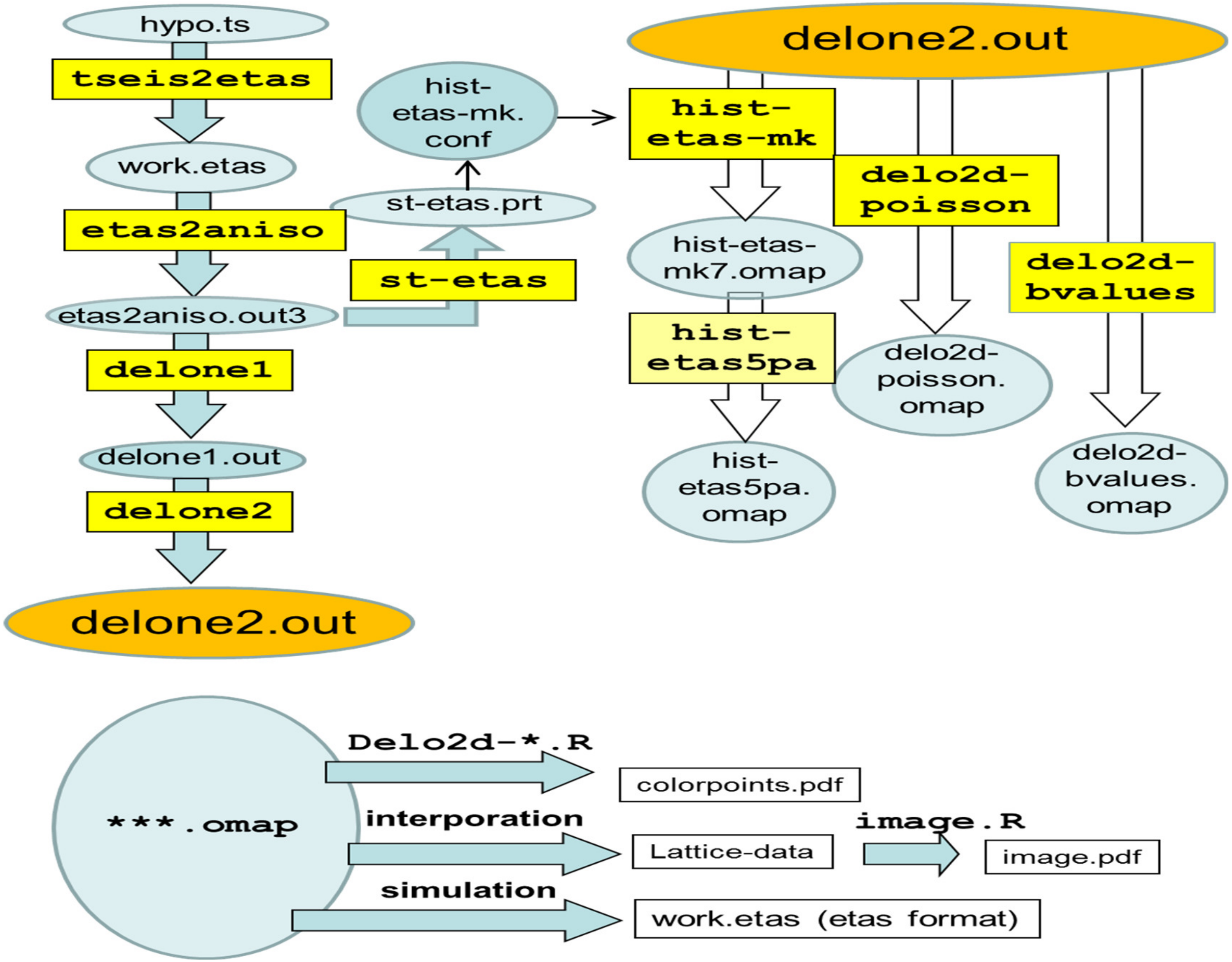
Space-Time Point-Process Models:
Users guide to this program package

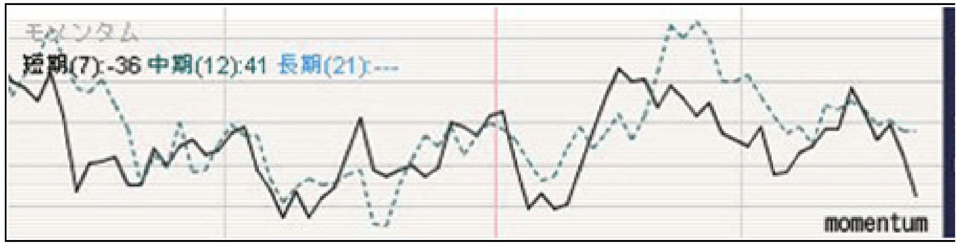
Yosihiko Ogata, Koichi Katsura,
David Harte, Masaharu Tanemura and
Jiancang Zhuang


hypo.ts
tseis2etas
work.etas
etas2aniso
etas2aniso.out3
delone1
delone1.out
delone2
delone2.out
hist-
etas-mk.
conf
st-etas.prt
st-etas
delone2.out
hist-
etas-mk
hist-etas-
mk7.omap
hist-
etas5pa
hist-
etas5pa.
omap
delo2d-
poisson
delo2d-
poisson.
omap
delo2d-
bvalues
delo2d-
bvalues.
omap
***.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

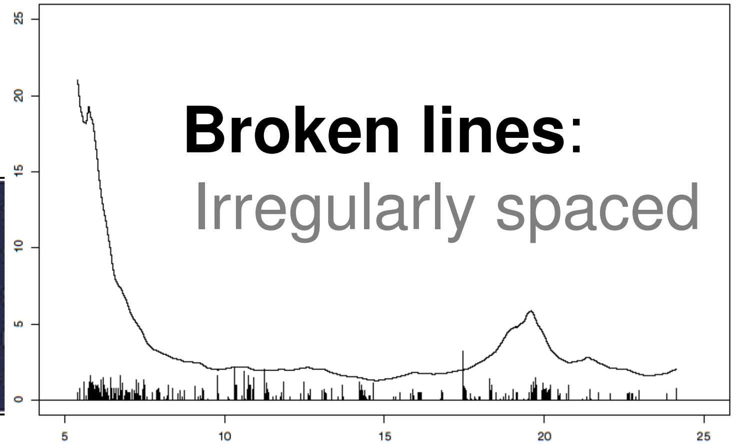
Delaunay-based linear interpolation for function

1-dim.

Broken lines: Regularly spaced

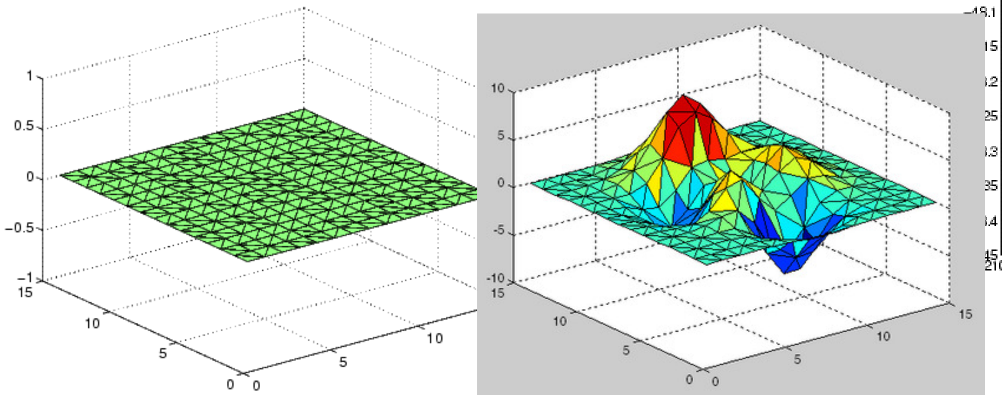


Broken lines: Irregularly spaced

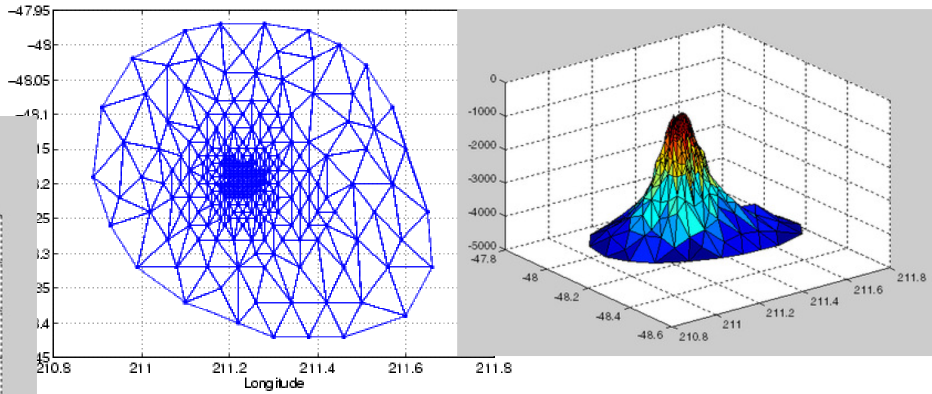


2-dim. Triangulation:

Regularly located



Irregularly located



Spatial non-homogeneous Poisson Process

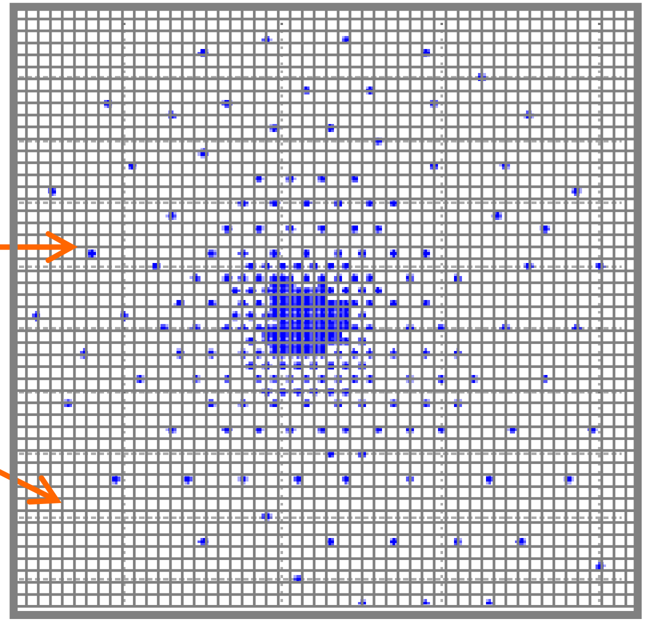
$$\Pr\{\text{an event in } [x, x+dx)\times[y, y+dy)\} = \lambda_\theta(x, y)dxdy + o(dxdy)$$

$$\lambda_\theta(x, y) = \text{Intensity function}$$

$\{(x_i, y_i); i = 1, 2, \cdots, N\}$ Point coordinates

$$\boxed{\lambda_\theta(x_i, y_i)dx_i dy_i}$$

$$\boxed{1 - \lambda_\theta(x, y)dxdy \approx e^{-\lambda_\theta(x,y)dxdy}}$$



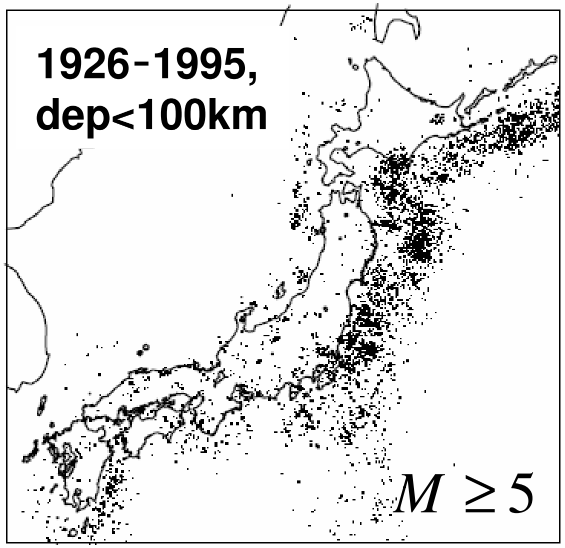
Likelihood function

$$L(\theta) = \left\{\prod_{i=1}^{N} \lambda_\theta(x_i, y_i)\right\} e^{-\int_A \lambda_\theta(x,y)dxdy}$$

Log-likelihood

$$\log L(\theta) = \sum_{i=1}^{N} \log \lambda_\theta(x_i, y_i) - \iint_A \lambda_\theta(x, y)\, dxdy$$

maximize $\Rightarrow$ $\hat{\theta}$ Maximum likelihood principle



Bayesian setting

$$\textit{prior}(\theta \mid w) \propto \exp\{-\textit{penalty}(\theta \mid w)\}$$

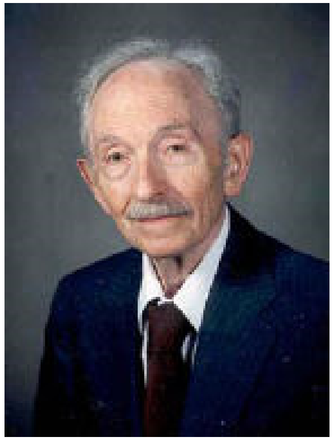
$$\textit{posterior}(\theta \mid w) = \frac{L(\theta) \cdot \textit{prior}(\theta \mid w)}{\Lambda(w)}$$

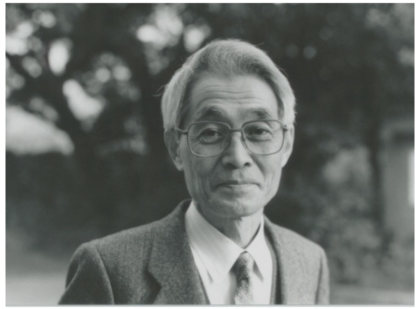
$$\boxed{\Lambda(w) = \int \cdots \int L(\theta) \cdot \textit{prior}(\theta \mid w) d\theta}$$

(**Likelihood** of a Bayesian model; *Good* 1965)

Spatial Poisson model and occurrence data $\{(x_i, y_i);\ i = 1, \cdots, n\}$ in A are given. Then **Log Likelihood** is

$$\boxed{\begin{aligned} &\log L(\theta) \\ &= \sum_{i=1}^{n} \log \lambda_\theta(x_i, y_i) - \iint_A \lambda_\theta(x, y) dx dy. \end{aligned}}$$





