World Automation Congress
Third International Symposium on Intelligent Automation and Control

Maui, Hawaii
June 11-16, 2000

Automatic Identification Of Large-Scale Field-Aliend Current Structures: Statistical Approach

Tomoyuki Higuchi and Shin-Ichi Ohtani
AUTOMATIC IDENTIFICATION OF LARGE-SCALE FIELD-ALIGNED CURRENT STRUCTURES: STATISTICAL APPROACH

Tomoyuki HIGUCHI
The Institute of Statistical Mathematics, Japan

Shin-Ichi OHTANI
The Johns Hopkins University, Applied Physics Laboratory, U.S.A.

ABSTRACT

Characteristics of large-scale field-aligned currents (LSFACs) observed above the Earth’s ionosphere are highly variable, and we have been depending on visual examination to identify LSFAC systems. The objective of this paper is to report a new procedure that we developed to automatically identify the spatial structure of LSFACs from satellite magnetic field measurements. Depending on the number of LSFAC sheets crossed by a satellite and also on the intensity and flow direction (upward/downward) of each LSFAC, a plot of the east-west (E-W) magnetic component can have any shape. The required task is to automatically fit line segments to the plot. The procedure is based on the concept of the first-order B-spline fitting with variable node positions. The number of node points, which determines the number of FAC sheets, is optimized for each orbit so that the Akaike Information Criterion (AIC) is minimized. We applied the developed procedure to the whole data set of magnetic field measurements made by the Defense Meteorological Satellite Program-F7 (DMSP-F7) satellite during a total of 1339 days of its mission.

KEYWORDS: AIC, Data mining, Discovery science, Linear-spline, Principal component analysis, Variable node positions,

INTRODUCTION

The Earth’s magnetic field interacts with a plasma stream from the Sun, solar wind, and forms a magnetic cavity called the magnetosphere. Because plasma in the magnetosphere is collisionless, conductance along the magnetic field is much higher than that across the magnetic field. Therefore, the large-scale electric field in the magnetosphere is mapped along magnetic field lines to the ionosphere, where a current is driven in the direction of the electric field. As a consequence, a three-dimensional current system is formed [6]. The source and sink of energy are in the magnetosphere and the ionosphere, respectively, and the two regions are connected by field-aligned currents (FACs), currents flowing along magnetic field lines. Thus, FACs play an important role in the energy transfer from the magnetosphere to the ionosphere. FACs are also related to the dynamics of aurorae. In case the reader is more interested in magnetospheric physics, see textbooks [9] for an introductory guide.

FACs are generally examined by making use of magnetic field, rather than particle flux, measurements made by satellites above the ionosphere. The latitudinal width
(one to several degrees) of each FAC system is often much smaller than their characteristic scale in the azimuthal direction. The associated magnetic perturbation, $\mathbf{B}$, tends to lie in the direction of a FAC sheet. That magnetic component increases or decreases as the satellite crosses the FAC sheet, depending on the direction (upward or downward) in which a LSFAC flows and on how (poleward or equatorward) the satellite crosses it. The FAC intensity can be determined from the magnitude of the magnetic variation. Thus, the basic idea of the analysis of FACs is straightforward. However, previous studies of FACs depended on visual examination of the plot of magnetic field measurements, and therefore required significant efforts. In addition, visual determination of boundaries of FAC systems is subjective. In this study, we have developed an automatic procedure to determine a structure of LSFACs without any interactive operation.

The rest of the article is organized as follows. Section 2 explains the coordinate transformation from the original coordinate system to one suited for analyzing the LSFACs structure. Section 3 describes a model for magnetic field perturbations associated with LSFACs and defines a procedure to estimate parameters included in this model. Section 4 briefly summarizes an undergoing and future application of the proposed procedure.

**PRINCIPAL COMPONENT ANALYSIS**

**Spacecraft coordinate system**

In this study we use three-component magnetic field measurements $[A_{X}, A_{Y}, A_{Z}]$, sampled every second by the Defense Meteorological Satellite Program–F7 (DMSP–F7) satellite during the entire interval of its mission from December 1983 to January 1988. DMSP–F7 is a Sun-synchronous satellite with a nearly circular polar orbit at about 835 km in altitude, with its ascending and descending nodes at 1030 LT (local time) and 2230 LT, respectively. The orbital period is about 101 minutes. We divide a data file of each polar pass into two parts, dayside and nightside files, by the data point of the highest-latitude satellite position. We have a total of 71,594 data files. Each data file, $A_{n}$ ($n = 1, \ldots, N$), usually contains from 600 to 800 vector measurements.

