Automatic Detection of Geomagnetic Jerks by Applying a Statistical Time Series Model to Geomagnetic Monthly Means

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Abstract. A geomagnetic jerk is defined as a sudden change in the trend of the time derivative of geomagnetic secular variation. A statistical time series model is applied to monthly means of geomagnetic eastward component obtained at 124 geomagnetic observatories to detect geomagnetic jerks objectively. The trend component in the model is expressed by a second order spline function with variable knots. The optimum parameter values of the model including positions of knots are estimated by the maximum likelihood method, and the optimum number of parameters is determined based on the Akaike Information Criterion. The geomagnetic jerks are detected objectively and automatically by regarding the determined positions of knots as the occurrence epochs. This analysis reveals that the geomagnetic jerk in 1991 is a local phenomenon while the 1969 and 1978 jerks are confirmed to be global phenomena.

1 Introduction

A geomagnetic secular variation observed on the ground is believed to have several origins. One is the dynamo action in the outer core, which is the liquid metallic layer in the interior of the earth, and the others are the currents flowing external of the earth such as the magnetopause currents, the ionospheric currents, the ring current, the tail current, and the field-aligned currents [17]. The geomagnetic field of the core origin is much larger than that of the external origins [8].

It has been reported that the trends of the time derivative of geomagnetic secular variations in the eastward component changed suddenly at Europe around 1969 [6], 1978 [18], and 1991 [5] [15] as shown in Fig. 1. This phenomenon is termed "geomagnetic jerk" [16], which mathematically means that the secular variation has an impulse in its third order time derivative. It is discussed for a couple of decades whether the origin of the jerks is internal or external of the earth and whether their distribution is worldwide or local [3] [7] [16].

In analyzing the geomagnetic jerks, most of the papers assume that the jerks occurred around 1969, 1978, and 1991, then fit several lines to the time derivative of the secular variation, and obtain a jerk amplitude as the difference of the trends of two lines connected at the occurrence epoch of the jerk. Their methods are subjective in the determination of occurrence epochs [3]. Two objective methods using the optimal piecewise regression analysis [24] and the wavelet analysis [1] [2] were developed for determination of the jerk occurrence epochs. The former method was applied to the geomagnetic annual means, but it is better to adopt monthly means rather than annual means for jerk analyses because it takes only a few years to change the trends of the time derivative of geomagnetic secular variation as seen in Fig. 1, and the temporal resolution of annual means is not sufficient to analyze the jerk. The latter method, on the other hand, was applied to geomagnetic monthly means, but this method still has room for improvement because seasonal adjustment is not taken into account. In this paper, a statistical time series model, in which the adjustments of both seasonal and short time scale variations are taken into account, is applied to geomagnetic monthly means to estimate the trend component of geomagnetic secular variation. The geomagnetic time series is decomposed into the trend, the seasonal, the stationary autoregressive (AR), and the observational noise components. The model parameters are estimated by the maximum likelihood method and the best model is selected based on the Akaike Information Criterion (AIC). Finally, the occurrence epochs of the jerks are determined automatically.

The data description shall be given in Section 2 and the method of our analysis is developed in Section 3. A result of the decomposition, the occurrence index of jerks, and the global distribution of jerk amplitudes are shown in Section 4.

2 Geomagnetic Data

We use monthly means of geomagnetic eastward component obtained at observatories distributed worldwide. The data are collected through the World Data Center system (e.g., see `http://swdcdb.kugi.kyoto-u.ac.jp/`). The time series at an observatory should be continuously maintained for more than ten years, since the geomagnetic jerks are the sudden changes in the trends of geomagnetic decadal changes. The time series of 124 geomagnetic observatories are selected according to this criterion. The distribution of these observatories is shown in Fig. 2. Since the worldwide coverage of the observatories in operation is not enough before International Geophysical Year (IGY) 1957, we use the time series from 1957 to 1999. Obvious artificial spikes and baseline jumps in the data are corrected manually before the analysis. The influences of small errors in the data at an observatory, which are difficult to be corrected manually, to result of