Penalized Log Likelihood

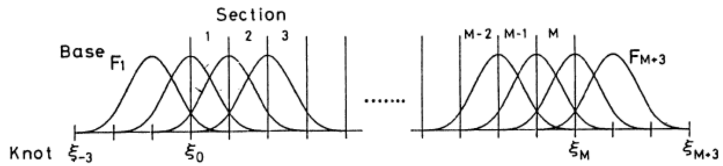
$$Q(\theta \mid w) = \log L(\theta) - \textit{penalty}(\theta \mid w)$$

where the $\textit{penalty}$ is

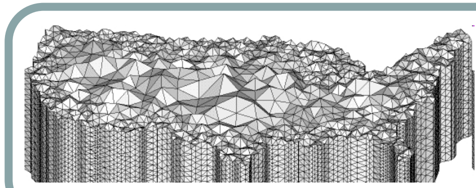
$$w \iint_A dx dy \left\{ \left(\frac{\partial \lambda_\theta}{\partial x} \right)^2 + \left(\frac{\partial \lambda_\theta}{\partial y} \right)^2 \right\}$$

Choose *w* that maximize the Λ, and then maximize the penalized log-likelihood

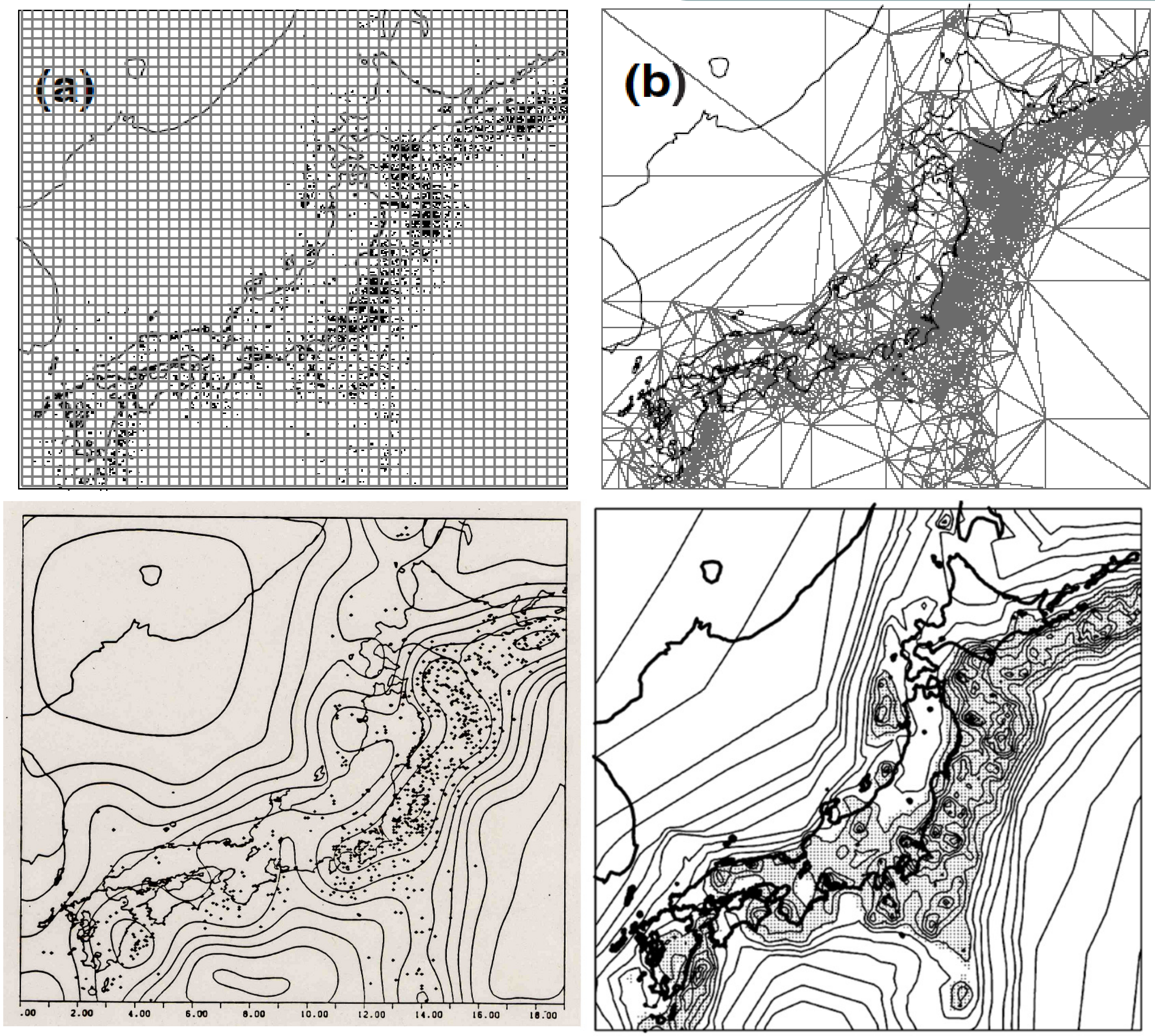
Bayesian smoothing



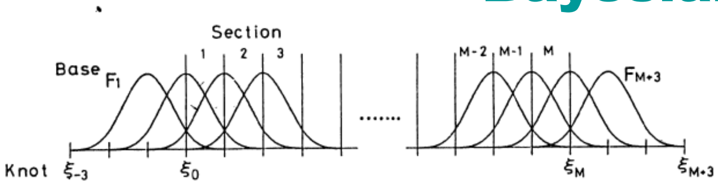
Cubic B-spline function



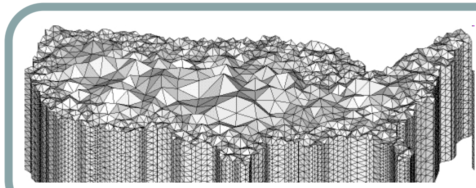
Delaunay-based function:



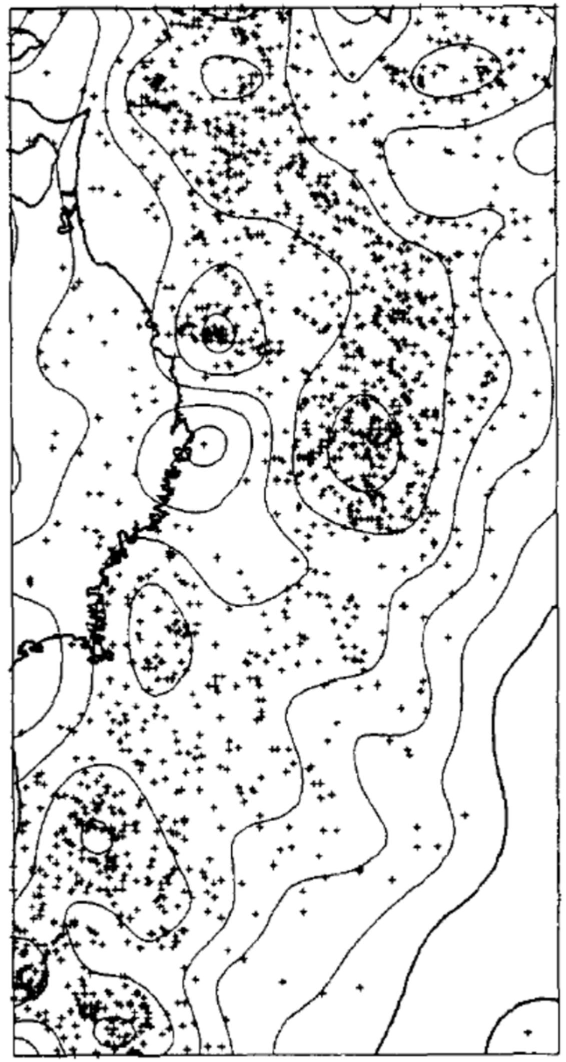
Bayesian smoothing

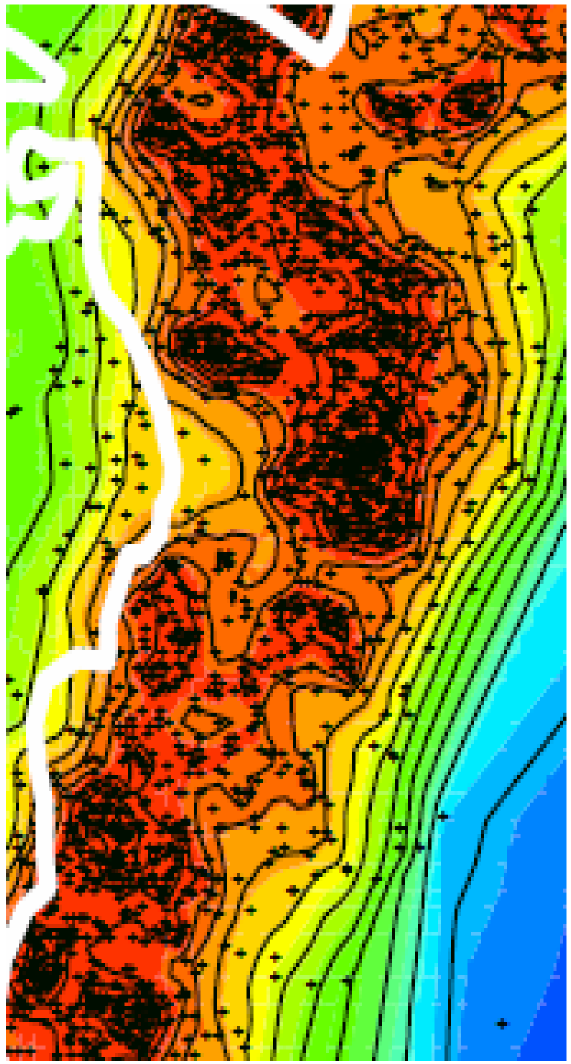


Cubic B-spline function

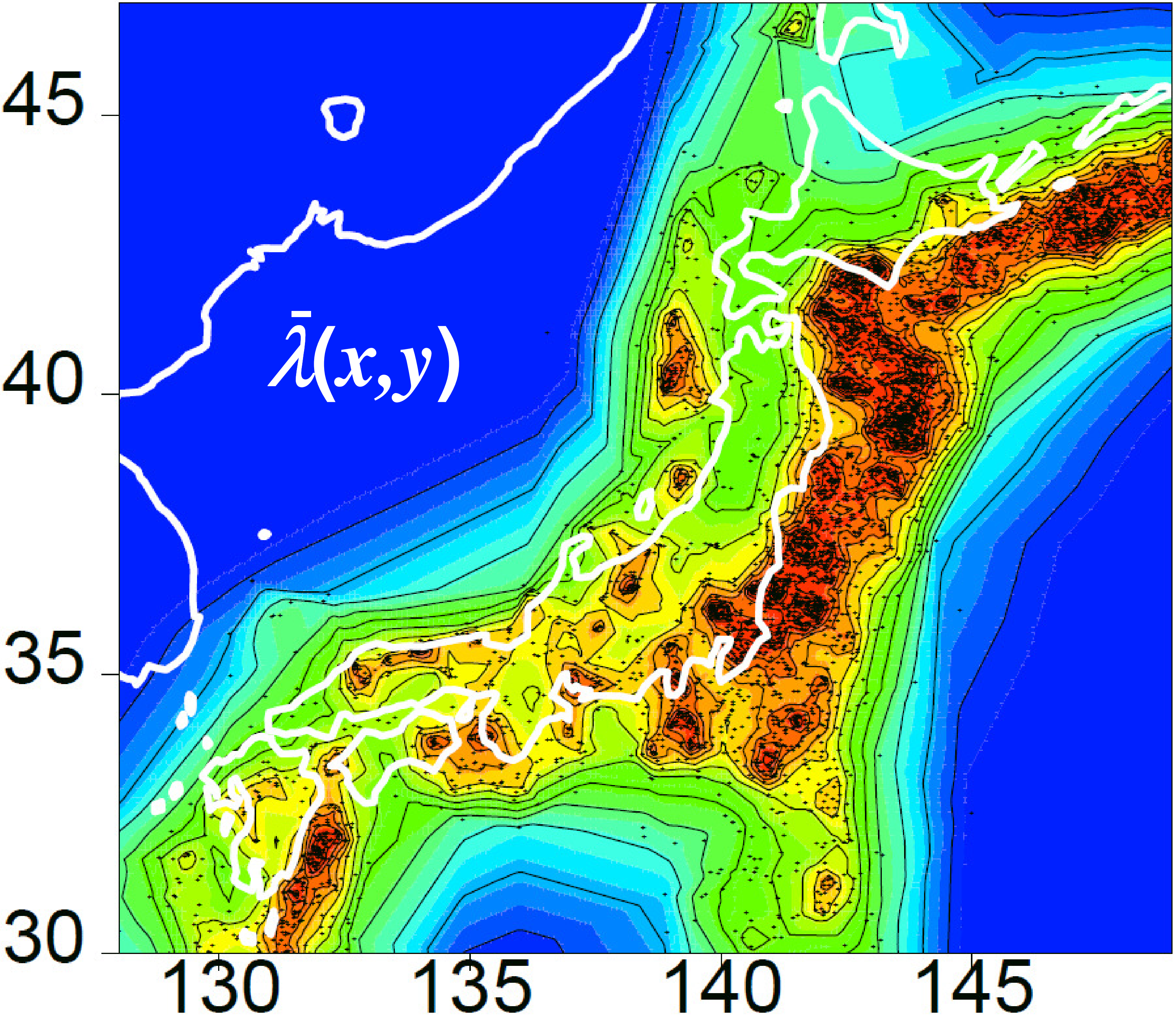


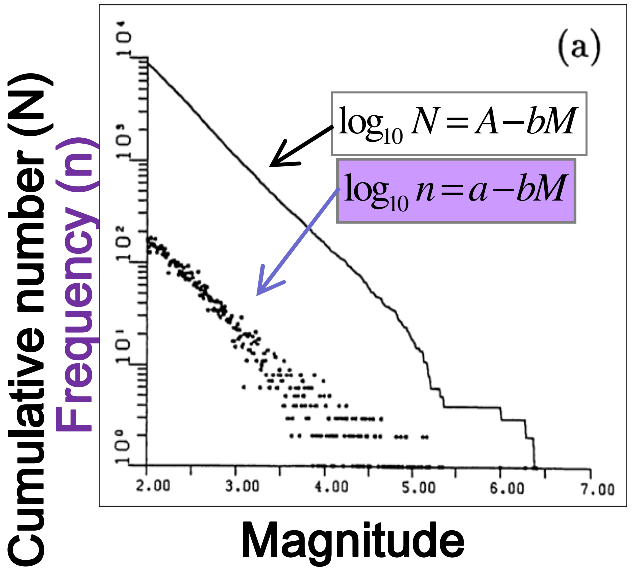
Delaunay-based function:

