Automatic and Accurate Determination of the Onset Time of the Quasi-periodic Oscillation

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1 Introduction

The identification of the start (onset) time of the quasi-periodic oscillation (QPO), which is called the Pi 2 pulsations in a magnetospheric physics, from the ground magnetic field observation is usually carried out by focusing on a wave-like component obtained by applying a linear band-pass filter [5, 6]. When the background magnetic field (i.e. time-dependent mean value structure) and/or the amplitude of high-frequency components (i.e., time-dependent variance structure) change rapidly around the initial period of Pi 2 pulsations, any linear band-pass filter, which also includes the procedure based on a simple modification of the wavelet analysis, always generates a pseudo precursor prior to a true onset time. In such a case, an accurate determination of onset time requires a nonlinear filter which enables us to separate only the wavy-like component associated with Pi 2 pulsations from the time-varying mean and/or variance structures with various discontinuities. In this study we introduce a locally fixed time series model which partitions the time series into three segments and to model each segment as the linear combination of several possible components. An optimal partition obtained by the minimum AIC procedure allows us to determine an onset time precisely even for the above-mentioned case. We illustrate this procedure by showing an application to actual data sets.

2 Treatment of Rapid Decrease in Trend

The time series $Y_{1:N} = [y_1, \ldots, y_N]$ is a scaler observation which is the H component recorded by a magnetometer at the ground station [8]. We sometimes observe an extremely rapid decrease in the background magnetic field measured at the high latitude stations. A preparatory removement of such rapid change in the trend from the original observations enhances efficiency and accuracy in an estimation of parameters involved in describing a time series model, because an onset time determination in our approach is based on a representation of the time series by a flexible model with many unknown parameters. Prior to an analysis of an onset determination we therefore apply a detrending procedure which fits a parametrically described function $\mu_n(\theta)$ to y_n, where θ is a parameter vector for representing μ_n that is a function of n.

The detrending procedure begins by examining a sequence of the first difference of original time series and identifying intervals each of which is defined by consecutive data points with the first difference value smaller than a certain threshold, γ_{th}. We denote the jth interval by $D_j = [i_{j,A},\ i_{j,B}]$ $(j = 1, \ldots, J)$, where J is the number of intervals with a rapid decrease. A detailed examination of y_n for $n \in D_j$ founds that a rapid decrease can be approximated by the first quarter of the cycle of a cosine function. Specifically a function form for the jth rapid decrease, f_n^j, is given by

$$f_n^j = (g_{j,A} - g_{j,B}) \cos \left(\frac{2\pi(n - i_{j,A})}{4(i_{j,B} - i_{j,A})} \right) + g_{j,B} \quad \text{for } n \in D_j. \tag{1}$$

μ_n for an interval between D_j and D_{j+1}, specified by h_n^j, is simply given by a linear function:

$$h_n^j = \left(\frac{g_{j+1,A} - g_{j,B}}{i_{j+1,A} - i_{j,B}} \right) (n - i_{j,B}) + g_{j,B} \quad \text{for } n \in C_j, \tag{2}$$

where $C_j = (i_{j,B},\ i_{j+1,A})$. μ_n for an interval before D_1 is given by a constant: $h_n^0 = g_{1,A}$. Similarly, for an interval after D_J, i.e., $C_J = (i_{J,B}, N]$, μ_n is given by $h_n^J = g_{J,B}$.

For given set of $D_1, \ldots, D_J$, an optimal set of $(g_{j,A},\ g_{j,B})$ $(j = 1, \ldots, J)$ is easily obtained by applying the least squares fit. Actually a minor adjustment of a location of D_j itself is carried out by minimizing the squared residuals. Eventually μ_n is represented with a parameter vector θ which consists of $4J$ variables:

$$\theta = \Big[(i_{1,A}, g_{1,A}),\ (i_{1,B}, g_{1,B}),\ \cdots\cdots,\ (i_{J,A}, g_{J,A}),\ (i_{J,B}, g_{J,B}) \Big]'. \tag{3}$$

As a result, a procedure for obtaining an optimal θ, θ^*, turns out to become nonlinear. The detrended signal, e_n, is defined by $e_n = y_n - \mu_n(\theta^*)$ $(n = 1, \ldots, N)$.

