Maneuvering Target Tracking by Using Particle Filter

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Abstract

The aim of this research is to track a maneuvering target, e.g. ship, aircraft, and so on. We use a state space representation to model this situation. Dynamics of the target is represented by system model, firstly in continuous time. Discretized system model is actually used. Position of the target is measured by radar, and this process is described by nonlinear observation model in polar coordinate. To follow abrupt changes of the target's motion due to sudden operation of acceleration pedal, break, and steering, we propose a use of heavy-tailed non-Gaussian distribution for the system noise. Consequently, the model we use here is a nonlinear non-Gaussian state space model. Particle filter is used to estimate the target's state of the nonlinear non-Gaussian model. Usefulness of the method is shown by simulation.

Key Words: Target tracking, Particle filter, State space model, Nonlinear, Non-Gaussian, Heavy-tailed distribution.

1. Introduction

Target tracking is one of the classical problems using state space representation and filtering/smoothing techniques. In this framework, dynamics of the target is represented by a system model and measurement process is written by observation model. By assuming all features of these two models are known, the remaining problem is to estimate a state of the models. Here, the state is a vector consists of position, velocity, and acceleration of the target. To solve this problem, Kalman filter is popularly used in case of all formulae are linear and all noises are Gaussian. When nonlinear formulae appear in the models, e.g., target position observed by a radar can be considered, extended Kalman filter is widely used for the state estimation. In both filters, probability distribution of the state is estimated by Gaussian, i.e., mean vector and covariance matrix are estimated.

There is a problem in tracking a maneuvering target that the target might have abrupt change of its state(acceleration) by sudden operation of acceleration pedal, break, or steering. In the conventional researches, Gaussian distribution is used for system noise, and this causes blunt estimation to such abrupt changes of the state. In this paper, we propose a use of uni-modal heavy-tailed non-Gaussian distribution for system noise to overcome this problem. Cauchy distribution is typical as the heavy-tailed distribution. The introduction of heavy-tailed distribution is interpreted that usual(frequently occur) movement is denoted by around the uni-mode and the abrupt change of the target is represented by the heavy-tail with low probability, which is relatively higher than Gaussian one.

In this situation, the state distribution can be multi-modal. This means that a use of extended Kalman filter might fail since the mode of Gaussian approximation might be placed at low probability area between modes of multi-modal distribution. Consequently, we have to use more exact approximation of non-Gaussian distribution for the state estimation. There are several methods of nonlinear non-Gaussian filter/smoother depending on the approximation of state distribution such as Gaussian sum[2], numerical representation[5], and using particles[7]. The use of particles is computationally effective among them, and we have employed it. This technique is called particle filter, and there are several researches e.g., bootstrap filter[3], conditional density propagation(CONDENSATION) [4] from a field of computer vision, and Monte Calro filter[6].

In the following sections, we will begin to define a model for maneuvering target tracking according to [9]. Continuous time model is firstly defined and discrete time model is derived from it. Heavy-tailed distribution, Cauchy distribution here, is introduced into the discrete time model. Next, we will explain a state estimation method for the nonlinear non-Gaussian model by using particle filter according to [6]. After that, to show the efficiency of the proposed method, a simulational experiment of maneuvering target has been done. In this experiment, the estimation result is compared with that of the Gaussian model with extended Kalman filter. At the end of this paper, we will conclude by making some remarks on our method.

2. Model

Firstly, dynamics of a maneuvering target is described by continuous time model with Gaussian white system noise according to [9]. After that, the model is discretized with respect to time, and we use it as a system model. Heavy-tailed non-Gaussian distribution is introduced to the system noise of the model.

2.1 Continuos time model

Let the dynamics of a target be written in differential equation

$$\dot{\mathbf{x}}(t) \quad = \quad \mathbf{F} \quad \mathbf{x}(t) \quad + \quad \mathbf{G} \quad \mathbf{u}(t), \tag{1}$$

where $t \in \Re$ is continuous time index,

$$\mathbf{x}(t) = [r_x(t), r_y(t), s_x(t), s_y(t), a_x(t), a_y(t)]^T \tag{2}$$

is a state vector consists of position $\mathbf{r}(t) = [r_x(t), r_y(t)]^T$, velocity $\mathbf{s}(t) = [s_x(t), s_y(t)]^T$, and acceleration $\mathbf{a}(t) = [a_x(t), a_y(t)]^T$ ($\mathbf{x}^T$ stands for transpose of $\mathbf{x}$, in this paper). $\mathbf{F}$ is a state transition matrix

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha \end{bmatrix}, \tag{3}$$

$\mathbf{G}$ is a matrix multiplyed to a system noise vector

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}} \tag{4}$$

and

$$\mathbf{u}(t) = [u_x(t), u_y(t)]^T \tag{5}$$

is the system noise vector.