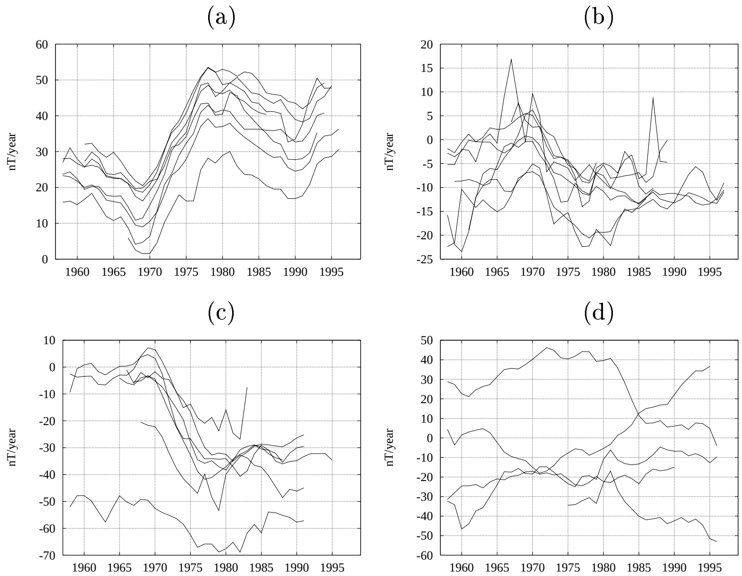

Fig. 1. The time derivative of annual means of the geomagnetic eastward component in (a) Europe, (b) Eastern Asia, (c) North America, and (d) the Southern Hemisphere. The trends sometimes change suddenly in a few years around 1969, 1978, and 1991. These phenomena are called geomagnetic jerks.

the analysis are considered to be identified by comparing with results obtained from the data at other observatories as shown in Section 4.

3 Method of Analysis

3.1 Statistical Time Series Model

A geomagnetic time series of monthly means in the eastward component obtained at an observatory is described by the following statistical time series model:

$$Y_n = t_n + s_n + p_n + w_n \ , \tag{1}$$

where Y_n is a monthly mean value at the consecutive month n starting from January 1957, t_n is the trend component, s_n is the seasonal component, p_n is the stationary AR component, and w_n is the observational noise component.

Trend Component The time derivative of decadal geomagnetic time series of the eastward component is approximated by a piecewise linear curve (first order spline) as seen in Fig. 1. The trend component t_n of the geomagnetic time series, therefore, is expressed by the second order spline function with knots $\xi_1, \xi_2, \cdots, \xi_K$, where K is the number of knots. Here $\xi_1, \xi_2, \cdots, \xi_K$ are integer

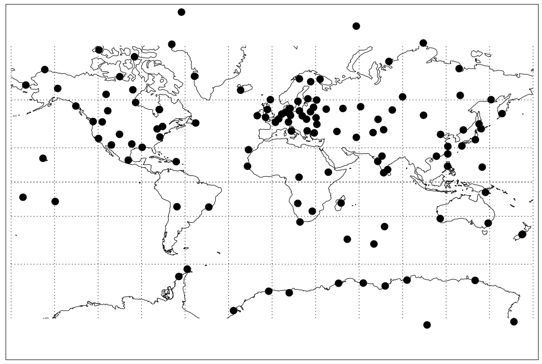

Fig. 2. The distribution of the 124 geomagnetic observatories used in this paper.

values, that is, a knot locates at a data point. The position of a knot corresponds to the occurrence epoch of a geomagnetic jerk.