3 Data Partition

Suppose that a wave train of the Pi 2 pulsation is observed in $E_{1:N} = [e_1, \ldots, e_N]$, and denote its starting and ending points by $k_1 + 1$ and k_2, respectively. Accordingly, a total interval is divided into three sub-intervals:

$$E_{1:N} = [\overbrace{e_1, \ldots, e_{k_1}}^{I^{(1)}} \mid \overbrace{e_{k_1+1}, \ldots, e_{k_2}}^{I^{(2)}} \mid \overbrace{e_{k_2+1}, \ldots, e_N}^{I^{(3)}}]. \tag{4}$$

A presence of the Pi 2 pulsations is assumed only for an interval $I^{(2)}$. The Akaike Information Criterion (AIC) [1] for $E_{1:N}$, AIC_N, is given by

$$\mathrm{AIC}(k_1, k_2) = \mathrm{AIC}_N = \mathrm{AIC}^{(1)} + \mathrm{AIC}^{(2)} + \mathrm{AIC}^{(3)}, \tag{5}$$

that is a function of k_1 and k_2, where $\mathrm{AIC}^{(m)}$ is the AIC for the mth interval [7]. The onset and offset time of the Pi 2 pulsations are given by the optimal dividing points, k_1^* and k_2^*, respectively, which are determined by minimizing the AIC_N.

4 Time Series Model for Each Segment

Suppose that the time series e_n for the m-th interval is given by the following observation model

$$e_n = t_n^{(m)} + s_n^{(m)} + w_n^{(m)}, \; w_n^{(m)} \sim N(0, \sigma^{2,(m)}) \quad (m = 1, 2, \text{ and } 3), \tag{6}$$

where $t_n^{(m)}$ is a stochastic trend component and is assumed to follow a system model [4]

$$t_n^{(m)} = 2t_{n-1}^{(m)} - t_{n-2}^{(m)} + v_n^{t,(m)}, \quad v_n^{t,(m)} \sim N(0, \tau_t^{2,(m)}). \tag{7}$$

$w_n^{(m)}$ is the observation noise. $s_n^{(2)}$ corresponds to the signal associated with the Pi 2 pulsations which is assumed to be a stochastic process with colored power spectrum. Obviously, $s_n^{(1)} = s_n^{(3)} \equiv 0$.

In this study $s_n^{(2)}$ is furthermore decomposed into the quasi-periodic oscillation (QPO) component q_n and autoregressive (AR) component r_n: $s_n^{(2)} = q_n + r_n$. q_n and r_n are are modeled by

$$q_n = 2\cos(2\pi f_c)\, q_{n-1} - q_{n-2} + v_n^q, \quad v_n^q \sim N(0, \tau_q^2), \tag{8}$$

and

$$r_n = \sum_{j=1}^{J_{AR}} a_j r_{n-j} + v_n^r, \quad v_n^r \sim N(0, \tau_r^2), \tag{9}$$

respectively. f_c corresponds to a reciprocal of a period of the Pi 2 pulsations in unit of data points. In this study it is treated as unknown parameter and need not be given beforehand. The presence of system noise in (8) makes the cycle stochastic rather than deterministic, and thus the QPO model allows us to represent a periodic component of distinct frequency f_c with stochastically time-varying amplitude and phase [3].

The AR component is introduced to represent the locally stationary component in s_n. Namely, whereas q_n describes a signal with an eminent peak in power spectrum (i.e., line spectrum), r_n accounts for a signal having a continuous spectrum. Several trials with changing J_{AR} in applications founds that a simple treatment of fixing $J_{AR} = 4$ is sufficient in our study.

5 Parameter Estimation Procedure

The time series model presented in previous section can be formulated by a state space model (SSM) [2] as follows:

$$\mathbf{x}_n^{(m)} = F^{(m)} \mathbf{x}_{n-1}^{(m)} + G^{(m)} \mathbf{v}_n^{(m)}, \tag{10}$$

$$e_n = D^{(m)} \mathbf{x}_n^{(m)} + w_n^{(m)}. \tag{11}$$

For example, the time series model for $I^{(2)}$ can be represented by the SSM in which the corresponding vectors and matrices are