The system model eq.(1) represents that variation of the target position is determined by velocity, variation of the velocity is determined by acceleration, and the acceleration is driven by the input $\mathbf{u}(t)$. The use of Gaussian white noise process $\mathbf{v}(t)$ as the input $\mathbf{u}(t)$ when the variation of acceleration is unknown has been proposed by [9].

2.2 Discretization

Solution of differential equation (1) is

$$\mathbf{x}(t) = \mathbf{A}(t - t_0)\mathbf{x}(t_0) + \int_{t_0}^{t} \mathbf{A}(t-\tau)\mathbf{G}\mathbf{v}(\tau)d\tau, \tag{6}$$

where t_0 is initial time, $\mathbf{v}(t)$ is Gaussian white noise process with mutually independent components, and

$$\mathbf{A}(t) \equiv e^{\mathbf{F}t} = \mathbf{I} + \mathbf{F}t + \frac{1}{2!}\mathbf{F}^2t^2 + \frac{1}{3!}\mathbf{F}^3t^3 + \cdots . \tag{7}$$

Let Δt is sampling time of discretization and use discrete time points $t = t_0 + k\Delta t$ for $k = 0, 1, 2, \cdots$, then we can use integer k to identify the discrete time points such as $\mathbf{x}(t_k) \equiv \mathbf{x}_k$. For these time points, we have

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}(\Delta t)\mathbf{x}_k \\ &\quad + \int_{t_k}^{t_{k+1}} \mathbf{A}(t_{k+1} - \tau)\mathbf{G}\mathbf{v}(\tau)d\tau. \end{aligned} \tag{8}$$

By assuming zero-th order hold to system noise, i.e., $\mathbf{v}_k = \mathbf{v}(\tau), \tau \in [t_k, t_{k+1})$, we have the discretized formula

$$\mathbf{x}_{k+1} = \mathbf{A}(\Delta t)\mathbf{x}_k + \mathbf{B}(\Delta t)\mathbf{v}_k \tag{9}$$

where

$$\begin{aligned} \mathbf{B}(t) &\equiv \int_0^t \mathbf{A}(\tau)\mathbf{G}d\tau \\ &= \mathbf{G}t + \tfrac{1}{2!}\mathbf{F}\mathbf{G}t^2 + \tfrac{1}{3!}\mathbf{F}^2\mathbf{G}t^3 + \cdots . \end{aligned} \tag{10}$$

2.3 System model

In eq.(9), by simply denoting $\mathbf{A}(\Delta t)$ and $\mathbf{B}(\Delta t)$ as $\mathbf{A}$ and $\mathbf{B}$ respectively, we have the system model

$$\mathbf{x}_k \quad = \quad \mathbf{A} \quad \mathbf{x}_{k-1} \quad + \quad \mathbf{B} \quad \mathbf{v}_k, \tag{11}$$

where,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 & a_1 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & a_1 \\ 0 & 0 & 1 & 0 & a_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 0 & \mathrm{e}^{-\alpha\Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathrm{e}^{-\alpha\Delta t} \end{bmatrix}, \tag{12}$$

$$\mathbf{B} = \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & b_1 & 0 & b_2 & 0 & b_3 \end{bmatrix}^{\mathrm{T}}. \tag{13}$$

The entities of eq.'s (12) and (13) are as follows;

$$b_1 = \frac{1}{\alpha}\left(\frac{(\Delta t)^2}{2} - a_1\right), \tag{14}$$

$$a_1 = b_2 = \frac{1}{\alpha}(\Delta t - a_2), \tag{15}$$

$$a_2 = b_3 = \frac{1}{\alpha}\left(1 - e^{-\alpha\Delta t}\right). \tag{16}$$

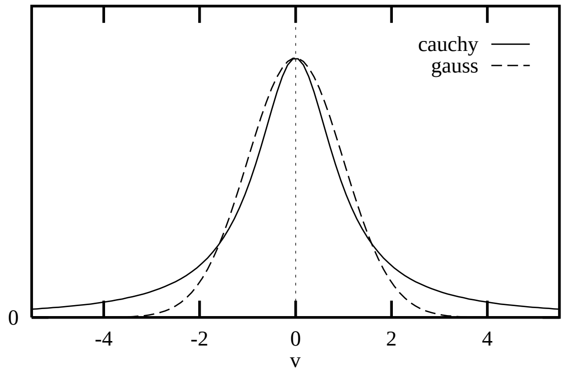

Figure 1: Probability density functions of Cauchy distribution and Gauss distribution