To derive models for the trend component, we present the continuous time model:

$$f(t) = f(t-\Delta t) + f'(t-\Delta t)\Delta t + \frac{1}{2}f''(t-\Delta t)(\Delta t)^2 + O\left((\Delta t)^3\right) \quad (2)$$

$$f'(t) = f'(t-\Delta t) + f''(t-\Delta t)\Delta t + O\left((\Delta t)^2\right) \quad (3)$$

$$f''(t) = f''(t-\Delta t) + O(\Delta t)\ . \quad (4)$$

The models for the trend component t_n, the first time differential component δt_n, and the second time differential component $\delta^2 t_n$ can be derived by discretizing the equations (2), (3), and (4), respectively:

$$t_n = t_{n-1} + \delta t_{n-1} + \frac{1}{2}\delta^2 t_{n-1} \quad (5)$$

$$\delta t_n = \delta t_{n-1} + \delta^2 t_{n-1} \quad (6)$$

$$\delta^2 t_n = \delta^2 t_{n-1} + v_{n1}\ , \quad (7)$$

where v_{n1} is the system noise sequence which obeys the normal distribution function with mean zero and variance τ_1^2, i.e., $v_{n1} \sim N(0, \tau_1^2)$. The system noise sequences for the trend component model (5) and the first time differential component model (6) are assumed to be zero. Here each interval between successive knots is assumed to be more than five years to avoid too many knots concentrating in a short period [24], and an initial and end knots are kept to be separated from $n = 1$ and $n = N$ by more than three years, respectively. Similar related approaches to a trend component model can be found in [9] and [20]. τ_1^2 has a large value at every knot where the second order time derivative of the spline function is not continuous while the value of τ_1^2 is zero elsewhere. The values of τ_1^2 at the knots $\xi_1, \xi_2, \cdots, \xi_K$ are written as $\tau_{11}^2, \tau_{12}^2, \cdots, \tau_{1K}^2$, respectively. The parameters which should be estimated in the trend component are the number and the positions of the knots and the variances of the system noise at each knot. An alternative way to realize an abrupt change in $\delta^2 t_n$ is to replace $N(0, \tau_{1k}^2)(k = 1, 2, \cdots, K)$ by the heavy-tailed non-normal distribution [10] [12]. Namely, we employ the non-normal linear trend model to detect an abrupt change automatically and objectively. Meanwhile this approach mitigates a difficulty of estimating the parameter values $(\xi_k, \tau_{1k}^2)(k = 1, 2, \cdots, K)$, an identification of jerks is not straightforward from the estimated $\delta^2 t_n$ obtained by applying the non-normal trend model. Therefore we did not adopt this approach in this study. If the jerks occurred simultaneously in the global extent, it is better to consider a time series model to treat all of observations simultaneously instead of analyzing data from each observatory separately. However we did not adopt this generalized model because the jerks were reported to occur, in the southern hemisphere, a few years after the occurrence in the northern hemisphere [2] [7].

Seasonal Component The seasonal component s_n, which represents the annual variation in the data, should have a twelve months periodicity, i.e., $s_n \approx s_{n-12}$. This condition can be rewritten as

$$\sum_{i=0}^{11} s_{n-i} = v_{n2} \ , \tag{8}$$

where v_{n2} is the system noise sequence which obeys $v_{n2} \sim N(0, \tau_2^2)$ [12]. τ_2^2 is a parameter to be estimated.

Stationary Autoregressive Component Unless the stationary AR component is included in the model (1), the best model tends to have more knots than expected as shown in the next section. This component may represent short time scale variations less than one year, such as the solar effects, or responses to the external field which reflect the mantle conductivity structures. The stationary AR component is expressed as

$$p_n = \sum_{i=1}^{m} a_i p_{n-i} + v_{n3} \ , \tag{9}$$

where m is the AR order and v_{n3} is the system noise sequence which obeys $v_{n3} \sim N(0, \tau_3^2)$. τ_3^2 is a parameter to be estimated.

Observational Noise The observational noise w_n is assumed to be a white noise sequence which obeys the normal distribution function with mean zero and variance σ^2, i.e., $w_n \sim N(0, \sigma^2)$, where σ^2 is a parameter to be estimated.