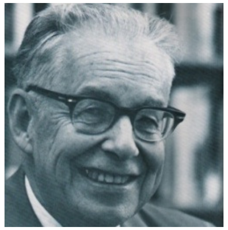




Earthquakes of M>= 5.0 occurred during 1926 - 1995







Gutenberg-Richter Law :

$$n = 10^{a-bM} = 10^{a} e^{-\beta M}, \quad \beta = b \ln 10$$

$$f_\theta(M) = \beta_\theta e^{-\beta_\theta (M - M_0)}, M \geq M_0, \beta_\theta = b_\theta \ln 10$$

Location-dependent b-value

$$f_\theta(M \mid x, y) = \beta_\theta(x, y) e^{-\beta_\theta(x,y)(M - M_0)}$$
$$\beta_\theta(x, y) = b_\theta(x, y) \ln 10$$

Penalized log likelihood

$$Q(\theta \mid w) = \sum_{i=1}^{N} \log f_\theta(M \mid x, y) - penalty(\theta \mid w)$$

$$penalty(\theta \mid w) = w \iint_A dxdy \left\{ \left(\frac{d\beta_\theta}{dx} \right)^2 + \left(\frac{d\beta_\theta}{dy} \right)^2 \right\}$$

Flatness constraints

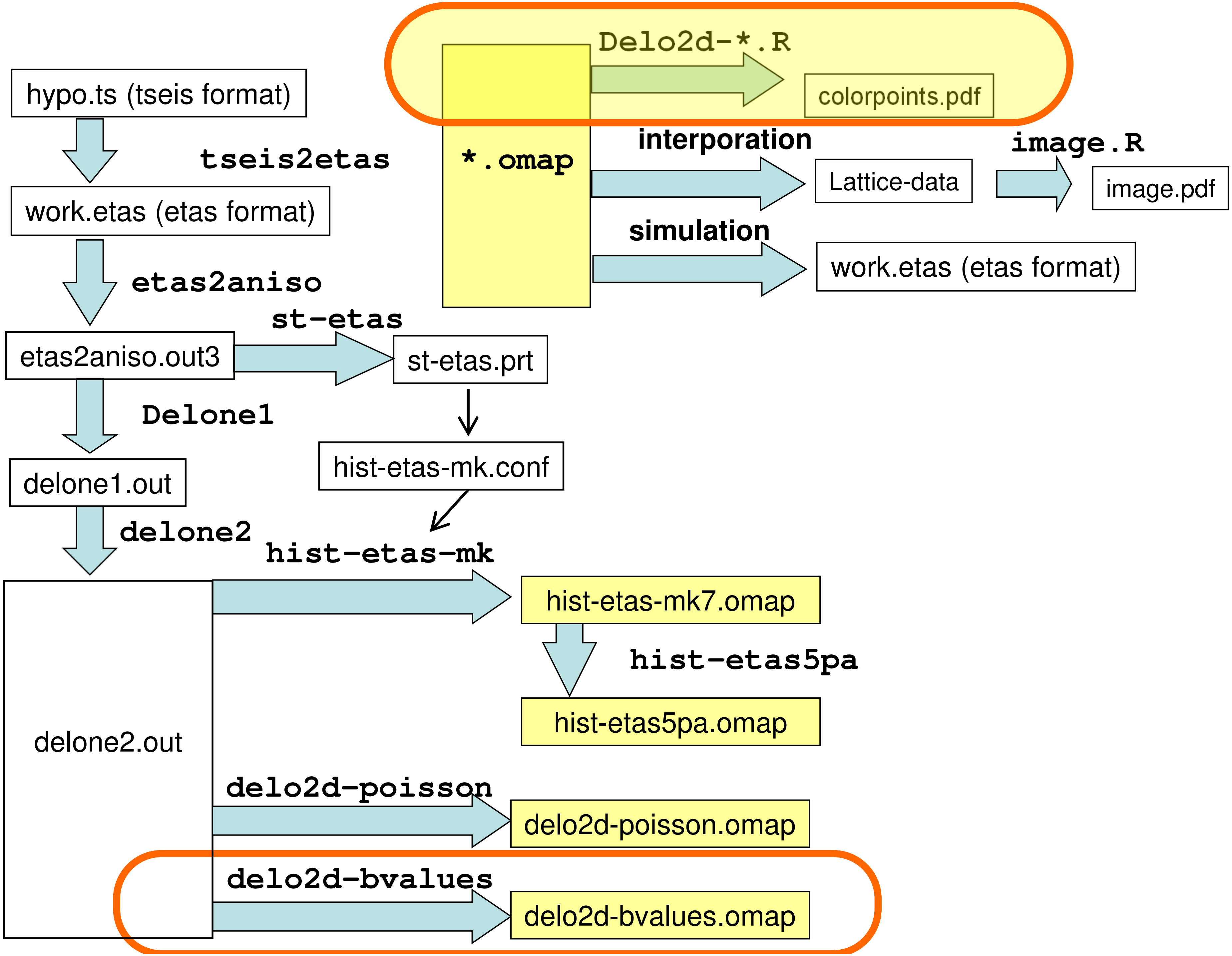
Bayesian setting

$$\textbf{prior}(\theta \mid w) \propto \exp\{ -\textbf{penalty}(\theta \mid w) \}$$

$$posterior(\theta \mid w) = \frac{L(\theta) \cdot prior(\theta \mid w)}{\Lambda(w)}$$

$$\Lambda(w) = \int \cdots \int L(\theta) \cdot prior(\theta \mid w) d\theta$$

Choose *w* that maximize the Λ, and then maximize the penalized log-likelihood

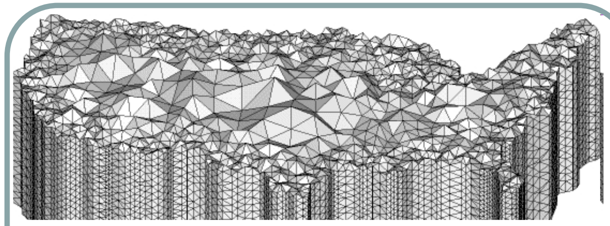


Delo2d-*.R
colorpoints.pdf
hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
*.omap
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)
etas2aniso
st-etas
etas2aniso.out3
st-etas.prt
Delone1
hist-etas-mk.conf
delone1.out
delone2
hist-etas-mk
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delone2.out
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap

Magnitude frequency

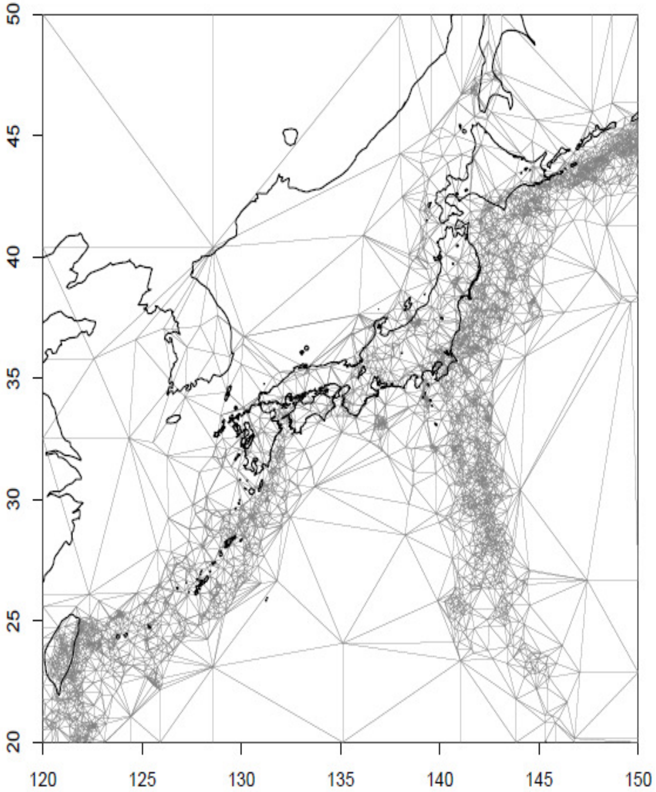
$$f(M \mid \vartheta) = \beta(x, y)e^{-\beta(x,y)(M-5.35)}$$

$$M \geq 5.4, b(x, y) = \frac{\beta(x, y)}{\ln 10}$$



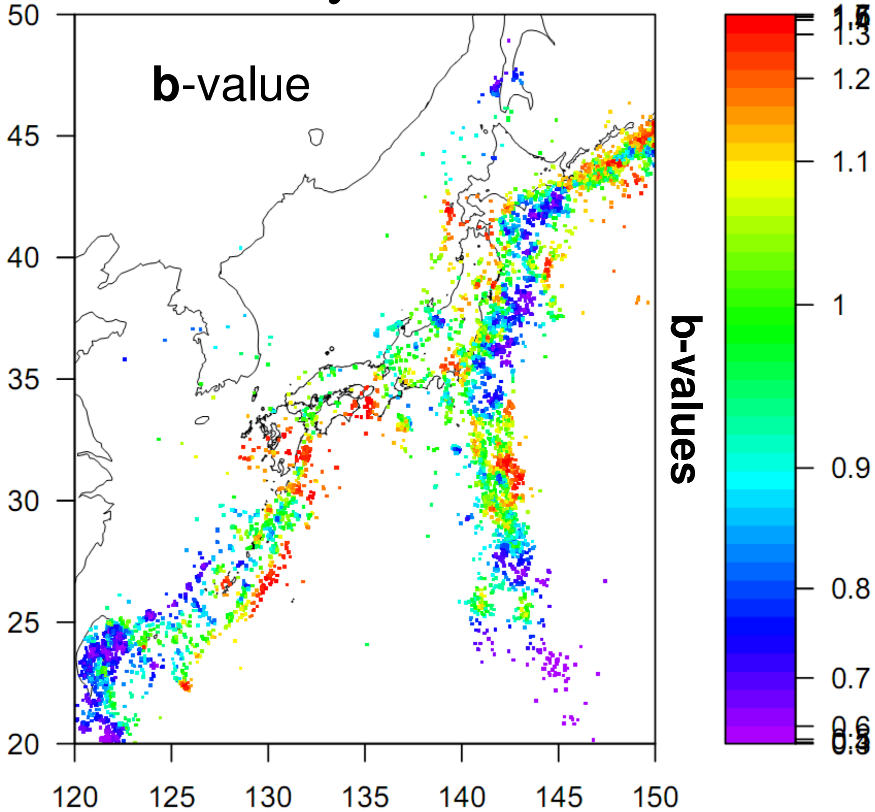
Delaunay-based function
An illustration

Delaunay triangulation

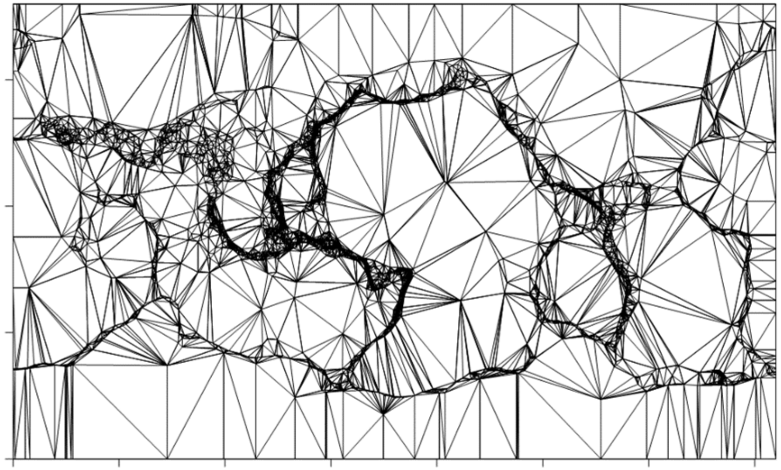


M >= 4, 1997.10 - 2008

Optimum maximum Posterior solution of Delaunay-based function



Harvard global CMT catalog; Mag >= 5.4

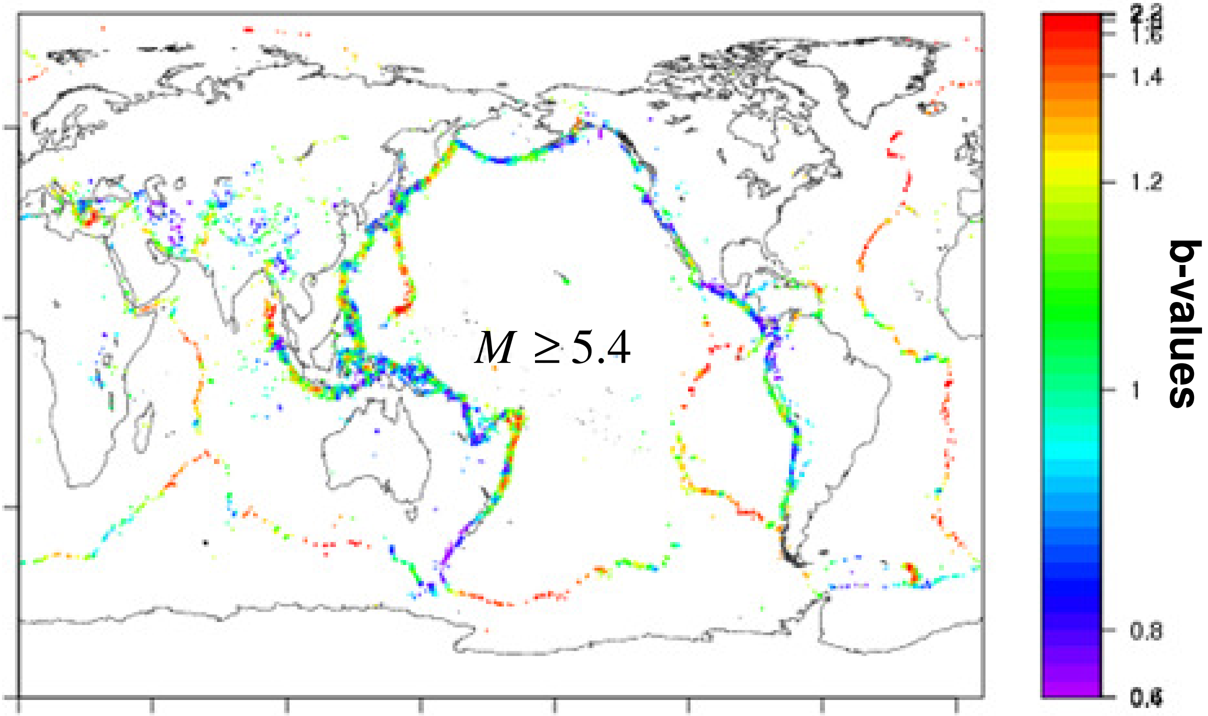


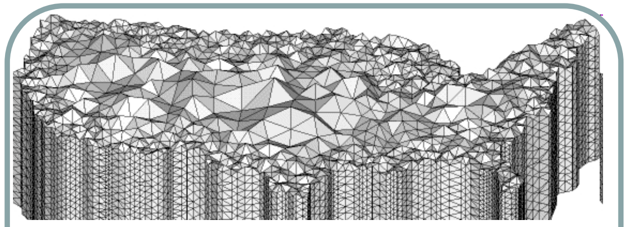
Penalized Log Likelihood

$$Q(\theta|\ w) = \log L(\theta) - penalty(\theta|\ w)$$

where the $penalty$ is

$$w \int\int_A dxdy \left\{ \left(\frac{\partial \lambda_\theta}{\partial x}\right)^2 + \left(\frac{\partial \lambda_\theta}{\partial y}\right)^2 \right\}$$