2.4 Heavy-tailed system noise

For the aim to track the maneuvering target with abrupt change of its acceleration, we propose a use of heavy-tailed non-Gaussian distribution

$$\mathbf{v}_k \sim C\left(\mathbf{0}, \mathbf{Q}_c\right), \quad \mathbf{Q}_c = \begin{bmatrix} {q_x}^2 & 0 \\ 0 & {q_y}^2 \\ o & \end{bmatrix}, \tag{17}$$

where, $C\left(\mathbf{0}, \mathbf{Q}_c\right)$ stands for heavy-tailed distribution with central position $\mathbf{0}$ and dispersion $\mathbf{Q}_c$. Since Cauchy distribution is typical one of the heavy-tailed distribution, so we have employed it as C. In scalar case, its probability density function (with central position is 0 and dispersion q) is given by

$$p_c(v) = \frac{q}{\pi(v^2 + q^2)} \tag{18}$$

and is shown in Fig.1. We can seen that Cauchy distribution has higher probability than Gaussian distribution for relatively large values of $|v|$.

2.5 Observation model

Assume that the target position is measured by a radar, and is denoted by observation model

$$\mathbf{y}_k \quad = \quad \mathbf{h}\left(\mathbf{x}_k\right) \quad + \quad \mathbf{w}_k, \tag{19}$$

where $\mathbf{y}_k = \left[y_\theta(k), y_g(k)\right]^T$ is observation vector consists of bearing $y_\theta(k)$ and range $y_g(k)$, and

$$\mathbf{w}_k = \left[w_\theta(k), w_g(k)\right]^T \tag{20}$$

is observation noise vector of Gaussian, such that

$$\begin{bmatrix} w_\theta(k) \\ w_g(k) \end{bmatrix} \sim N\left(\mathbf{0}, \mathbf{R}\right), \quad \mathbf{R} = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_g^2 \end{bmatrix}. \tag{21}$$

Measurement process by radar is denoted by nonlinear formula

$$\mathbf{h}\left(\mathbf{x}_k\right) = \begin{bmatrix} \tan^{-1}\left\{\dfrac{r_x(k)}{r_y(k)}\right\} \\ \sqrt{r_x(k)^2 + r_y(k)^2} \end{bmatrix} \tag{22}$$

3. State estimation

To estimate the state of non-Gaussian nonlinear state space model defined above, non-Gaussian nonlinear filtering method is needed. In the filtering method, non-Gaussian distribution of the state is approximated by some kind of fashion. There are several conventional method, e.g., Gaussian sum approximation[2], numerical representation[5], and approximation by particle[7].

The use of particle has computational advantage compared with other, i.e., Gaussian-sum approximation [2] has combinatorial problem in the computation, and numerical approximation of distribution [5] has exponential order of computation. Contrary to them, computational cost of approximation by particles is the order of number of particles.

The filter using particles is called Sequential Monte Carlo method[7]. There are several methods in SMC, such as bootstrap filter[3], conditional density propagation (CONDENSATION) [4], and Monte Carlo filter [6]. We have employed Monte Carlo filter (MCF) among them, and will be is explained in the following subsections.

3.1 General state space representation

MCF can estimate the state for general class of state space representation[6]. We use a subset of the class of general state space representation defined as follows;

$$\mathbf{x}_k = \mathbf{g}(\mathbf{x}_{k-1}, \mathbf{v}_k), \qquad \mathbf{v}_k \sim q(\ \cdot\ ; \mathbf{Q}) \tag{23}$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{w}_k), \qquad \mathbf{w}_k \sim r(\ \cdot\ ; \mathbf{R}) \tag{24}$$

In eq.'s (23) and (24), $\mathbf{g}(\mathbf{x}, \mathbf{v})$ and $\mathbf{h}(\mathbf{x}, \mathbf{w})$ are nonlinear functions, where we assume the inverse of the function $\mathbf{h}(\mathbf{x}, \mathbf{w})$ with respect to $\mathbf{w}$ exists (which is denoted by $\mathbf{h}^{-1}(\mathbf{y}, \mathbf{x})$), and $q(\ \cdot\ ; \mathbf{Q})$ and $r(\ \cdot\ ; \mathbf{R})$

are probability density function of appropriate distribution (which is non-Gaussian distributions, in general).