The original DMSP–F7 magnetometer data are given in the spacecraft coordinate system in which the X axis is directed vertically downward, the Y axis is in the direction of the projection of the satellite orbital velocity onto the plane perpendicular to the X axis, and the Z axis completes a right-hand orthogonal system. Since in the high-latitude region the magnetic field is approximately vertical, magnetic perturbations, $\mathbf{B}$, caused by FACs should lie mostly in the YZ plane in the high-latitude region. If LSFACs have a sheet structure, the associated magnetic perturbation, $\mathbf{B}_{n} = A_{n} - C_{n} = [B_{X,n}, B_{Y,n}, B_{Z,n}]^{T}$, is expected to be parallel to the current sheet where $C_{n}$ is the terrestrial-origin magnetic field at the satellite position. The inclination of the orientation of FAC sheets from the Z axis, $\Phi_{Z}$, is different for different orbits.

We determine the orientation of LSFAC sheets by applying the principal component analysis (PCA) to $\mathbf{B}_{n}$ for the **interval of the satellite crossing of LSFACs**; the sheet orientation is defined as the orientation of the principal axis. We found that the measurement of the X (vertical) component occasionally includes artificial level shifts. However, because magnetic perturbations caused by LSFACs are horizontal in a good approximation, we can simply exclude the X-component from the PCA.
Preparatory determination of the interval of the satellite crossing of LSFACs

To explain how to determine the interval of the satellite crossing of LSFACs, we take a dayside data file of the Northern-Hemisphere as an example. Such a data file begins with measurements made equatorward (E) of LSFAC, followed by those inside (F), and then poleward (P) of LSFAC systems. We will denote those intervals as $I^E_Z$, $I^F_Z$, and $I^P_Z$, in this order. Here the subscript Z explicitly indicates that the division is made on the basis of the Z-component measurement. Note that because of the high orbital inclination of the DMSP-F7 satellite, the Z axis is directed approximately azimuthally in the direction we expect to see magnetic perturbations associated with LSFACs.

The characteristics of the magnetic variation such as a variance are significantly different between the two regions: inside and outside regions of LSFAC systems. A plot of $B_Z$ for outside regions ($I^E_Z$ and $I^P_Z$), is approximated by a linear trend and therefore a variance of the detrend signal is small. In contrast, the variance of the variation for $I^F_Z$ should be large, because the variation of $B_Z$ is structured.

We evaluate the variance of the linearly detrended signal for $I^E_Z$ and $I^P_Z$ and denote them by $U^E_Z$ and $U^P_Z$, respectively. Since the smaller-scale magnetic variations are embedded in large-scale ones associated with LSFACs, we evaluate a variance of the smoothed data for the interval of $[L^E_Z, L^P_Z]$. The smoothing procedure we use in this study is based on the state space model with Kalman filter and smoothing algorithms [2,8]. A trade-off parameter to control a smoothness of the result, specified by $\tau^2$, is given by the mean length of the 300 satellite crossings LSFACs systems [3]. We denote the smoothed data by $\tilde{B}_{Z,n}$. We use the variance of the first-order difference of $\tilde{B}_{Z,n}$, $\tilde{U}^E_Z$, as a quantity to characterize the disturbances observed in $I^E_Z$. The variance of magnetic field variations outside of LSFAC systems is represented by the average of $U^E_Z$ and $U^P_Z$ weighted with the number of data points of the corresponding interval and is denoted by $U^{E,P}_Z$. The optimal interval is determined by maximizing the ratio of $\tilde{U}^E_Z$ to $U^{E,P}_Z$, $Q(L^E_Z, L^P_Z) = \tilde{U}^E_Z / U^{E,P}_Z$.

Maximum variance direction

We perform the PCA for measured magnetic field vectors in the YZ plane, $(B_{Y,n}, B_{Z,n})$ ($n = L^E_Z, \ldots, L^P_Z$), for the selected interval, $I^E_Z$. Although the PCA determines the maximum variance orientation, an ambiguity remains concerning the direction. We choose the direction of the new coordinate axis so that the angle from the positive Z direction is smaller. We represent this angle as $\Phi_Z$. The square root of the ratio of the maximum eigen value to the minimum one is denoted by $\alpha$. Fluctuations are isotropic if $\alpha = 1$. Increasing in $\alpha$, the observed magnetic fluctuations tend to lie in a certain direction, which corresponds to the orientation of LSFAC systems.

