribution itself. Notation of particles are as follows; for one-step-ahead prediction,

$$\left\{\mathbf{p}_1^{(k)}, \mathbf{p}_2^{(k)}, \cdots, \mathbf{p}_M^{(k)}\right\} \sim p(\mathbf{x}_k|Y_{k-1}), \tag{30}$$

filtering,

$$\left\{\mathbf{f}_1^{(k)}, \mathbf{f}_2^{(k)}, \cdots, \mathbf{f}_M^{(k)}\right\} \sim p(\mathbf{x}_k|Y_k), \tag{31}$$

and smoothing(with lag L)

$$\left\{\mathbf{s}_1^{(k|k+L)}, \mathbf{s}_2^{(k|k+L)}, \cdots, \mathbf{s}_M^{(k|k+L)}\right\} \sim p(\mathbf{x}_k|Y_{k+L}). \tag{32}$$

3.3 Filtering procedure

Following two procedures are are alternatively used.
One-step-ahead prediction:

$$\mathbf{p}_i^{(k)} = \mathbf{g}(\mathbf{f}_i^{(k-1)}, \mathbf{v}_i^{(k)}) \tag{33}$$

where $\left\{\mathbf{v}_1^{(k)}, \mathbf{v}_2^{(k)}, \cdots, \mathbf{v}_M^{(k)}\right\} \sim q(\mathbf{v};\mathbf{Q})$.
Filtering:
Calculate likelihood of each particle by

$$\alpha_i^{(k)} = p(y_k|\mathbf{p}_i^{(k)}) = r\left(\mathbf{h}^{-1}(y_k, \mathbf{p}_i^{(k)}); \mathbf{R}\right). \tag{34}$$

Resample according to

$$\mathbf{f}_i^{(k)} = \begin{cases} \mathbf{p}_1^{(k)} & \text{with prob.} & \alpha_1^{(k)} / \sum_{j=1}^{M} \alpha_j^{(k)} \\ \vdots & \vdots & \vdots \\ \mathbf{p}_M^{(k)} & \text{with prob.} & \alpha_M^{(k)} / \sum_{j=1}^{M} \alpha_j^{(k)} \end{cases} \tag{35}$$

Starting from particles of $p(\mathbf{x}_0|Y_0)$, alternatively applying (33) and (35) in the order of $k = 1, 2, \cdots, N$, particles of $p(\mathbf{x}_k|Y_{k-1})$ and $p(\mathbf{x}_k|Y_k)$ are obtained for all k.

3.4 Smoothing

By augmenting the particle described as below, we have smoothing estimation using the same algorithm iterating the filtering and the one-step-ahead prediction. The augmented particle consists of smoothing particles(i.e. for the past times) and filtering/one-step-ahead prediction particle(for the current time). Assume that fixed lag smoothing with lag L is performed, then, the i-th augmented particle for one-step-ahead prediction is

$$\mathbf{P}_i^{(k)} \equiv \left\{\mathbf{p}_i^{(k)}, \mathbf{s}_i^{(k-1|k-1)}, \mathbf{s}_i^{(k-2|k-1)}, \cdots, \mathbf{s}_i^{(k-L|k-1)}\right\}, \tag{36}$$

and for filtering

$$\mathbf{F}_i^{(k)} \equiv \left\{\mathbf{f}_i^{(k)}, \mathbf{s}_i^{(k-1|k)}, \mathbf{s}_i^{(k-2|k)}, \cdots, \mathbf{s}_i^{(k-L|k)}\right\}. \tag{37}$$

Note that $\mathbf{f}_i^{(k)}$ can be rewritten by $\mathbf{s}_i^{(k|k)}$ according to its definition.

By applying the same algorithm of one-step-ahead prediction and filtering to particles of $\mathbf{F}_i^{(k)}$ and $\mathbf{P}_i^{(k)}$, we obtain particles for the fixed lag(L) smoothing of time $k-L$ by extracting $\mathbf{s}_i^{(k-L|k)}$ from eq.(37).

Remark that by theoretical point of view, the augmented particles approximate the joint distribution

$$\left\{\mathbf{F}_i^{(k)}\right\} \sim p(\mathbf{x}_k, \mathbf{x}_{k-1}, \cdots, \mathbf{x}_{k-L}|Y_k), \tag{38}$$

and

$$\left\{\mathbf{P}_i^{(k)}\right\} \sim p(\mathbf{x}_k, \mathbf{x}_{k-1}, \cdots, \mathbf{x}_{k-L}|Y_{k-1}). \tag{39}$$

3.5 Likelihood

Likelihood of the model to the given data set can be approximately obtained by

$$\begin{aligned} l(\vartheta) &= \sum_{k=1}^{N} \log p(y_k|Y_{k-1}) \\ &\simeq \sum_{k=1}^{N} \log \left(\frac{1}{M}\sum_{j=1}^{M}\alpha_j^{(k)}\right). \end{aligned} \tag{40}$$

Where, $\vartheta = \{\mathbf{R}, \mathbf{Q}\}$ is called "hyperparameter" that governs the performance of state estimation. The optimal value of hyperparameter, denoted by $\hat{\vartheta}$, is determined by maximizing the log-likelihood, eq.(40) [1].