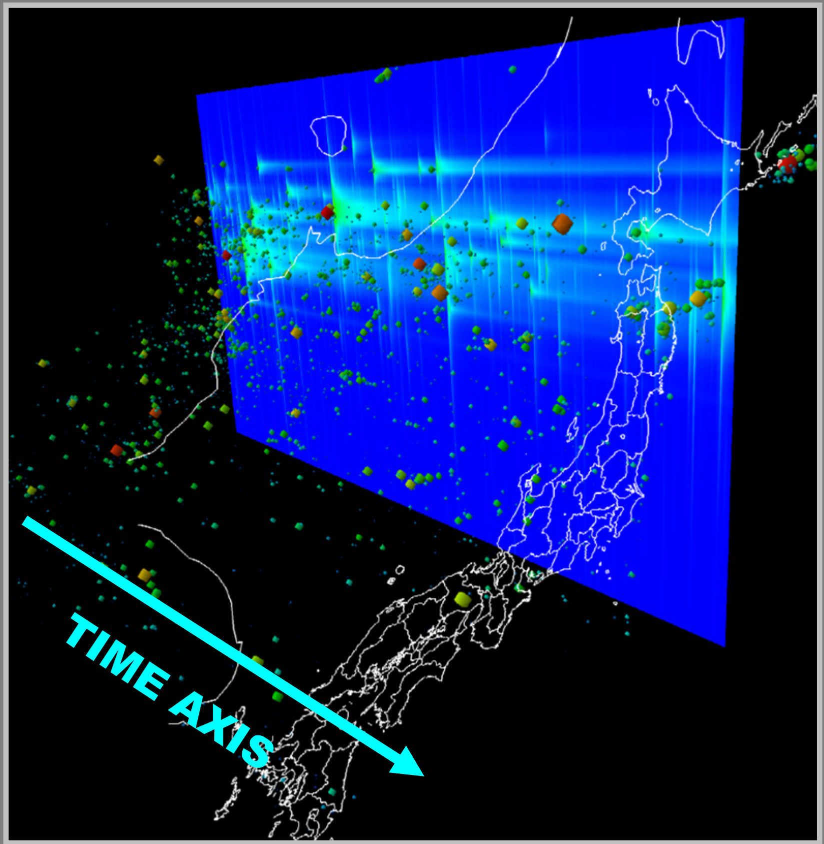
Delaunay-based function
an illustration

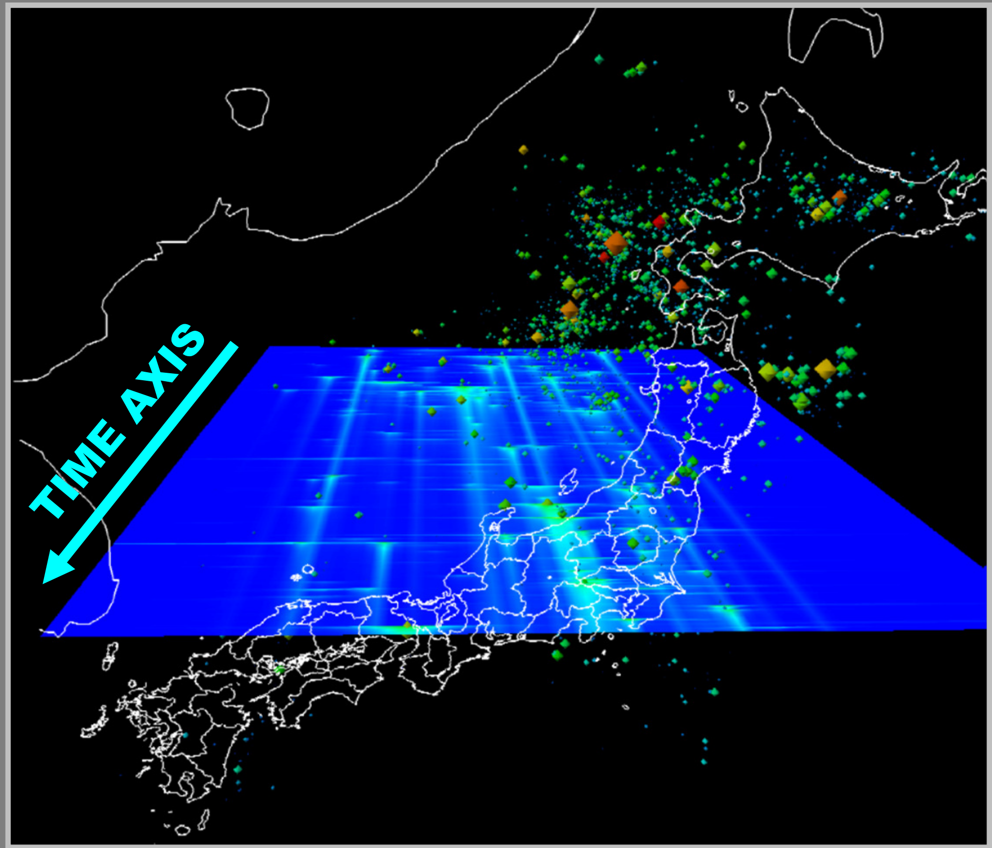
Likelihood based on location-dependent exponential distribution

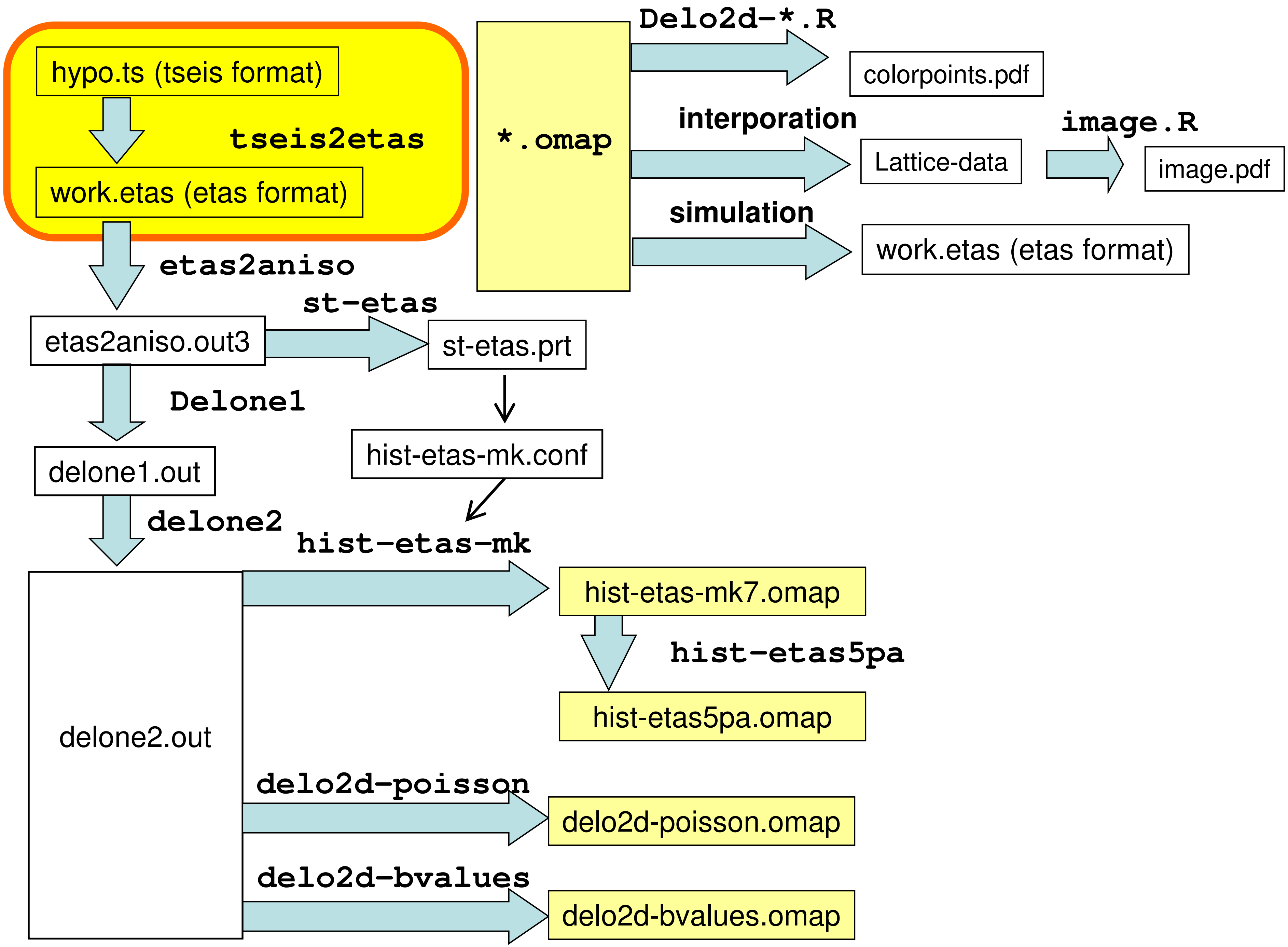
$$f(M \mid \vartheta) = \beta(x, y) e^{-\beta(x,y)(M-5.35)}$$

$$M \geq 5.4,\ b(x, y) = \frac{\beta(x, y)}{\ln 10}$$

Space-time earthquake data






hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
Delone1
delone1.out
hist-etas-mk.conf
delone2
hist-etas-mk
delone2.out
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

Catalog → Data

TSEIS format

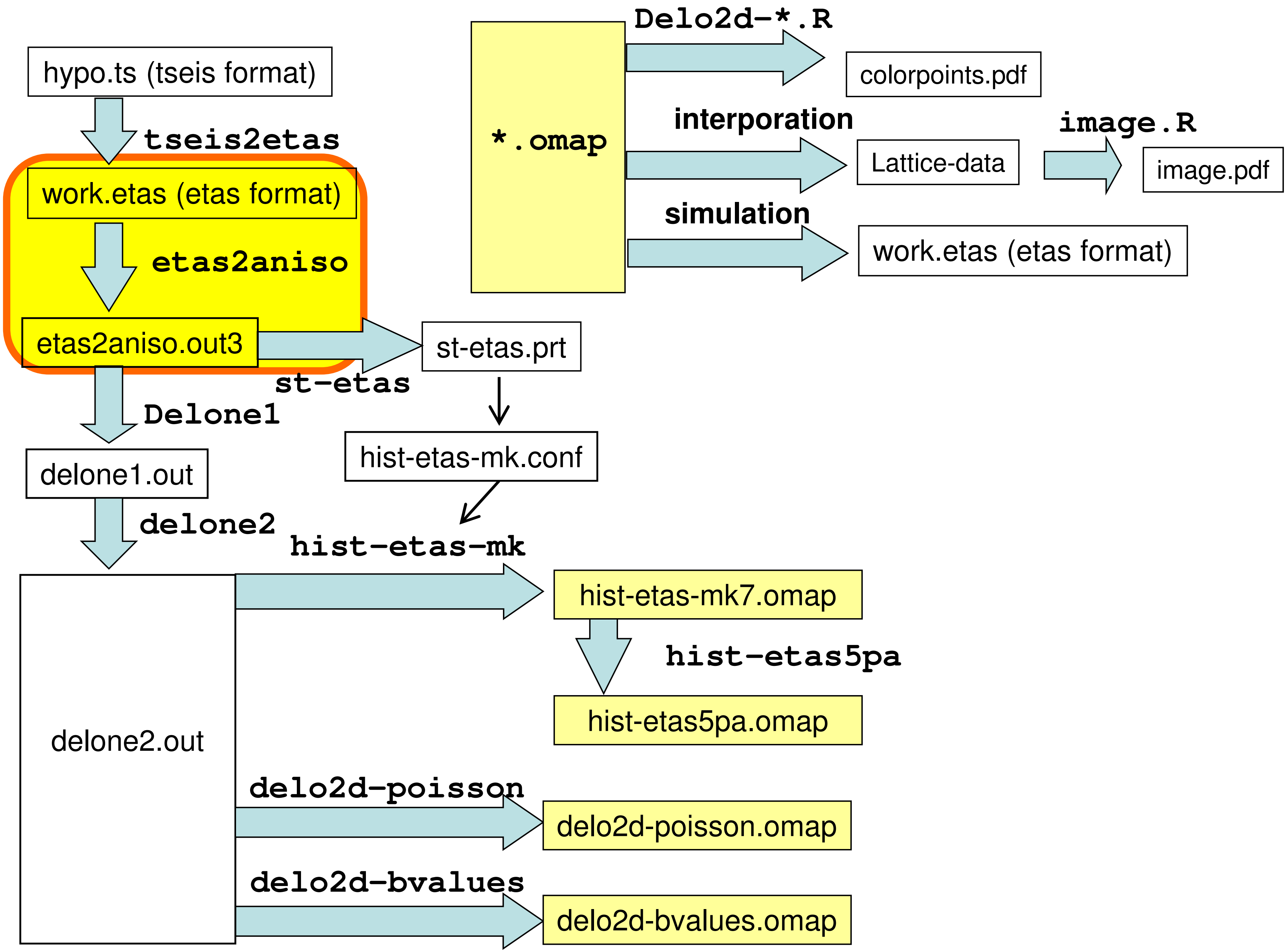
1976	01	01	00	00	00.00	182.360	-28.610	59.00	-7.3
1976	01	01	01	29	39.60	182.360	-28.610	59.00	7.3
1976	01	05	02	31	36.30	285.100	-13.290	95.00	5.7
1976	01	06	21	08	19.30	159.330	51.600	33.00	6.1
1976	01	13	13	29	19.50	343.420	66.160	33.00	6.3
.....									
2005	12	22	04	28	23.60	224.340	-54.500	10.00	5.5
2005	12	22	12	20	2.90	224.130	-54.720	10.00	6.4
2005	12	26	13	48	1.80	140.650	26.820	7.40	5.6
2005	12	30	18	26	43.90	277.730	7.530	10.00	6.1
2005	12	31	24	00	00.00	277.730	7.530	10.00	-6.1

tseis2etas

ETAS format

formatted_for_etas

1	182.36000	-28.61000	-7.3	0.00000	-59.00	1976	1	1
2	182.36000	-28.61000	7.3	0.06226	-59.00	1976	1	1
3	285.10000	-13.29000	5.7	4.10528	-95.00	1976	1	5
4	159.33000	51.60000	6.1	5.88078	-33.00	1976	1	6
5	343.42000	66.16000	6.3	12.56203	-33.00	1976	1	13
.....								
10562	224.34000	-54.50000	5.5	10948.18638	-10.00	2005	12	22
10563	224.13000	-54.72000	6.4	10948.51392	-10.00	2005	12	22
10564	140.65000	26.82000	5.6	10952.57502	-7.40	2005	12	26
10565	277.73000	7.53000	6.1	10956.76856	-10.00	2005	12	30
10566	277.73000	7.53000	-6.1	10958.00000	-10.00	2005	12	31



hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
Delone1
delone1.out
delone2
hist-etas-mk.conf
hist-etas-mk
delone2.out
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