Measured magnetic vectors are transformed from satellite coordinates to maximum–minimum variance coordinates by rotating by $\Phi_Z$ in the YZ plane. In this new coordinate system, the component in the maximum variance direction is denoted by $y_n$ ($n = 1, \ldots, N$) and will undergo a further procedure. If LSFACs systems have a sheet structure, the associated magnetic variation should be confined in $y_n$.

A MODEL FOR MAGNETIC FIELD PERTURBATION

Linear spline with variable nodes

We assume that a measurement $y_n$ ($i = 1, \ldots, N$) consists of a true value $f_n$
and an observation noise \( e_n \), that is, \( y_n = f_n + e_n \). The observation noise \( e_n \) is an independently and identically distributed (i.i.d.) white noise sequence that has a Gaussian (normal) distribution with mean 0 and variance unknown \( \sigma^2 \). In this study, we approximate \( f_n \) by a first-order B-spline function, that is, a sequence of straight lines \([15]\), which is sometimes called a linear spline \([7]\). In this study we call it the linear spline henceforth. For a given set of node positions, \( T_J = \{t_j | j = 1, \ldots, J\} \), where a linear spline has a kink, \( f_n \) can be specified by a set of \( J \) values at the node points, \( g_j (j = 1, \ldots, J) \), together with the values at both ends, \( g_0 = f_1 \) and \( g_{J+1} = f_N \). A set of \( g_j (j = 0, \ldots, J+1) \) for a linear spline with \( J \) nodes is denoted by \( G_J \) henceforth.

A set of \( g_j \) can be determined by minimizing the residual sum of squares (RSS), \( \text{RSS}_J \). This minimization of RSS can be reduced to a linear least-squares problem after an appropriate linear transformation of \( g_j \), which can then be solved by a normal numerical procedure \([5]\). Although node points are fixed in usual spline applications, the benefit of the spline function can be maximized when node points are allowed to move. We therefore treat a set of node positions, \( T_J \), as variables to be optimized. The minimization of \( \text{RSS}_J \) is nonlinear with respect to \( T_J \) and requires a numerical technique. We initially tried to use the quasi-Newton method, specifically speaking, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) updating formula \([3]\). However, this approach was found to require significant amount of computation and turned out to be infeasible for analyzing the whole DMSP-17 data set. Furthermore, in that procedure, two node positions sometimes intersect in the course of iteration. We therefore developed a method to give a better initial (starting) point for \( T_J \) in the \( J \)-dimensional space.

**Initial guess on node position**

Optimal node points of the linear spline fit should be located near local maximum and minimum points of large-scale variations. We select the positions of local maxima and minima of smoothed data as candidates for optimal node points. Two difficulties exist in smoothing the data. First, the characteristic length of large-scale variations is different from an event to an event, and therefore we cannot use a fixed value as the halfband-width of the smoothing; the halfband-width must be determined for each data file to remove smaller-scale variations embedded in signatures associated with LSFACs. Second, magnetic fluctuations observed in the field-aligned current region have different characteristics, such as their spatial scale, from those outside the field-aligned current region.

The procedure begins with dividing the interval of a data file into three subintervals: \( I_y^E = [1, L_y^E - 1] \), \( I_y^F = [L_y^E, L_y^F] \), and \( I_y^P = [L_y^F + 1, N] \). We determine the interval associated with LSFACs observed in \( y_n, [L_y^E, L_y^F] \), in the same way we did in Section 2 except that we use \( y \) instead of \( B_y \). The next step is to determine a characteristic spatial scale by calculating the auto-correlation function for each subinterval. The obtained characteristic spatial scale is used to design the low-pass filter \([3]\). Then, the data for each interval are smoothed with the corresponding low-pass filter. The results, which hereafter will be presented as \( y_n^E, y_n^F, y_n^P \).

As the candidates of node points, we select points of a local maximum or a local minimum of the smoothed data. Such candidates are denoted as follows: \( T^E = [t_1^E, \ldots, t_{M_E}^E] \), \( T^F = [t_1^F, \ldots, t_{M_F}^F] \), and \( T^P = [t_1^P, \ldots, t_{M_P}^P] \), where \( M_E, M_F, \) and \( M_P \) are the sum of the numbers of local maximum and minimum points for each
The intensity and density of a LSFAC are given by the level shift, $H_j$, and the slope of a segment, $\delta_j$, respectively. We denote the maximum absolute $H_j$ by $H_{\text{max}}$. In this study, a segment satisfying the several conditions in terms of a geophysical point of view is regarded as a LSFAC.