3.2 Parameter Estimation and Model Identification

When a parameter vector involved in the equations (1)-(9)

$$\boldsymbol{\theta} = (\xi_1, \xi_2, \cdots, \xi_K, a_1, a_2, \cdots, a_m, \tau_{11}^2, \tau_{12}^2, \cdots, \tau_{1K}^2, \tau_2^2, \tau_3^2, \sigma^2) \tag{10}$$

is given, the equations (1)-(9) can be represented by the state space model [14]

$$\boldsymbol{x}_n = F\boldsymbol{x}_{n-1} + G\boldsymbol{v}_n \tag{11}$$
$$Y_n = H\boldsymbol{x}_n + w_n \tag{12}$$

with the state space vector $\boldsymbol{x}_n$ and the system noise vector $\boldsymbol{v}_n$

$$\boldsymbol{x}_n = (\, t_n \quad \delta t_n \quad \delta^2 t_n \quad s_n \quad \cdots \quad s_{n-10} \quad p_n \quad \cdots \quad p_{n-m+1}\,)^t \tag{13}$$
$$\boldsymbol{v}_n = (\, v_{n1} \quad v_{n2} \quad v_{n3}\,)^t \ , \tag{14}$$

where the superscript t denotes the transpose operation, and the matrices F, G, and H are

$$F = \left(\begin{array}{ccc|cccc|cccc}
1 & 1 & 1/2 & & & & & & & & \\
0 & 1 & 1 & & & & & & & & \\
0 & 0 & 1 & & & & & & & & \\
\hline
 & & & -1 & \cdots & -1 & -1 & & & & \\
 & & & 1 & & & & & & & \\
 & & & & \ddots & & & & & & \\
 & & & & & 1 & & & & & \\
\hline
 & & & & & & & a_1 & \cdots & a_{m-1} & a_m \\
 & & & & & & & 1 & & & \\
 & & & & & & & & \ddots & & \\
 & & & & & & & & & 1 &
\end{array}\right) \tag{15}$$

$$G = \left(\begin{array}{c|c|c}
0 & & \\
0 & & \\
1 & & \\
\hline
 & 1 & \\
 & 0 & \\
 & \vdots & \\
 & 0 & \\
\hline
 & & 1 \\
 & & 0 \\
 & & \vdots \\
 & & 0
\end{array}\right) \tag{16}$$

$$H = (\underbrace{1 \ \ 0 \ \ 0}_{3} \mid \underbrace{1 \ \ 0 \ \ \cdots \ \ 0}_{11} \mid \underbrace{1 \ \ 0 \ \ \cdots \ \ 0}_{m}) \ . \tag{17}$$

The log-likelihood function of this model as a function of $\boldsymbol{\theta}$ is

$$\ell(\boldsymbol{\theta}) = -\frac{1}{2}\left[N \log 2\pi + \sum_{n=1}^{N}\left\{\log d_{n|n-1} + \frac{(Y_n - Y_{n|n-1})^2}{d_{n|n-1}}\right\}\right] \ , \tag{18}$$

where N is the number of observational data, $Y_{n|n-1}$ is the mean of the predictive distribution for Y_n given observational data $Y_i (i = 1, 2, \cdots, n-1)$, and $d_{n|n-1}$

is the variance of that distribution [12]. The quantities of $Y_{n|n-1}$ and $d_{n|n-1}$ in the equation (18) can be obtained by the Kalman filter algorithm recursively:

$$\boldsymbol{x}_{n|n-1} = F\boldsymbol{x}_{n-1|n-1} \tag{19}$$
$$V_{n|n-1} = FV_{n-1|n-1}F^t + GQG^t \tag{20}$$
$$K_n = V_{n|n-1}H^t(HV_{n|n-1}H^t + \sigma^2)^{-1} \tag{21}$$
$$\boldsymbol{x}_{n|n} = \boldsymbol{x}_{n|n-1} + K_n(Y_n - H\boldsymbol{x}_{n|n-1}) \tag{22}$$
$$V_{n|n} = (I - K_nH)V_{n|n-1} \ , \tag{23}$$

where $\boldsymbol{x}_{n|j}$ is the mean vector and $V_{n|j}$ is the variance covariance matrix of the state vector $\boldsymbol{x}_n$ given observational data $Y_i (i = 1, 2, \cdots, j)$, I is the unit matrix, and Q is the variance covariance matrix of the system noise:

$$Q = \begin{pmatrix} \tau_1^2 & 0 & 0 \\ 0 & \tau_2^2 & 0 \\ 0 & 0 & \tau_3^2 \end{pmatrix} \ . \tag{24}$$