Using all detected earthquakes during a short period after a large event, we identify:

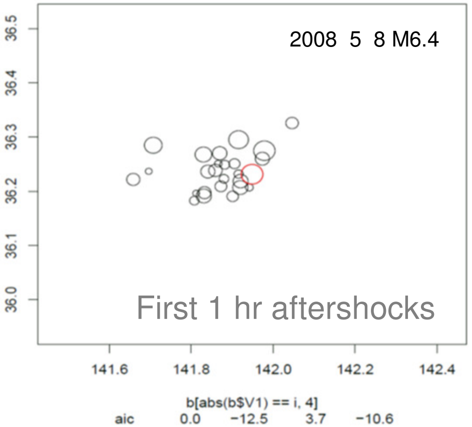
Isotropic kernel

$$Q_j(x,y) = (x-x_j)^2 + (y-y_j)^2 = (x-x_j, y-y_j) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x-x_j \\ y-y_j \end{pmatrix}$$

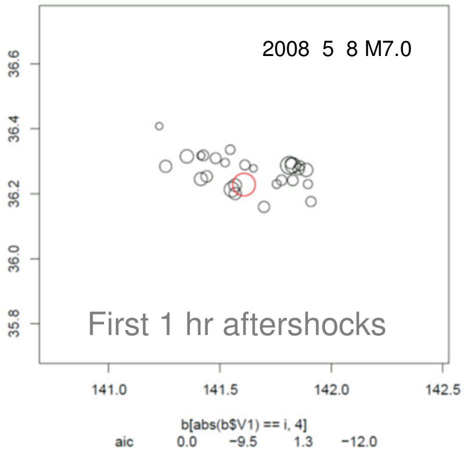
OR

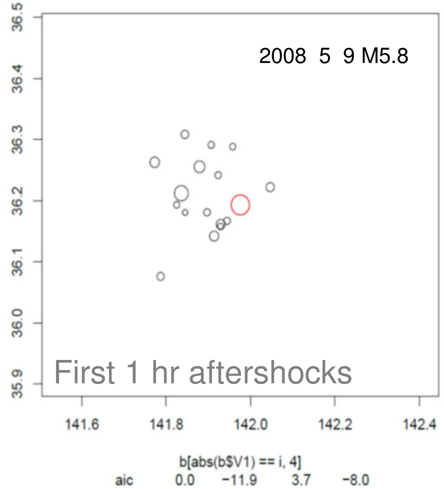
Anisotropic kernel

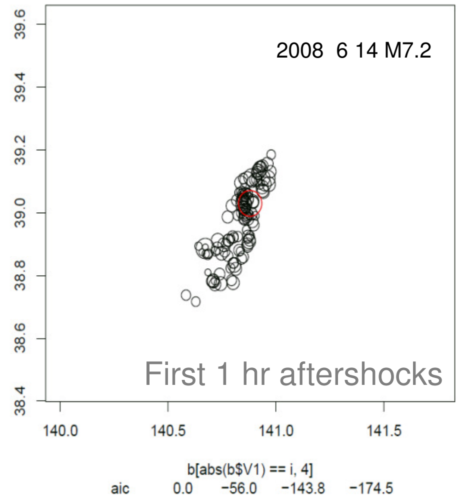
$$Q_j(x,y) \quad = \quad \sigma_1^2 \sigma_2^2 \sqrt{1-\rho^2}\,(x-\overline{x}_j\,, y-\overline{y}_j\,) \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}^{-1} \begin{pmatrix} x-\overline{x}_j \\ y-\overline{y}_j \end{pmatrix}$$

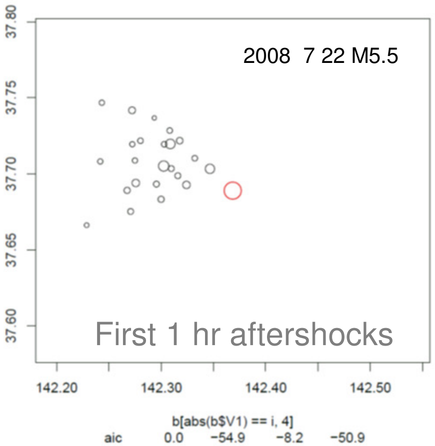


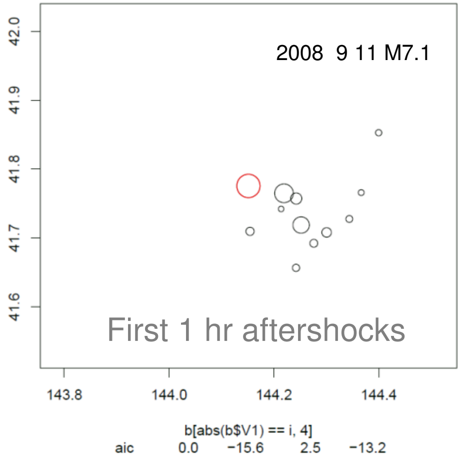
Spatial distributions of the first 1 hour aftershocks (Red circle = mainshock)

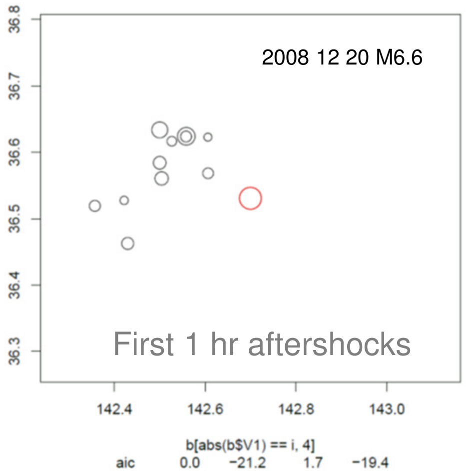








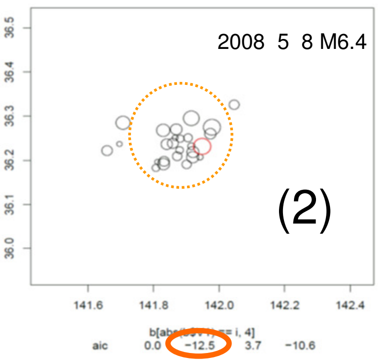


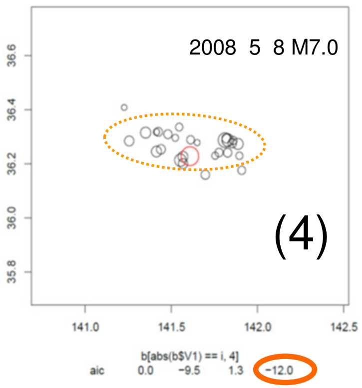


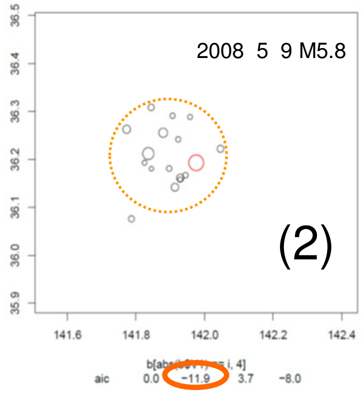
$$\log L\left(\mu_x,\mu_y;\sigma_1,\sigma_2,\rho \mid \begin{pmatrix}x_1\\ y_1\end{pmatrix},\cdots,\begin{pmatrix}x_n\\ y_n\end{pmatrix}\right) \propto \frac{n}{2}\log_e \det\begin{pmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{pmatrix}^{-1} - \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu_x,\, y_i-\mu_y)\begin{pmatrix}\sigma_x^2 & \rho\sigma_x\sigma_y\\ \rho\sigma_x\sigma_y & \sigma_y^2\end{pmatrix}^{-1}\begin{pmatrix}x_i-\mu_x\\ y_i-\mu_y\end{pmatrix}$$

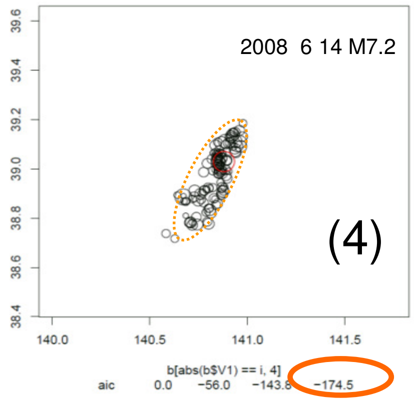
Selection by AIC = - 2 max(log L) + 2#(parameters)

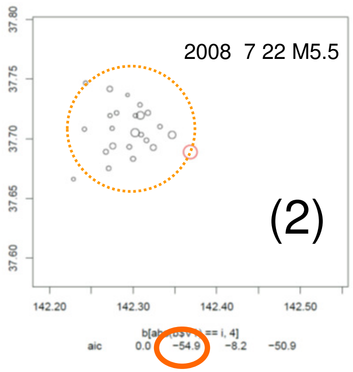
Models	(1)	(2)	(3)	(4)
	$N\left(\begin{pmatrix}x_1\\ y_1\end{pmatrix},\begin{pmatrix}\hat\sigma^2 & 0\\ 0 & \hat\sigma^2\end{pmatrix}\right)$	$N\left(\begin{pmatrix}\bar x\\ \bar y\end{pmatrix},\begin{pmatrix}\hat\sigma^2 & 0\\ 0 & \hat\sigma^2\end{pmatrix}\right)$	$N\left(\begin{pmatrix}x_1\\ y_1\end{pmatrix},\begin{pmatrix}\hat\sigma_x^2 & \hat\rho\hat\sigma_x\hat\sigma_y\\ \hat\rho\hat\sigma_x\hat\sigma_y & \hat\sigma_y^2\end{pmatrix}\right)$	$N\left(\begin{pmatrix}\bar x\\ \bar y\end{pmatrix},\begin{pmatrix}\hat\sigma_x^2 & \hat\rho\hat\sigma_x\hat\sigma_y\\ \hat\rho\hat\sigma_x\hat\sigma_y^2 & \hat\sigma_y\end{pmatrix}\right)$

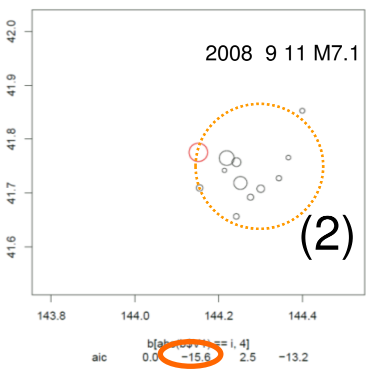


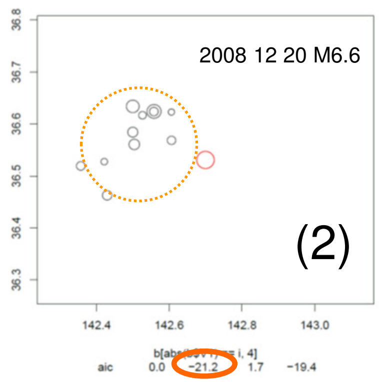












etas2aniso

ETAS format

etas2aniso format

formatted_for_etas

1	142.00000	37.50000	6.0	39.08333	-50.00	1885	2	9
2	139.00000	35.50000	6.0	78.54167	-20.00	1885	3	20
3	143.00000	40.50000	6.9	161.38889	-50.00	1885	6	11
4	141.00000	35.00000	6.5	165.06944	-50.00	1885	6	15
5	142.00000	42.00000	6.0	209.22917	-50.00	1885	7	29
6	146.50000	43.00000	6.5	247.82639	-50.00	1885	9	5
7	145.00000	42.50000	6.0	250.97222	-50.00	1885	9	8
8	139.00000	34.00000	6.8	268.50000	-20.00	1885	9	26
9	139.00000	34.00000	6.7	270.22917	-20.00	1885	9	28

. . . .

