リスク解析戦略研究センター・セミナー

【日時】　平成２９年2月２３日（木）　１３：３０－１４：２０

【場所】　セミナー室７（5 階, A504)

【講演者】　Sunyoung Kim　　(Ewha W. University, Seoul, Korea)

【講演題目】Polynomial Optimization and Conic Programming Relaxation

【講演概要】

We consider the problem: Given a real valued function f_0 defined on a set U and a subset S of U, find an $\boldsymbol{x}^* \in S$ that minimizes f_0 over S, *i.e.*, an $\boldsymbol{x}^* \in S$ such that $f_0(\boldsymbol{x}) \geq f_0(\boldsymbol{x}^*)$ for all $\boldsymbol{x} \in S$. If we embed U in the n-dimensional Euclidean space $\mathbb{R}^n$ and describe the feasible region S in terms of inequalities:

$$S = \left\{ \boldsymbol{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n : \begin{array}{l} f_p(\boldsymbol{x}) \leq 0 \ (p = 1, 2, \ldots, m), \\ x_i \text{ is integer } (i = 1, \ldots, r) \end{array} \right\},$$

for some $r \in \{0, 1, \ldots, n\}$, where $f_p : \mathbb{R}^n \to \mathbb{R}$ $(p = 1, \ldots, n)$, then a general optimization problem can be written as

$$\text{(P)} \quad \text{minimize} \quad f_0(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{x} \in S.$$

The optimization problems of the form (P) are classified into three types: continuous, mixed discrete and discrete, if $r = 0$, $1 \leq r < n$ and $r = n$, respectively. For quadratic optimization problems (QOPs) with integer variables, it is fair to say that successful numerical methods have not been developed yet in comparison to mixed integer linear programs (LPs) and integer LPs. For polynomial optimization problems (POPs), a basic principle of "lifting to the higher dimensional space of symmetric matrices" is applied to the problem, before the relaxation principle is used. Additional structures and assumptions on QOPs are necessary to strengthen the conic LP relaxation, and additional techniques need to be incorporated for designing efficient numerical methods. We can extend the relaxation and lifting principles to a general POP. In my talk, I will briefly introduce global optimization and discuss recent developments on copositive programs and Lagrangian-conic relaxations, a joint work with Masakazu Kojima and Kim-Chuan Toh.