The positions of knots $\xi_i (i = 1, 2, \cdots, K)$ move to other data points in the course of this iterative estimation. Other parameters in the parameter vector (10) is iteratively searched by employing the quasi-Newton method [19] which makes the log-likelihood (18) larger. This procedure is iterated until the parameter vector $\boldsymbol{\theta}$ makes the log-likelihood maximum. The best model among the models with different number of the parameters is selected by minimizing the AIC:

$$\begin{aligned} \mathrm{AIC} &= -2\ell(\hat{\boldsymbol{\theta}}) + 2 \dim \hat{\boldsymbol{\theta}} \\ &= \begin{cases} -2\ell(\hat{\boldsymbol{\theta}}) + 2(2K + 2) & (m = 0) \\ -2\ell(\hat{\boldsymbol{\theta}}) + 2(2K + m + 3) & (m \neq 0) \end{cases} \ . \end{aligned} \tag{25}$$

The exceptional handling of $m = 0$ is necessary to avoid the identification problem between innovation variance τ_3^2 of the 0-th AR component and variance σ^2 of the observational noise. See [11] for an application of the AIC to determination of the number and positions of the knots. There are various kinds of information criteria besides the AIC [13]. Minimum description of length (MDL) for the selection of the orders of the models is discussed in [21]. Modeling and simplicity in a field of scientific discovery and artificial intelligence are discussed in [22] and [23].

The final estimate of the state vector variable for the best model is obtained by the fixed interval smoother algorithm [12]:

$$A_n = V_{n|n}F_{n+1}^t V_{n+1|n}^{-1} \tag{26}$$
$$\boldsymbol{x}_{n|N} = \boldsymbol{x}_{n|n} + A_n(\boldsymbol{x}_{n+1|N} - \boldsymbol{x}_{n+1|n}) \tag{27}$$
$$V_{n|N} = V_{n|n} + A_n(V_{n+1|N} - V_{n+1|n})A_n^t \ . \tag{28}$$

The estimated positions of the knots are regarded as the occurrence epochs of jerks. The jerk amplitude $\delta^3 Y_n$ at a knot ξ_i is defined as:

$$\delta^3 Y_{\xi_i} = \delta^2 t_{\xi_i} - \delta^2 t_{\xi_i - 1} \ . \tag{29}$$

The unit of this jerk amplitude is not nT(nano tesla)/year^3 but nT/year^2 because it is defined as a simple difference between successive second time differential components. The definition of the jerk amplitude (29) follows that of the previous papers (e.g., [7]).