878626	131.37880	33.31280	0.0	45836.91136	-7.42	2010	6	30
878627	137.60930	35.12800	0.0	45836.91890	-14.48	2010	6	30
878628	140.38950	39.87070	0.5	45836.92130	-1.81	2010	6	30
878629	140.38680	39.86650	0.3	45836.92144	-2.28	2010	6	30
878630	140.39000	39.86970	0.9	45836.92151	-2.12	2010	6	30
878631	140.38680	39.86670	0.9	45836.92157	-3.98	2010	6	30
878632	140.38470	39.86680	0.3	45836.92242	-3.83	2010	6	30
878633	141.69300	45.06570	0.8	45836.92817	-25.53	2010	6	30
878634	140.38500	39.86880	0.4	45836.92949	-5.29	2010	6	30
878635	140.91420	39.06220	0.9	45836.93190	-6.58	2010	6	30
878636	129.89900	32.58320	0.4	45836.94384	-10.85	2010	6	30
878637	135.88000	34.42550	0.7	45836.94456	-9.40	2010	6	30
878638	138.19130	36.96350	0.3	45836.96106	-15.29	2010	6	30
878639	134.98520	33.83080	1.0	45836.96420	-12.10	2010	6	30
878640	130.82330	32.98950	0.3	45836.96595	-9.20	2010	6	30
878641	140.84020	39.09670	0.7	45836.97037	-10.16	2010	6	30
878642	131.02080	31.88720	0.5	45836.97579	-9.48	2010	6	30
878643	134.99270	33.85970	0.5	45836.97898	-10.41	2010	6	30
878644	139.11430	35.02220	0.2	45836.98698	-6.79	2010	6	30

2	142.00000	37.50000	6.00000	39.08333	1.0000	1.0000	0.0000
3	139.00000	35.50000	6.00000	78.54167	1.0000	1.0000	0.0000
4	143.00000	40.50000	6.90000	161.38889	1.0000	1.0000	0.0000
5	141.00000	35.00000	6.50000	165.06944	1.0000	1.0000	0.0000
6	142.00000	42.00000	6.00000	209.22917	1.0000	1.0000	0.0000
7	146.50000	43.00000	6.50000	247.82639	1.0000	1.0000	0.0000
8	145.00000	42.50000	6.00000	250.97222	1.0000	1.0000	0.0000
9	139.00000	34.00000	6.80000	268.50000	1.0000	1.0000	0.0000
10	139.00000	34.00000	6.70000	270.22917	1.0000	1.0000	0.0000

. . . .

610591	139.19984	34.93534	5.80000	44305.11851	0.0473	0.0462	-0.5593
613215	139.33080	34.91680	5.10000	44316.76703	1.0000	1.0000	0.0000
657732	136.69450	37.23236	6.90000	44643.40414	0.1109	0.0863	0.6213
657733	136.72380	37.25580	5.10000	44643.40501	1.0000	1.0000	0.0000
658447	136.83950	37.30430	5.30000	44643.75816	1.0000	1.0000	0.0000
658973	136.48930	37.16680	5.30000	44644.30320	1.0000	1.0000	0.0000
665556	136.41345	34.79063	5.40000	44664.51354	0.0135	0.0134	-0.5479
678354	136.65470	37.24420	5.00000	44721.15641	1.0000	1.0000	0.0000
684230	138.59066	37.45124	6.80000	44756.42596	0.1464	0.0975	-0.2300
684639	138.64450	37.50400	5.80000	44756.65116	1.0000	1.0000	0.0000
692806	140.35014	35.35289	5.20000	44789.70496	0.0348	0.0309	0.2955
736597	140.85768	39.00530	7.20000	45090.36372	0.0899	0.1320	0.2104
736660	140.67300	38.88630	5.70000	45090.38903	1.0000	1.0000	0.0000
736963	140.94020	39.14130	5.20000	45090.51913	1.0000	1.0000	0.0000
739547	140.83970	38.99720	5.30000	45092.96850	1.0000	1.0000	0.0000
786017	136.31936	35.65924	5.20000	45339.28272	0.0095	0.0084	0.3103
817808	138.42956	34.82938	6.50000	45513.21326	0.0711	0.0518	-0.4588
844846	139.13254	34.96575	5.00000	45641.98987	0.0108	0.0070	-0.4183
845846	139.11717	35.01632	5.10000	45642.36501	0.0226	0.0123	0.4680

$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

Immediately after a large event, we adopt:

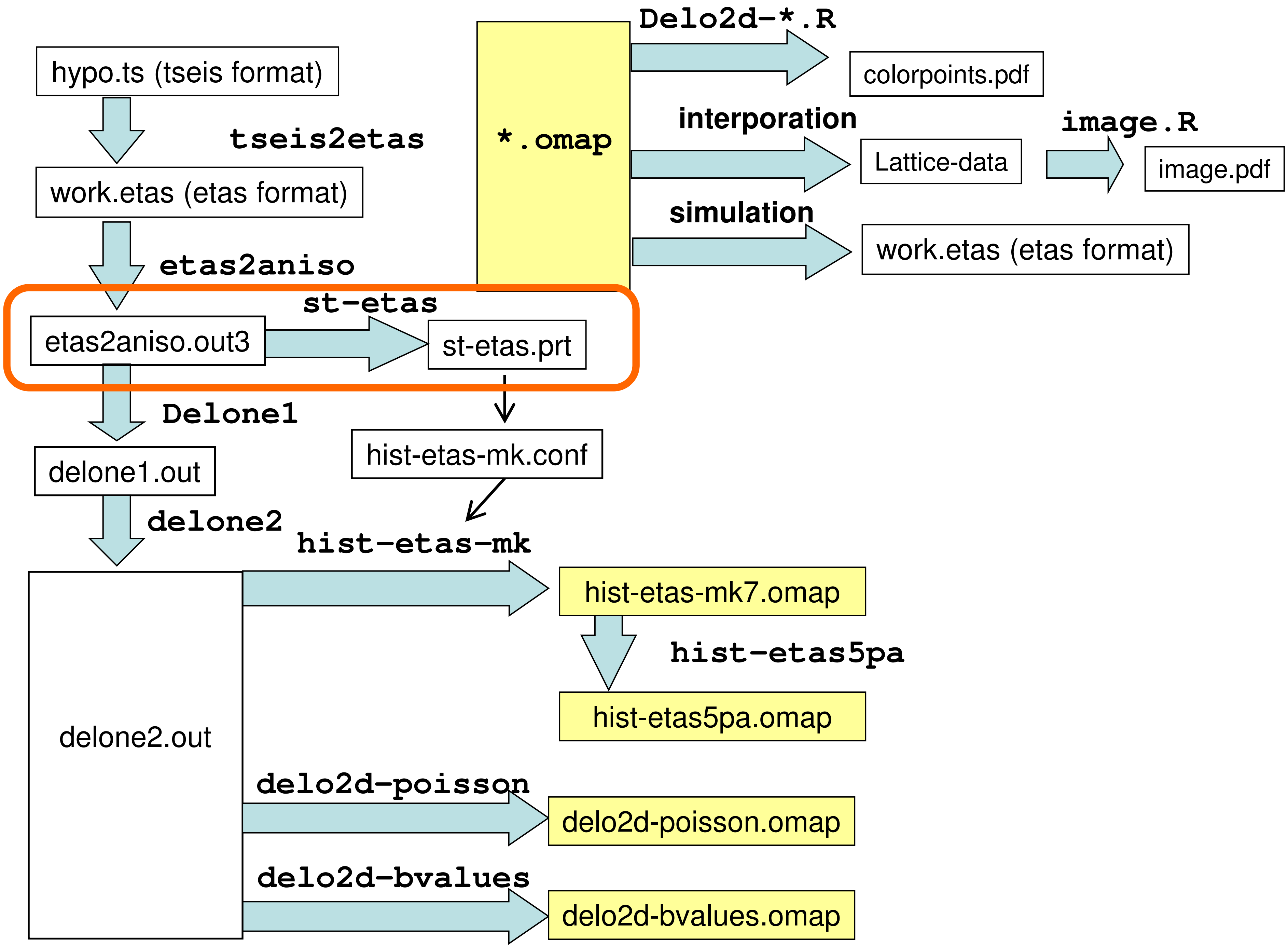
Isotropic kernel within 1 hour

$$Q_j(x,y) = (x-x_j)^2 + (y-y_j)^2 = (x-x_j, y-y_j) S_j \begin{pmatrix} x-x_j \\ y-y_j \end{pmatrix} \quad \text{where} \quad S_j = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Anisotropic kernel after 1 hour

$$Q_j(x,y) = \frac{1}{\sqrt{1-\rho^2}} \left(\frac{\sigma_2^2}{\sigma_1^2}(x-\bar{x}_j)^2 - 2\rho(x-\bar{x}_j)(y-\bar{y}_j) + \frac{\sigma_1^2}{\sigma_2^2}(y-\bar{y}_j)^2 \right)$$

$$= (x-\bar{x}_j, y-\bar{y}_j) S_j \begin{pmatrix} x-\bar{x}_j \\ y-\bar{y}_j \end{pmatrix} \quad \text{where} \quad S_j = \sigma_1^2\sigma_2^2\sqrt{1-\rho^2} \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}^{-1}$$


hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
Delone1
delone1.out
hist-etas-mk.conf
delone2
hist-etas-mk
delone2.out
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

Space-Time ETAS model

Ogata (1998, *AISM*)

$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \boxed{\left\{ \frac{Q_j(x,\, y)}{e^{\alpha M_j}} + d \right\}^{-q}}$$

where

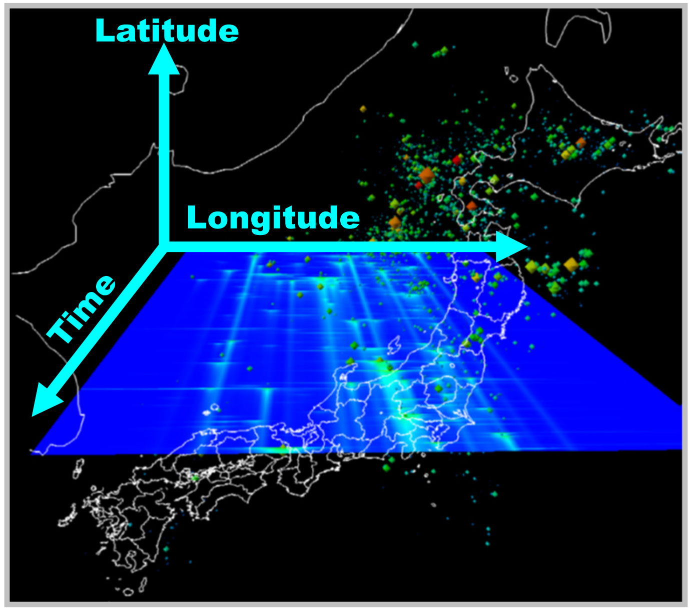
$$Q_j(x,\, y) = (x-x_j,\ y-y_j)\, S_j \begin{pmatrix} x-x_j \\ y-y_j \end{pmatrix}$$

Program **etaspaSelecAniso** maximize the log-likelihood function

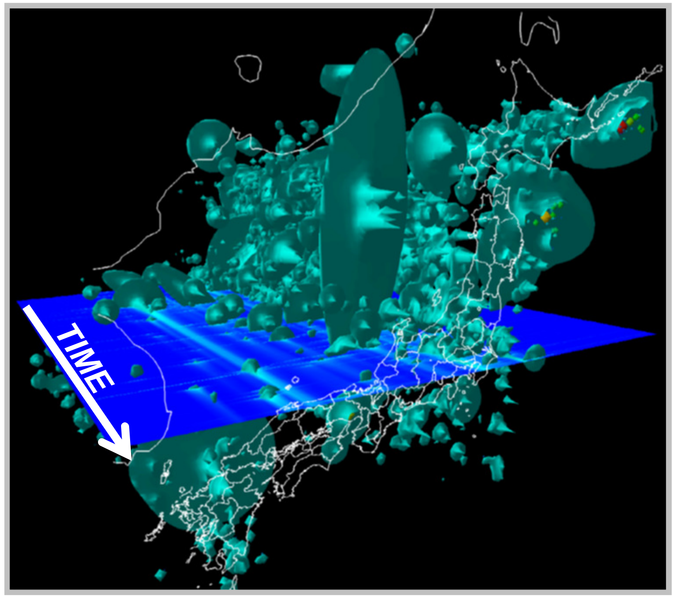
$$\log L(\theta) = \sum_{\{i;\, S<t_i<T\}} \log \lambda_\theta(t_i, x_i, y_i) - \int_S^T \iint_A \lambda_\theta(t,x,y)\, dt dx dy$$

where $\theta = (\mu, K, c, \alpha, p, d, q)$

Earthquakes (ti,xi,yi)
1926 - 1995; magnitude >= 5.0



Space-time ETAS model
Isosurface of an occurrence rate $\lambda(t, x, y)$



Space-Time ETAS model

$$\lambda(t, x, y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t - t_j + c)^p} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

where

$$Q_j(x,\ y) = (x - x_j,\ y - y_j)\, S_j \begin{pmatrix} x - x_j \\ y - y_j \end{pmatrix}$$

Heterogeneity in Space

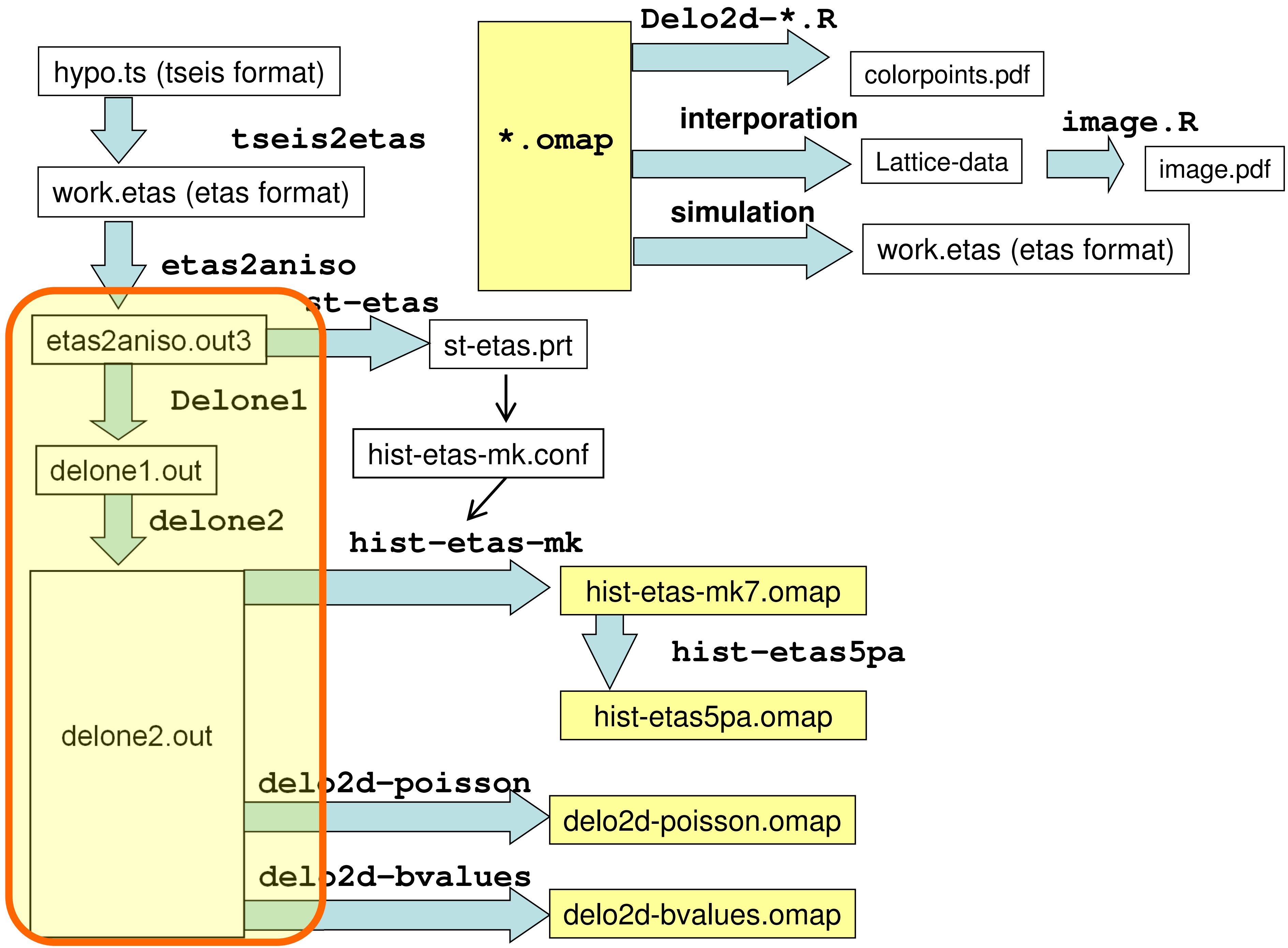
$$\mu \Longrightarrow \mu\,(x, y);$$

$$K \Longrightarrow K\,(x_j, y_j); \quad \alpha \Longrightarrow \alpha\,(x_j, y_j);$$