This general formula of state space representation is a wide class of the state space model. The target tracking model defined at the previous section is in this class of representation.

3.2 State estimation

Let the observation series be denoted by

$$\mathbf{Y}_N = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_N\}. \tag{25}$$

The problem of state estimation is to calculate conditional distribution of the state given the observations. It is divided into three types depending on the time relationship between the state and the observations. Assume that observations up to current time are given, then, estimation of the future state is called prediction, the current state is called filtering, and the past state is called smoothing. More detailed, to obtain $p(\mathbf{x}_k|Y_{k-1})$ is called one-step-ahead prediction, $p(\mathbf{x}_k|Y_k)$ is filtering, and $p(\mathbf{x}_k|Y_{k+L})$ is smoothing with fixed lag L.

3.3 State approximation by particles

MCF uses an approximation of non-Gaussian distribution by many number of its realizations, which are called "particles". Filtering and smoothing procedures are done by using these particles instead of distribution itself. Notation of particles are as follows. For one-step-ahead prediction,

$$\left\{\mathbf{p}_1^{(k)}, \mathbf{p}_2^{(k)}, \cdots, \mathbf{p}_M^{(k)}\right\} \sim p(\mathbf{x}_k|Y_{k-1}), \tag{26}$$

filtering,

$$\left\{\mathbf{f}_1^{(k)}, \mathbf{f}_2^{(k)}, \cdots, \mathbf{f}_M^{(k)}\right\} \sim p(\mathbf{x}_k|Y_k), \tag{27}$$

and smoothing(with lag L)

$$\begin{aligned}\left\{\mathbf{s}_1^{(k|k+L)}, \mathbf{s}_2^{(k|k+L)}, \cdots, \mathbf{s}_M^{(k|k+L)}\right\}& \\ \sim p(\mathbf{x}_k|Y_{k+L}).&\end{aligned} \tag{28}$$

3.4 Filtering procedure

Starting from particles of initial distribution $p(\mathbf{x}_0|Y_0)$, alternatively applying the following two procedures according to the order of time index $k = 1, 2, \cdots, N$, we have particles of one-step-ahead prediction $p(\mathbf{x}_k|Y_{k-1})$ and filtering $p(\mathbf{x}_k|Y_k)$ for all time $k = 1, 2, \cdots, N$.

<u>One-step-ahead prediction</u>:

$$\mathbf{p}_i^{(k)} = \mathbf{g}(\mathbf{f}_i^{(k-1)}, \mathbf{v}_i^{(k)}) \tag{29}$$

where

$$\left\{\mathbf{v}_1^{(k)}, \mathbf{v}_2^{(k)}, \cdots, \mathbf{v}_M^{(k)}\right\} \sim q(\mathbf{v}; \mathbf{Q}). \tag{30}$$

<u>Filtering</u>:

Calculate likelihood of each particle by

$$\alpha_i^{(k)} = p(y_k|\mathbf{p}_i^{(k)}) = r\left(\mathbf{h}^{-1}(y_k, \mathbf{p}_i^{(k)}); \mathbf{R}\right). \tag{31}$$

Resample particles according to

$$\mathbf{f}_i^{(k)} = \begin{cases} \mathbf{p}_1^{(k)} & \text{with prob.} \quad \alpha_1^{(k)} / \sum_{j=1}^{M} \alpha_j^{(k)} \\ \vdots & \qquad \vdots \qquad\qquad \vdots \\ \mathbf{p}_M^{(k)} & \text{with prob.} \quad \alpha_M^{(k)} / \sum_{j=1}^{M} \alpha_j^{(k)} \end{cases} \tag{32}$$

3.5 Smoothing

Smoothing is carried out by augmenting the particle as described below, and applying the filtering algorithm to the augmented particles. The augmented particle consists of smoothing particles for the past times and filtering/one-step-ahead prediction particle for the current time. Let us use fixed lag smoothing with lag L. In this case, the i-th augmented particle for one-step-ahead prediction is

$$\begin{aligned}\mathbf{P}_i^{(k)} \equiv \Big\{&\mathbf{p}_i^{(k)}, \mathbf{s}_i^{(k-1|k-1)}, \\ &\mathbf{s}_i^{(k-2|k-1)}, \cdots, \mathbf{s}_i^{(k-L|k-1)}\Big\},\end{aligned} \tag{33}$$

and that for filtering is

$$\begin{aligned}\mathbf{F}_i^{(k)} \equiv \Big\{&\mathbf{f}_i^{(k)}, \mathbf{s}_i^{(k-1|k)}, \\ &\mathbf{s}_i^{(k-2|k)}, \cdots, \mathbf{s}_i^{(k-L|k)}\Big\}.\end{aligned} \tag{34}$$

Note that $\mathbf{f}_i^{(k)}$ is equivalent to $\mathbf{s}_i^{(k|k)}$ according to its definition.