The fitness of a linear spline is often assessed by a root-mean-squared error: $\sigma^* = \sqrt{\text{RSS}_{j*}/N}$. However, because the intensity of LSFACs is highly variable, we found that a certain normalization is necessary. We decided to use $H_{\text{max}}$ for this. That is, we use $R_{j*} = \sigma^*/H_{\text{max}}$ to evaluate the fitness. Data files with $R_{j*} \leq 8$ undergo further investigation for a geophysical objective. In addition, $\alpha$ is also important in evaluating the structure of LSFACs, because $\alpha$ represents an extent to which a LSFAC has a sheet structure. For further investigation, $\alpha$ must be greater than 2.

We apply the linear spline fit with $J$ node points to $y_n$. We confine the value of $J$ to no more than 7. We first choose $J$ node points from $T_M$. The chosen set of $J$ node points is denoted by $T_j(k)$ ($k = 1, \ldots, K_j$), where $K_j$ is the number of the combination of $M$ and $J$: $K_j = \binom{M}{J}$. $T_j(k)$ is a subset of $T_M$.

For a given $T_j(k)$, we apply the linear spline fit to $y_n$. $G_j(k)$, which minimizes $\text{RSS}_j(k)$, the residual sum of squares for this set of node points, is regarded as optimal. The minimum $\text{RSS}_j(k)$ is denoted as $\text{RSS}_{j*}(k)$. The best $T_j(k)$ among the $K_j$ subsets, $T_j$, is defined as $T_j(k)$ giving the smallest $\text{RSS}_{j*}(k)$, $\text{RSS}_{j*}$. The additional refinement is carried out by the quasi-Newton method. The positions of the local maximum or minimum of the smoothed data do not always agree with those of the original data, and therefore this minor adjustment is required.

The final step of the linear spline fit is to determine the optimal number of node points, $J^*$. This task can be done by adopting the Akaike Information Criterion (AIC) $[1,14]$ for the model with $J$ node points defined by $AIC_J = N \log(\text{RSS}_J/N) + 4J + \text{constant}$, where a constant factor is independent of the selection of models $[5]$. The best fit is defined as the linear spline with the smallest $AIC_J$ among $AIC_J$ ($J = 1, \ldots, 7$). We designate that optimal node number by $J^*$. Correspondingly, we refer to RSS for the best model as $\text{RSS}_{J^*}$.

**Determination of number of sheets**

The procedure of the linear spline fit is to represent the observed sequence of magnetic field variations with a set of $(t_j, g_j)$ ($j = 0, \ldots, J^* + 1$), where $t_j$ and $g_j$ represent a node position and a value at that position, respectively. The $j$th current system is given by the segment for the $j$th interval, $I_j = t_{j+1} - t_j$ ($j = 0, \ldots, J^*$). The intensity and density of a LSFAC are given by the level shift, $H_j$, and the slope of a segment, $\delta_j$, respectively. We denote the maximum absolute $H_j$ by $H_{\text{max}}$. In this study, a segment satisfying the several conditions in terms of a geophysical point of view is regarded as a LSFAC.

The first subject we investigated with the developed procedure is the spatial structure of LSFACs. Ohtani et al. $[12]$ reported four events in which DMSP–F7 observed a four-FAC-sheet structure along dayside passes. Because of the rare occurrence of
the four-FAC-sheet structure, we need to look through a large amount of data for making a statistical study, which is extremely difficult to achieve without any computational procedure to identify the structure of FACs. The procedure we developed selected 517 northern and 436 southern passes along which DMSP-F7 observed four LSFACs [10].

Another example of the applications is a statistical study of the dependence of the intensity of FAC systems on solar wind conditions [11]. Understanding how a FAC system depends on solar wind conditions not only provides information on the generation mechanism of a FAC but also is important in predicting magnetospheric and ionospheric activities from the solar wind observation. Development of such a prediction system, space weather forecasting, is becoming an important element of space science, as space environment, which is influential to the operation of satellites, and more relevant to human activities. The developed automatic procedure to identify the structure of FAC systems can be used to conduct such scientific and environmental studies [4].

ACKNOWLEDGMENTS

The DMSP-F7 magnetometer data were provided by F. J. Rich, of National Space Science Data Center, through the World Data Center–A for Rockets and Satellites.

REFERENCES