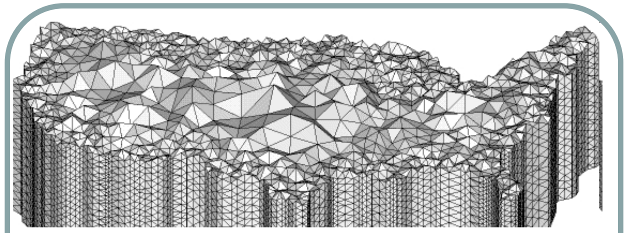
$$p \Longrightarrow p\,(x_j, y_j); \quad q \Longrightarrow q\,(x_j, y_j)$$

Hierarchical Space-Time (HIST) ETAS model

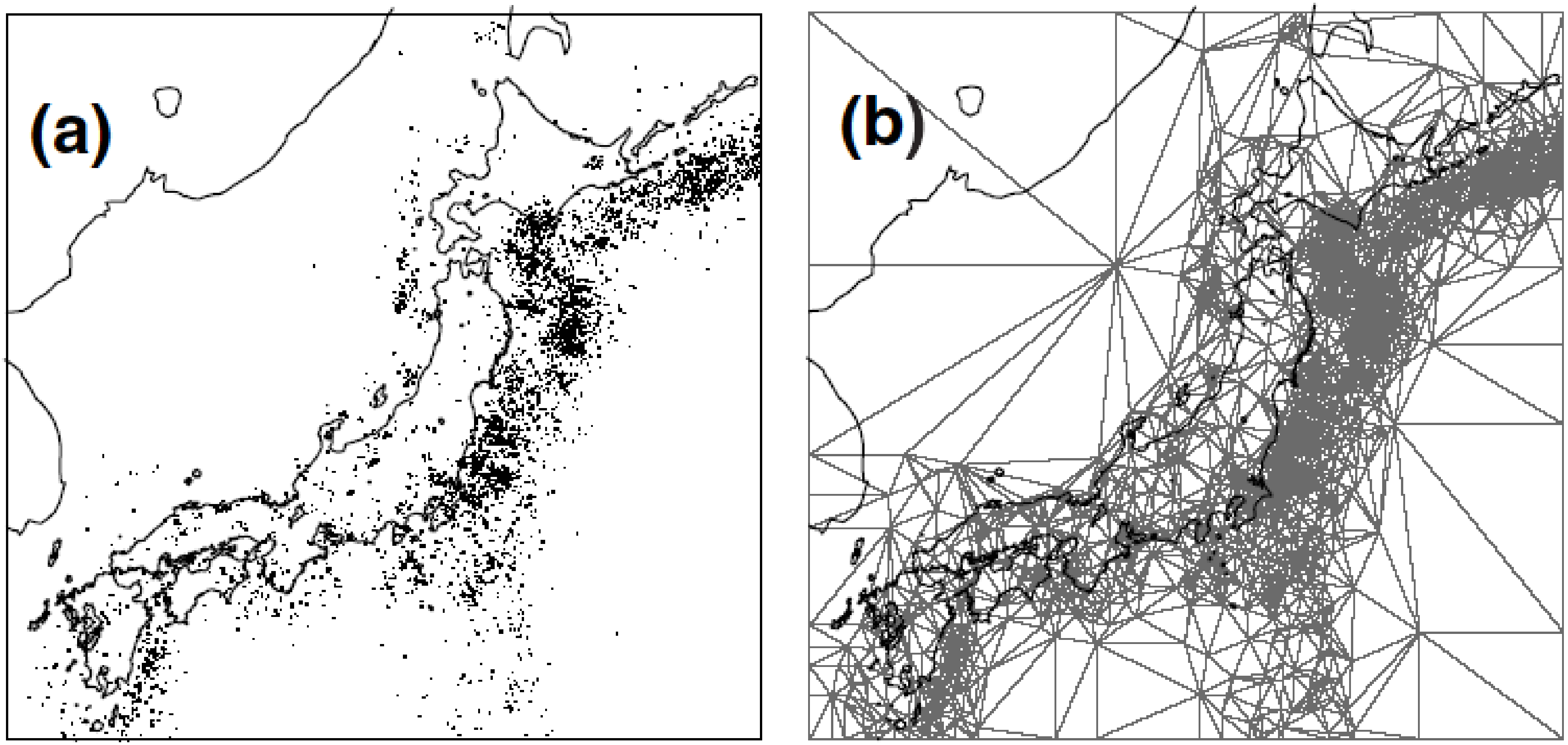
= location-dependent parameters

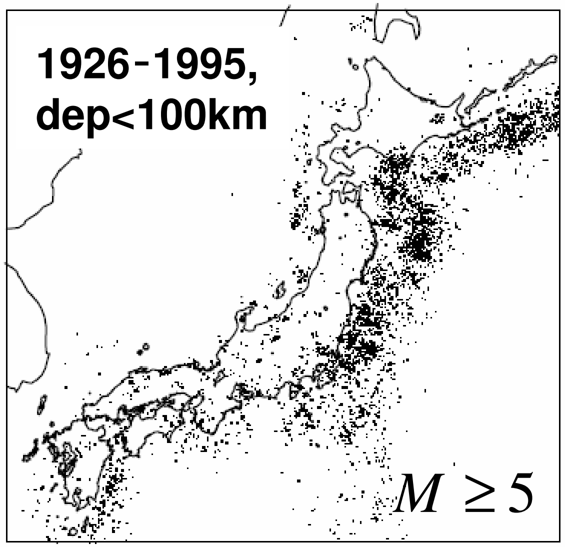

Delo2d-*.R
hypo.ts (tseis format)
colorpoints.pdf
interporation
image.R
tseis2etas
*.omap
Lattice-data
image.pdf
work.etas (etas format)
simulation
work.etas (etas format)
etas2aniso
st-etas
etas2aniso.out3
st-etas.prt
Delone1
delone1.out
hist-etas-mk.conf
delone2
hist-etas-mk
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delone2.out
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap

1926 - 1995, M >= 5.0, depth<100km

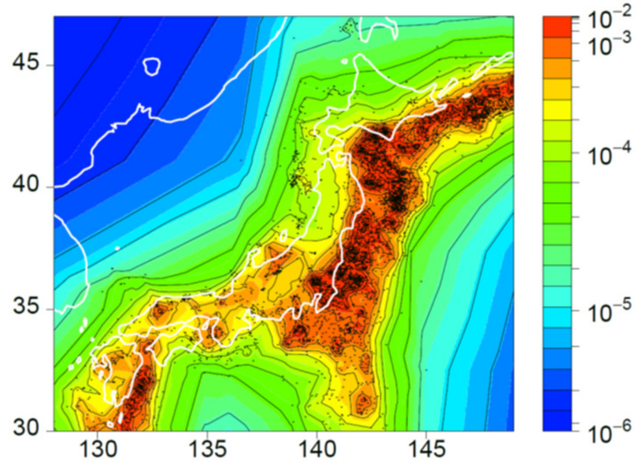


Delaunay-based function
an illustration





$\mu\,(x,y)$

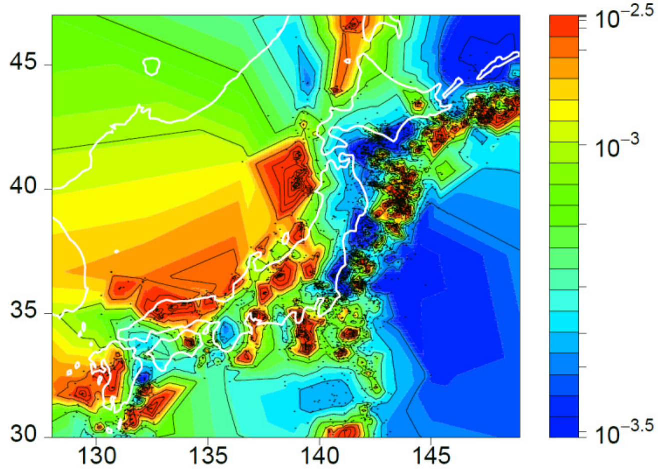


Background activity

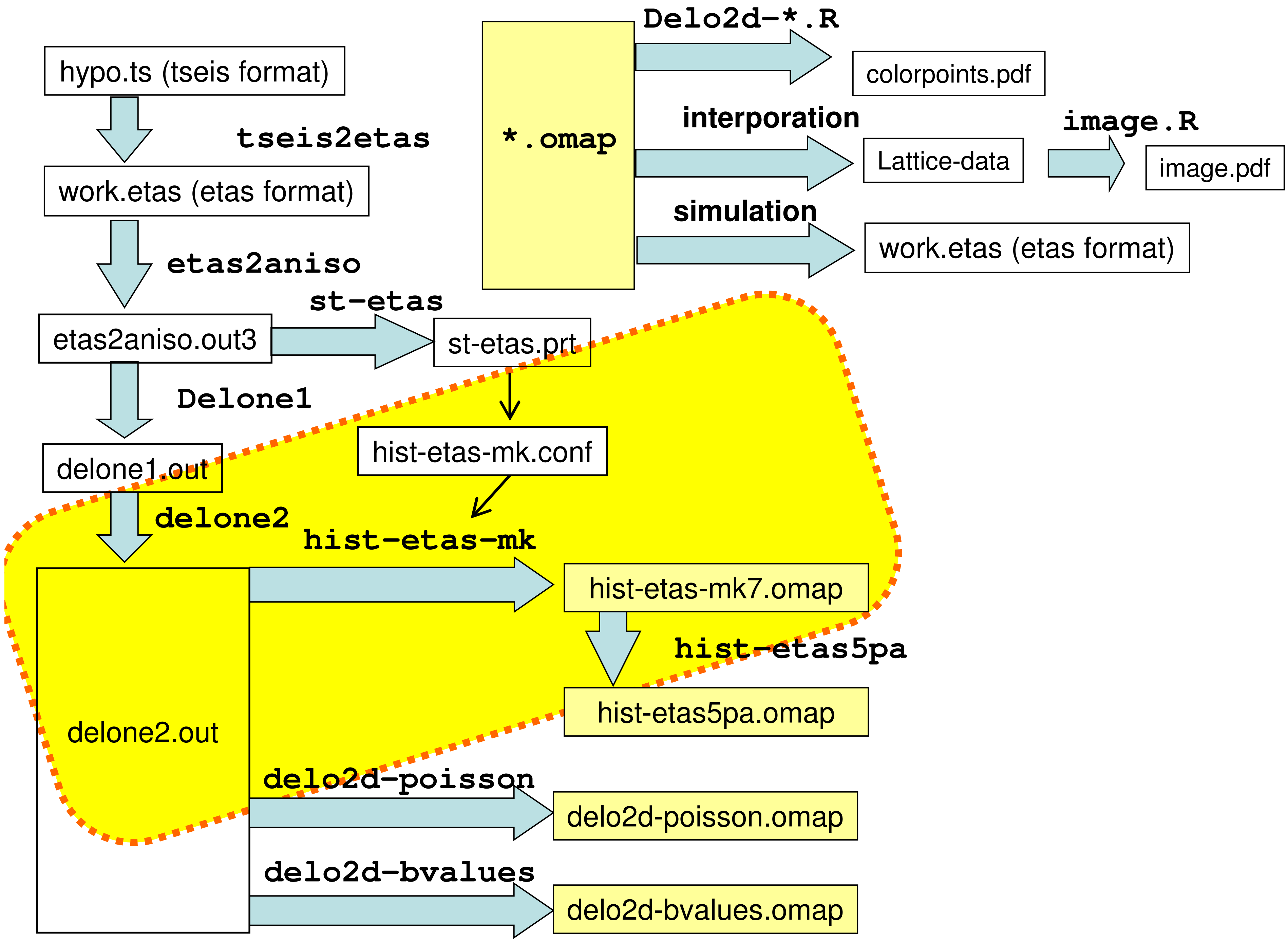
Aftershock productivity

$$\lambda(t,x,y\,|\,H_t) = \mu(x,y) + \sum_{\{j;\,t_j<t\}} \frac{K(x,y)}{(t-t_j+c)^p} \times \left[\frac{(x-x_j,\, y-y_j)\, S_j\, (x-x_j,\, y-y_j)^t}{e^{\alpha(M_j-M_c)}} + d \right]^{-q}$$

$K(x,y)$



→ Simultaneous estimation of $\mu(x,y)$, $K(x,y)$, c, α, p, d and q


hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
Delone1
delone1.out
hist-etas-mk.conf
delone2
hist-etas-mk
delone2.out
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

Log likelihood

$$\log L(\theta) = \sum_{\{i;\, S<t_i<T\}} \log \lambda_\theta(t_i, x_i, y_i) - \int_S^T \iint_A \lambda_\theta(t, x, y)\, dtdxdy$$