Apply the same algorithm of filtering, i.e., alternatively perform the one-step-ahead prediction and the filtering procedures to particles of $\mathbf{F}_i^{(k)}$ and $\mathbf{P}_i^{(k)}$. Then, we obtain particles for the fixed lag(L) smoothing of time $k-L$ by extracting $\mathbf{s}_i^{(k-L|k)}$ from eq.(34).

Note that in theoretical point of view, the augmented particles approximate the joint distribution

$$\left\{\mathbf{F}_i^{(k)}\right\} \sim p(\mathbf{x}_k, \mathbf{x}_{k-1}, \cdots, \mathbf{x}_{k-L}|Y_k), \tag{35}$$

and

$$\left\{\mathbf{P}_i^{(k)}\right\} \sim p(\mathbf{x}_k, \mathbf{x}_{k-1}, \cdots, \mathbf{x}_{k-L}|Y_{k-1}). \tag{36}$$

3.6 Likelihood

Likelihood of the model to the observation series (25) can be approximately obtained by

$$\begin{aligned} l(\vartheta) &= \sum_{k=1}^{N} \log p(y_k|Y_{k-1}) \\ &\simeq \sum_{k=1}^{N} \log\left(\frac{1}{M}\sum_{j=1}^{M}\alpha_j^{(k)}\right), \end{aligned} \tag{37}$$

where, vector ϑ is called "hyperparameter" that governs the performance of state estimation. The optimal value of hyperparameter, denoted by $\hat{\vartheta}$, is determined by maximizing the log-likelihood, eq.(37) [1]. In the target tracking model, the hyperparameter consists of covariance matrices of observation noise and system noise such that

$$\vartheta = \{\mathbf{R}, \mathbf{Q}\}. \tag{38}$$

Let us explain the role of the elements, in scalar case to simplify. When $\mathbf{R}$ is taken to be small and $\mathbf{Q}$ to be large, the observations are reliable and the state variables change quickly. On the other hand, in the opposite case, the observations are rather ignored and the state variables evolve smoothly.

4. Simulation

Synthetic data have been generated by simulating the maneuvering target(ship) with sampling time 0.01[sec], Trajectory is the same to [8] (which has single turn), and they are shown in Fig.2 in Cartesian coordinate. We assume the use of radar to observe the target's position with sampling time Δt = 3.75[sec], then the data are actually given in polar coordinate. We assume in this experiment that observation noises are very small, such as variance of bearing, σ_θ^2, is 10^{-10}, and that of range, σ_g^2, is 10^{-2}.

The proposed model has been applied to the data. MCF is used for the state estimation with the number of particles $100,000$. Gaussian model is also

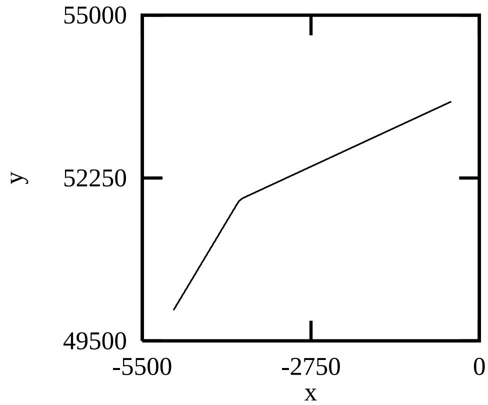

Figure 2: Trajectory

applied to the same data for the comparison. Extended Kalman filter(EKF) is used for the state estimation. For both models, we assume the variances of observation noise vector are known, and system noises are determined by log-likelihood with constraints ${q_x}^2 = {q_y}^2 \equiv q^2$. Then we have q^2 values 10^{-7} for Cauchy model and 10^{-6} for Gaussian model.