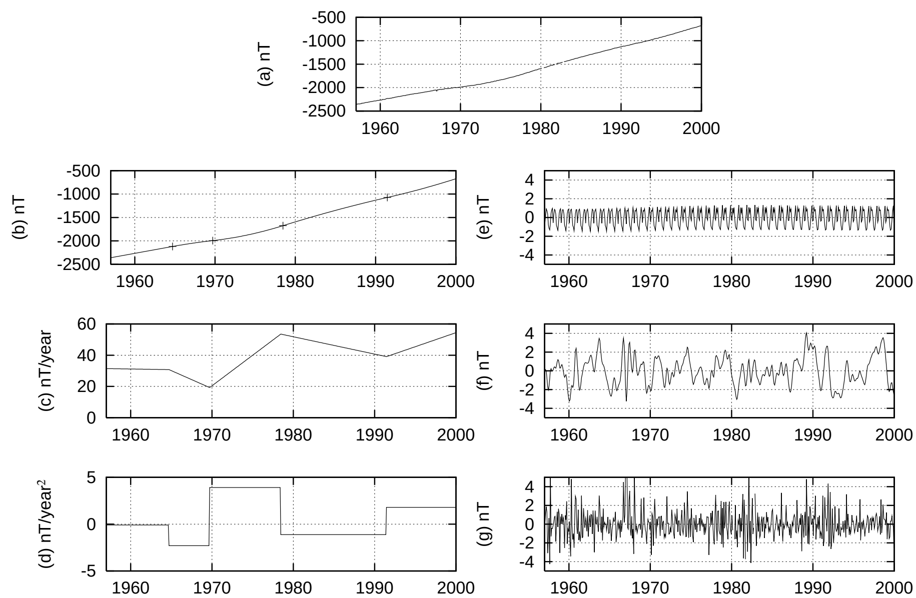

Fig. 3. (a) The time series of monthly means of geomagnetic eastward component Y_n at Chambon-la-Foret, France, (b) the trend component t_n, (c) the first time differential component δt_n, (d) the second time differential component $\delta^2 t_n$, (e) the seasonal component s_n, (f) the stationary AR component p_n, and (g) the observational noise w_n. The trend component is expressed by a second order spline function, and estimated positions of the knots (denoted by the plus symbols, i.e., September 1964, September 1969, June 1978, and June 1991) are regarded as the occurrence epochs of the geomagnetic jerks.

4 Results and Discussion

An example of the decomposition of geomagnetic time series by the method mentioned in Section 3 is illustrated in Fig. 3. The data used in Fig. 3 are the geomagnetic monthly means of the eastward component at Chambon-la-Foret (48.02°N, 2.27°E), France. Although there are no data from March to April 1980 and in October 1981, values of state vector (11) in the missing periods can be estimated by the Kalman filter and smoother algorithms [4]. The number of the knots for the best model is $\hat{K} = 4$ and they locate at $\hat{\xi}_1$ = September 1964,

$\hat{\xi}_2$ = September 1969, $\hat{\xi}_3$ = June 1978, and $\hat{\xi}_4$ = June 1991; our method can identify the jerks which are believed to have occurred except for 1964. Jerks are detected by our method around 1969, 1978, and 1991 from geomagnetic eastward component at most of the European observatories. On the other hand, the ratio of the number of observatories where a jerk is detected in 1964 to the number of observatories where a jerk is not detected in that year is much lesser than that in the years when the jerks are believed to have occurred. That is, the detected 1964 jerk may be a local phenomenon at Chambon-la-Foret. The number of the knots becomes 5 and the AIC increases from 2461.06 to 2558.86 unless the stationary AR component p_n is included in the model (1). This result indicates that an inclusion of the stationary AR component is reasonable. The same tendency is seen in the results obtained from the data at most of the observatories.

Fig. 4 shows the occurrence index of geomagnetic jerks detected by our method. The occurrence index in the year y is defined by

$$R_y = \frac{1}{n_y} \sum_i \sum_{\xi_j \in y} |\delta^3 Y_{\xi_j}(O_i)| \, , \tag{30}$$

where $\delta^3 Y_{\xi_j}(O_i)$ is the jerk amplitude obtained at the i-th observatory O_i and n_y is the number of observatories available in the year y. It can be confirmed from Fig. 4 that R_y has clear peaks in 1969 and 1977 when the jerks are believed to have occurred [6] [18], which is consistent with the result obtained by [2], and also has small peaks in 1989 and 1994. However there is no peak in 1991, when a jerk is also believed to have occurred [5] [15]. Fig. 5 shows the global distributions of the jerk amplitudes $\delta^3 Y_n$ defined by the equation (29) in the years when the geomagnetic jerks believed to have occurred (i.e., 1969, 1978, and 1991). The circles indicate positive jerk amplitudes and the triangles indicate negative ones. It should be noted from these figures that the 1991 jerk is not a worldwide phenomenon in comparison with the 1969 and 1978 ones. This result is inconsistent with the conclusion by the previous paper [7].