4 Simulation

To show the improvement by our method, a simulational experiment has been done as follows. At first, synthetic data have been generated to simulate the maneuvering target(ship), similarly to [9] but we use twice maneuver. They are shown in **Figure 2** by Cartesian coordinate(upper) and polar one(lower). The polar data are assumed to be actually observed and used for state estimation. In these figures, solid lines show the observation, and dashed lines show the true trajectory. It can be seen that noise features are different depending on the coordinate system.

Non-Gauss(Cauchy) model is applied to the data by using particle filter(MCF) with the number of particles $M = 100,000$. Gauss model by using extended Kalman filter(EKF) is also applied to the same data. Variances of observation noise vector are assumed to be known as $\sigma_\theta^2 = 9\times10^{-2}$ and $\sigma_g^2 = 3\times10^{-6}$. As a distribution of initial state, we assume that the average of position, velocity and acceleration are known and the variance are small(assume 1.0 here for all element). Hyperparameters are determined by maximizing the log-likelihood for both models as shown in **Table 1**, where underline shows the determined.

Estimation results of filtering of x-axis, $\hat{\mathbf{x}}_{k|k}$, are shown in **Figure 3** for position, **Figure 4** for velocity, and **Figure 5** for acceleration. In these figures, solid lines show the estimation result, and dashed lines show the true trajectory. The result of Gauss model(EKF) is the mean value of marginal distribution since it is Gaussian. There are several choice to show the result in case of non-Gauss model, here, the result of Cauchy(MCF) is the mean value of marginal distribution computed by averaging the particles for each element.

By looking the results, when no maneuver, Cauchy model has smooth trajectory but Gauss model has been fluctuated. On the other hand, when maneuvered, both model track the true trajectory with certain delay. The performances on the maneuvered period between two models are almost the same, exactly saying Cauchy model is slightly better than Gauss model. One may consider that the use of smaller variance τ^2 in Gauss model will have smoother one at the no maneuver period, however, then it will only have blunt estimation at the maneuvered period.

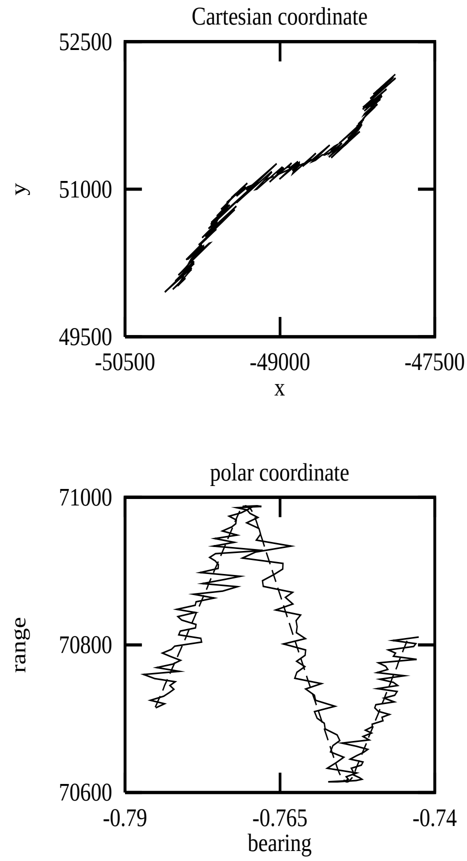

Figure 2. Trajectory and observation

Table 1. Log-likelihood

τ^2	log-likelihood	
	Cauchy(MCF)	Gauss(EKF)
1e-3	402.25	530.76
1e-4	470.36	$\underline{588.59}$
1e-5	514.91	566.87
1e-6	539.25	65.84
1e-7	$\underline{554.74}$	-3473.44
1e-8	246.48	-25113.5

5 Conclusion

By using heavy-tailed distribution(Cauchy distribution) instead of Gaussian distribution for the system noise of state space model, the improvement of tracking performance of a maneuvering target with abrupt change of its state(acceleration) has been shown through a simulational experiment by comparing with the Gaussian model using extended Kalman filter. Application to the real data is the future work.

Acknowledgement

This study was carried out under the ISM Cooperative Research Program (2000-ISM·CPR-1005).