$$\mathbf{x}_n^{(2)} = [t_n^{(2)}, t_{n-1}^{(2)}, q_n, q_{n-1}, r_n, r_{n-1}, r_{n-2}, r_{n-3}]'$$
$$D^{(2)} = [1, 0, 1, 0, 1, 0, 0, 0],$$

$$F^{(2)} = \left(\begin{array}{cc|cc|cccc} 2 & -1 & & & & & & \\ 1 & & & & & & & \\ \hline & & C & -1 & & & & \\ & & 1 & & & & & \\ \hline & & & & a_1 & a_2 & a_3 & a_4 \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \end{array}\right), \quad G^{(2)} = \left(\begin{array}{c|c|c} 1 & & \\ 0 & & \\ \hline & 1 & \\ & 0 & \\ \hline & & 1 \\ & & 0 \\ & & 0 \\ & & 0 \end{array}\right), \quad \mathbf{v}_n^{(2)} = \begin{bmatrix} v_n^{t,(2)} \\ v_n^q \\ v_n^r \end{bmatrix}$$

where $C = 2\cos(2\pi f_c)$. Here the empty entries of $F^{(2)}$ and $G^{(2)}$ are all zero and $\mathbf{v}_n^{(2)} \sim N(0, R^{(2)})$ with a diagonal variance matrix of $R^{(2)} = \mathrm{diag}(\tau_t^{2,(2)}, \tau_p^2, \tau_r^2)$. An optimal estimation for $t_n^{(2)}$, q_n, and r_n is given by the estimated $\mathbf{x}_n^{(2)}$ that is obtained by the Kalman filter and smoother [2]. Here $\tau_t^{2,(2)}$, τ_p^2, and τ_r^2 are unknown parameters to be optimized. Then the time series model for $I^{(2)}$ involves nine unknown parameters:

$$\lambda^{(2)} = [\sigma^{2,(2)}, \tau_t^{2,(2)}, f_c, \tau_p^2, \tau_q^2, a_1, a_2, a_3, a_4]'. \tag{12}$$

The optimal $\lambda^{(2)}$, $\lambda^{(2)*}$, can be determined by minimizing the log-likelihood, $\ell(\lambda^{(2)}) = \log p(E_{k_1+1:k_2}|\lambda^{(2)})$, where $E_{k_1+1:k_2} = [e_{k_1+1}, \ldots, e_{k_2}]$ [4]. The AIC value for $I^{(2)}$, $\mathrm{AIC}^{(2)}$, is also defined by

$$\mathrm{AIC}^{(2)} = -2\ell(\lambda^{(2)*}) + 2\mathrm{dim}\,(\lambda^{(2)}). \tag{13}$$

Similarly, $\mathrm{AIC}^{(1)}$ and $\mathrm{AIC}^{(3)}$ in (5) are also defined.

6 Result and Summary

Fig. 1 shows one of results of the decomposition obtained by applying our procedure to data sets in each of which a typical Pi2 pulsation is observed. The data set that we examined is the H component measured at Kotel'ney (Russia) from 1996/May/26 16:10.00–17:10.00. The sampling time is a second, and thus $N = 3,600$. The two vertical lines indicate the estimated k_1^* and k_2^*, respectively. The three lines are the estimated r_n, q_n, and original observation y_n, from the above, respectively. The horizontal arrow illustrates that the minimum AIC_N procedure finds an optimal k_1. The thick line is the estimated trend component: $\mu_n + t_n^{(m)}$. For this case, three rapid decrease are identified; namely, $J = 3$.

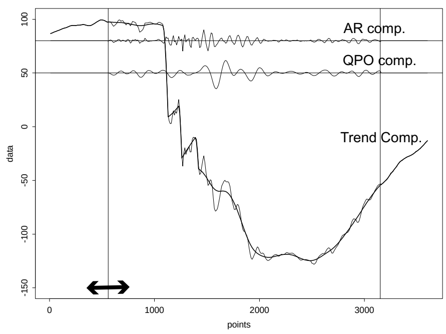

Fig. 1. Decomvolution of the magnetic field data.

The advantages of applying our procedure are summarized as follows. First, our model for decomposition is robust to a rapid change in trend, and then it gives us a good separation of the Pi2 wave component. Second, the onset time can be objectively determined by minimizing an information criterion, AIC_N. It turns out that our method is free from the ambiguity of onset time determination. Finally, our procedure is fully automatic.

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