5 Conclusions

We developed a method to analyze the time series of geomagnetic monthly means for identifying the geomagnetic jerks. Each geomagnetic time series is decomposed into the trend, the seasonal, the stationary AR, and the observational noise components by applying a statistical time series model. The trend component is expressed by a second order spline function because a jerk is an impulse in the third order time derivative of the geomagnetic time series. The model parameters including the positions of the knots of the spline function are estimated by the maximum likelihood method and the number of the knots and the AR order is selected based on the AIC. Distributions of jerk amplitudes are obtained by regarding the estimated positions of the knots as the occurrence epochs of the jerks and by defining a jerk amplitude at an optimum knot as the difference between the successive second time differential components in the

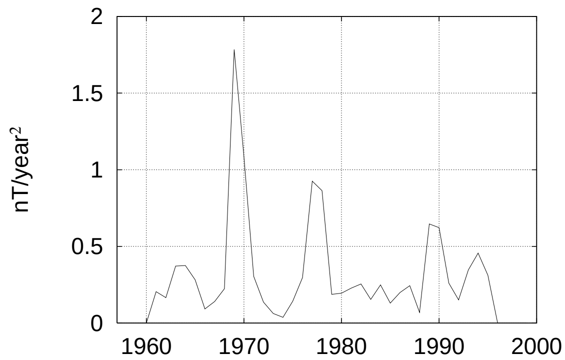

Fig. 4. The occurrence index of geomagnetic jerks defined by the equation (30).

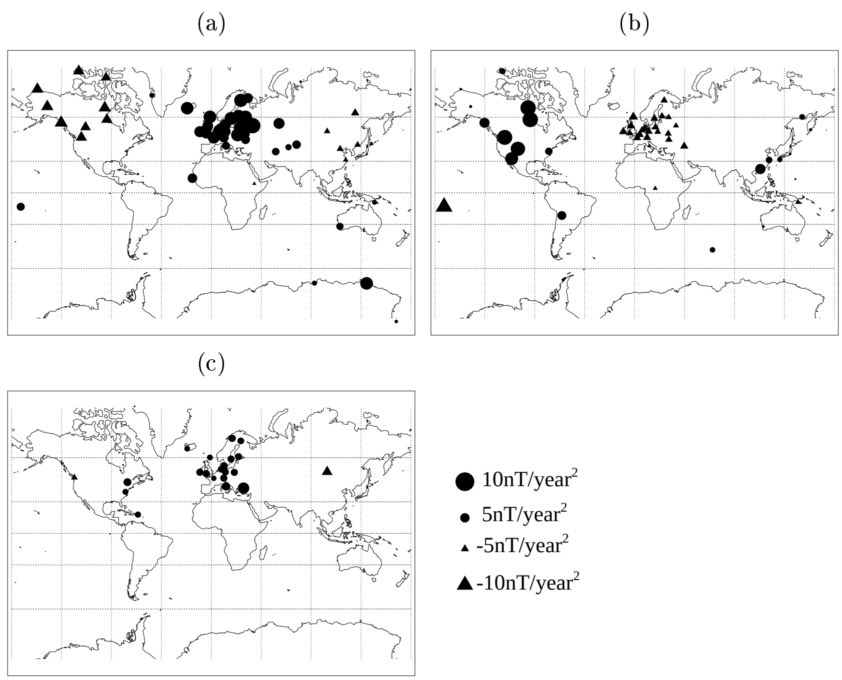

Fig. 5. The distributions of the jerk amplitudes $\delta^3 Y$ defined by the equation (29) around (a) 1969, (b) 1978, and (c) 1991. The circles show positive jerk amplitudes and the triangles show negative ones. The magnitude of a jerk amplitude is indicated by the radius of the symbol.

trend component connected at the knot. The 1969 and 1978 jerks are confirmed to be worldwide phenomena while the 1991 jerk is found to be localized. Our method developed in this paper gives a valuable contribution to discovery in the applied domain.

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