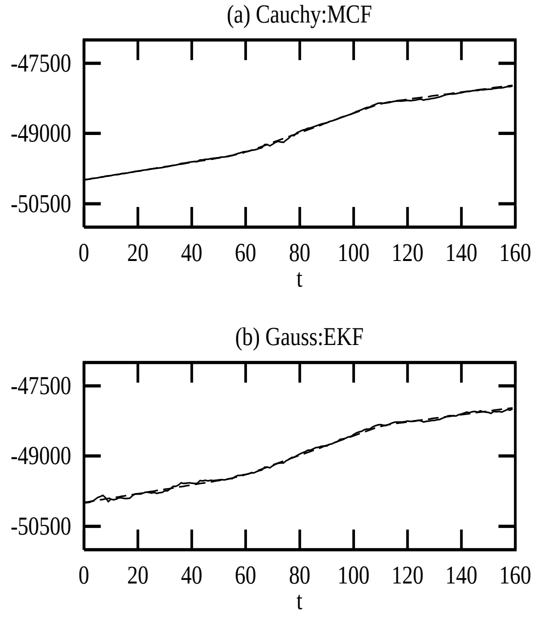

Figure 3. Estimation results, $\hat{r}_x(k|k)$

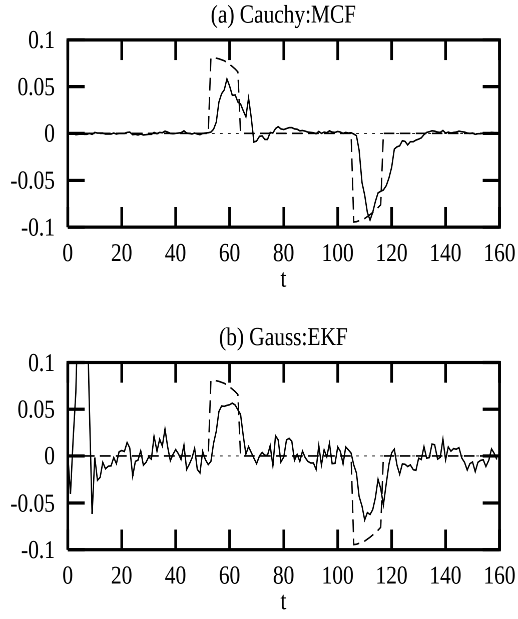

Figure 5. Estimation results, $\hat{a}_x(k|k)$

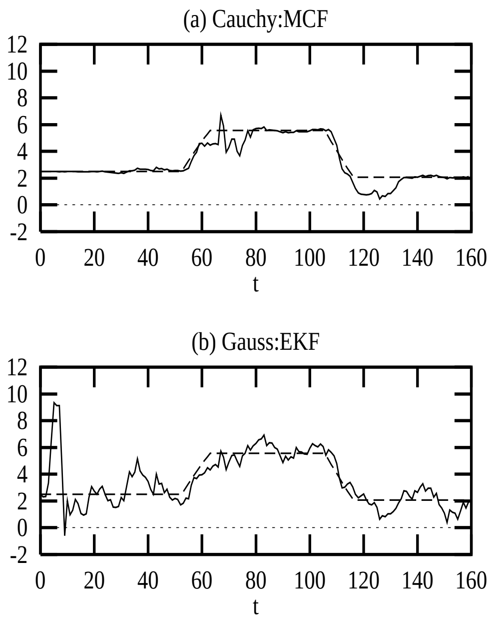

Figure 4. Estimation results, $\hat{v}_x(k|k)$

References

[1] H.Akaike. Likelihood and the Bayes procedure, *Bayesian Statistics*, Valencia, Spain: University Press, 1980.

[2] D.L.Alspach and H.W.Sorenson. Nonlinear Bayesian Estimation Using Gaussian Sum Approximations, *IEEE Trans. A.C.*,**17**, No.4, 439-448, 1972.

[3] N.J.Gordon, D.J.Salmond, and A.F.M.Smith. Novel approach to nonlinear/non-Gaussian Bayesian State Estimation, *IEE Proceedings-F*, **140**, No.2, 107-113, 1993.

[4] M.Isard and A.Blake. CONDENSATION – Conditional Density Propagation for Visual Tracking, *Journal of Computer Vision*, **29**, No.1, 5-28, 1998.

[5] G.Kitagawa. A Nonlinear Smoothing Method for Time Series Analysis, *Statistica Sinica*, **1**, No.2, 371-388, 1991.

[6] G.Kitagawa. Monte Carlo filter and smoother for non-Gaussian nonlinear state space models, *Journal of Computational and Graphical Statistics* **5**,No.1,1-25, 1996.

[7] J.S.Liu and R.Chen. Sequential Monte Carlo methods for dynamic systems, *Journal of the American Statistical Association*, **93**, 1032-1044, 1998.

[8] R.A.Singer. Estimating optimal tracking filter performance for manned maneuvering targets, *IEEE Trans. Aerospace and Electronics Systems*, **Vol.AES-6**, No.4, pp.473-483, 1970.

[9] A.Ohsumi and S.Yasui. Tracking of a maneuvering target considering its kinematic constrains, *Proc. of world automation congress, 3rd intl. Sympo. on Intelligent automation and control*, 2000.