Estimation(filtering) results of position, velocity, and acceleration for time interval $k = 100 \sim 150$ are shown in Fig.3, Fig.4, and Fig.5, respectively. In each figure, solid line shows the median(of marginal distribution) of Cauchy model with MCF, long-dashed line shows the mean of Gaussian model with EKF, and short-dashed line shows the true trajectory.

Looking at the estimation result of acceleration $a_x(k)$ and $a_y(k)$, we can see that Cauchy-MCF model can quickly follow the sudden changes. On the other hand, result of Gaussian-EKF model has delayed responses to them. It is more obvious in $a_x(k)$ than in $a_y(k)$.

5. Conclusion

Dynamics of a maneuvering target has been described in system model with random walk of its acceleration according to [9], and nonlinear observation model has been used to represent the radar measurement process. We have proposed a use of heavy-tailed distribution(Cauchy distribution) instead of Gaussian distribution for the system noise of the model. Through a simulational experiment with small measurement noise, the improvement of tracking performance of a maneuvering target with

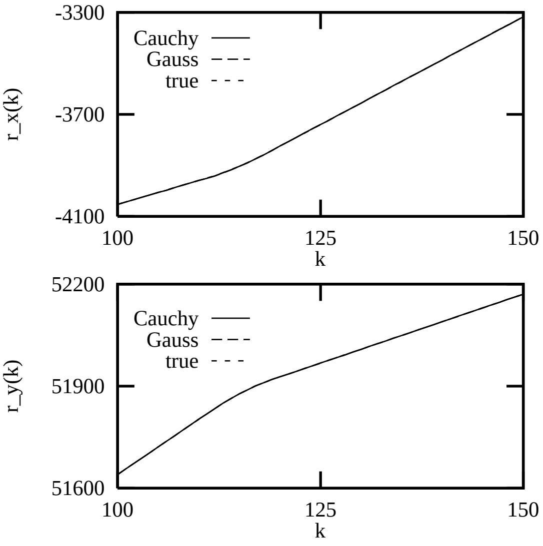

Figure 3: Estimation results, $\hat{\mathbf{r}}(k|k)$

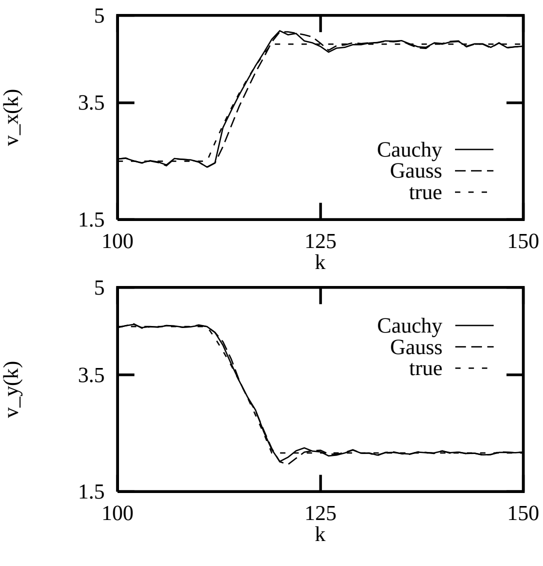

Figure 4: Filtering results of $s_x(k)$ and $s_y(k)$

abrupt change of its state(acceleration) has been shown by comparing with the Gaussian model using extended Kalman filter.

For the future work, large measurement noise and application to the real data are considered. However, there remain several problems to overcome in such worse SNR case since the acceleration is much sensitive to the noise in position.

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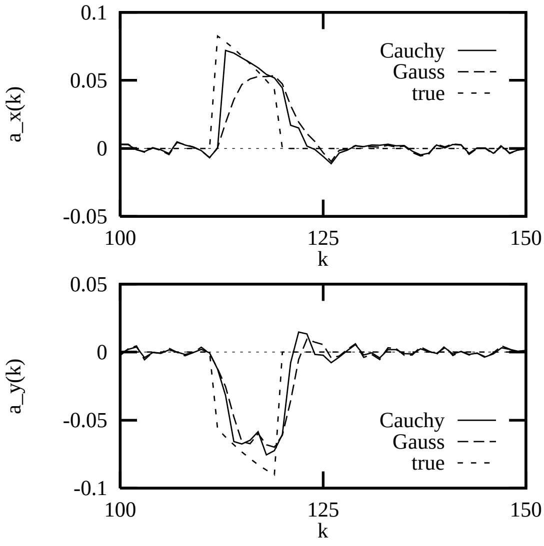

Figure 5: Filtering results of $a_x(k)$ and $a_y(k)$

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