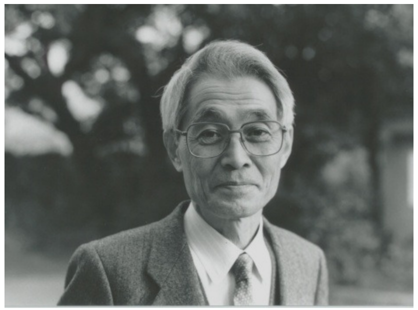
where $\theta = (\mu, K, c, \alpha, p, d, q)$

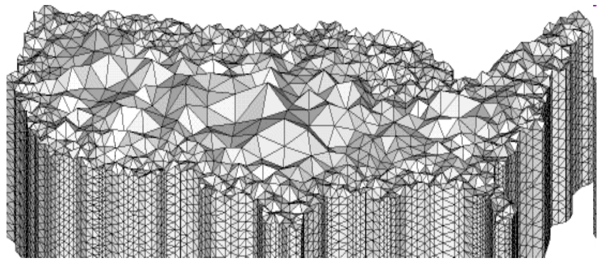
Penalized log likelihood

$$Q(\theta \mid w) = \log L(\theta) - penalty(\theta \mid w_\mu, w_K)$$

$$penalty(\theta \mid w_\mu, w_K) = w_\mu \iint_A dxdy \left\{ \left(\frac{d\mu}{dx}\right)^2 + \left(\frac{d\mu}{dy}\right)^2 \right\} + w_K \iint_A dxdy \left\{ \left(\frac{dK}{dx}\right)^2 + \left(\frac{dK}{dy}\right)^2 \right\}$$

Flatness constraints





Delaunay-based function: an illustration

$$posterior(\theta \mid \boldsymbol{\rho}) = \frac{L(\theta) \cdot prior(\theta \mid \boldsymbol{\rho})}{\Lambda(\boldsymbol{\rho})}, \quad \boldsymbol{\rho} = (w_\mu, w_K)$$

$$\Lambda(\boldsymbol{\rho}) = \int \cdots \int L(\theta) \cdot prior(\theta \mid \boldsymbol{\rho}) d\theta$$

Likelihood of a Bayesian model;
Good (1965)

Choose $\boldsymbol{\rho}$ that maximize $\Lambda(\boldsymbol{\rho})$
in addition to optimize the base-line parameters

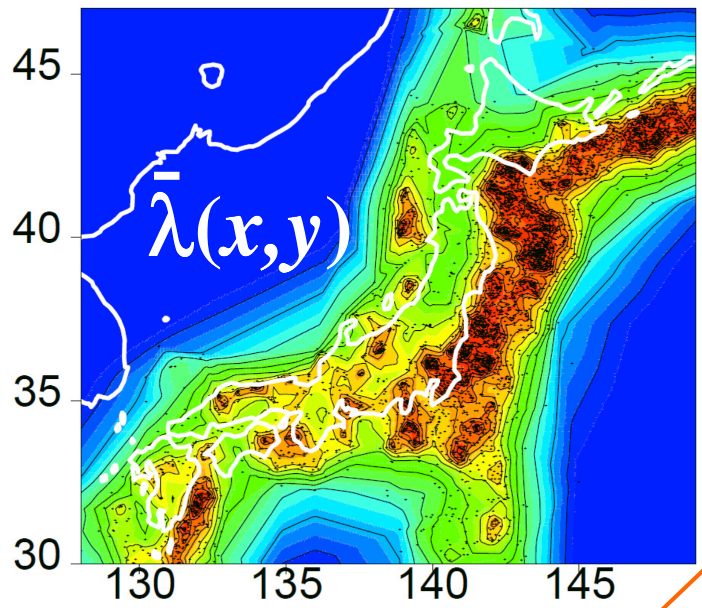
$$\hat{\theta} = (\hat{\mu}_0, \hat{K}_0, \hat{c}, \hat{\alpha}, \hat{p}, \hat{d}, \hat{q})$$

or minimize

$$ABIC = (-2) \max_{\boldsymbol{\rho}} \{\log \Lambda(\boldsymbol{\rho})\} + 2 \times \dim(\boldsymbol{\rho})$$

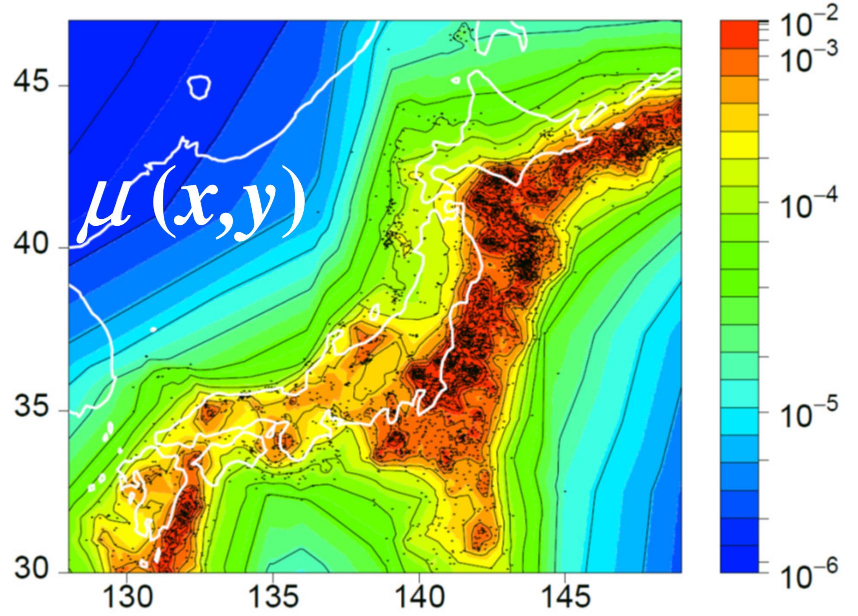
Akaike Bayesian information criterion (Akaike, 1980)

1926-1995 $M \geq 5.0$

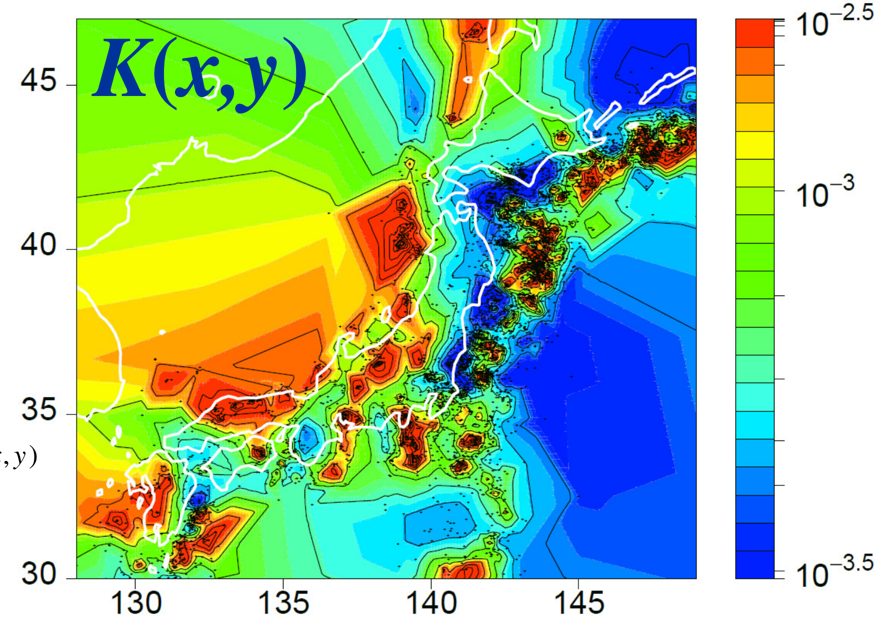


Spatial Poisson intensity

$$\boxed{\bar{\lambda}(x,y)} \approx \frac{1}{70}\int_{1926}^{1995} \lambda(t,x,y \mid H_t)\,dt$$



≈ +



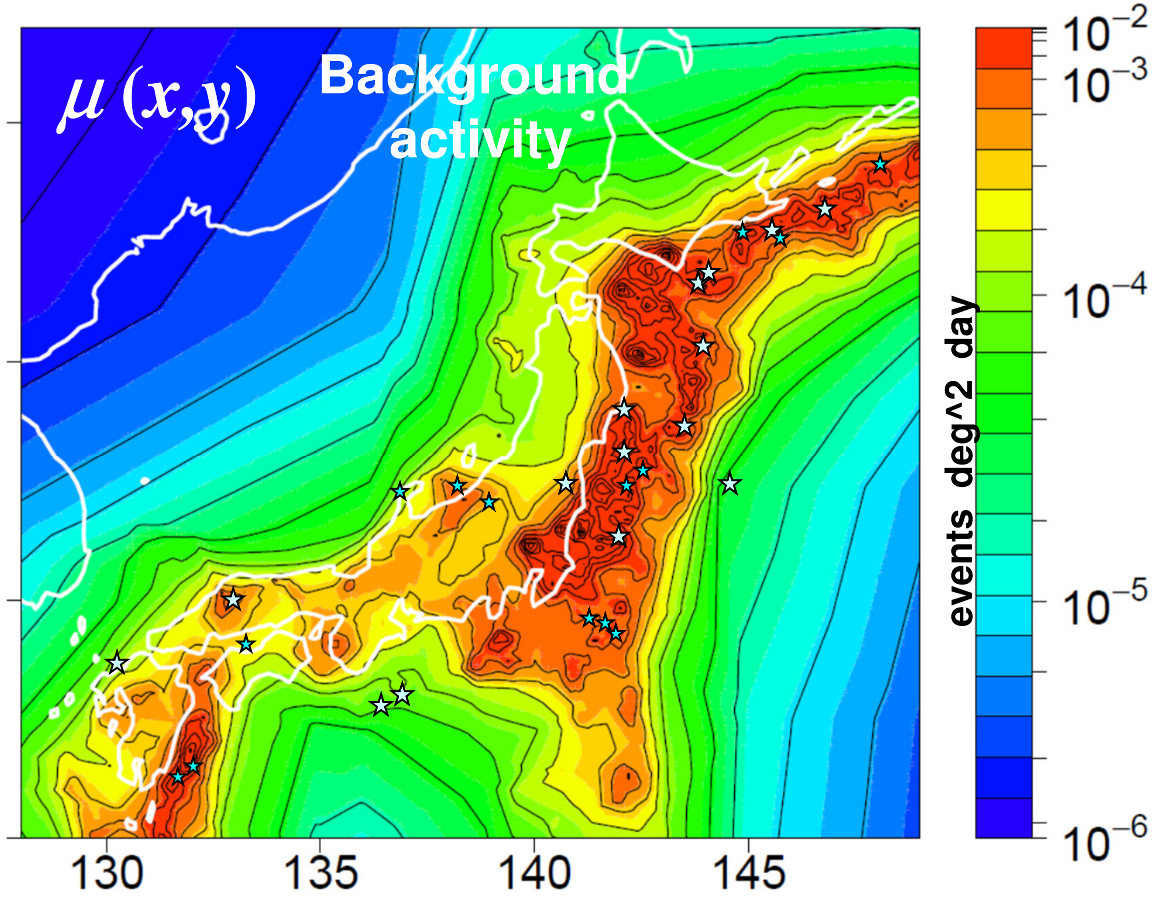
Background activity

Aftershock productivity

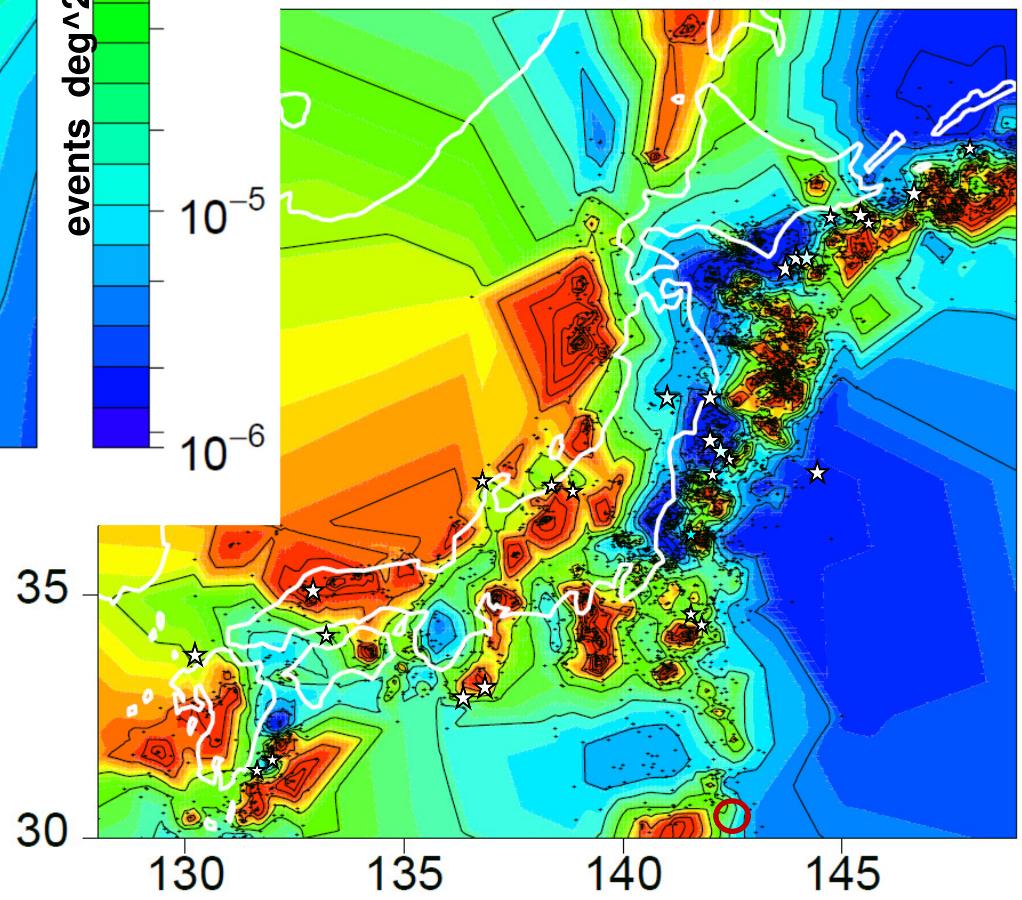
$$\lambda(t,x,y \mid H_t) = \mu(x,y) + \sum_{\{j;t_j<t\}} \frac{K(x,y)}{(t-t_j+c)^{p(x,y)}} \times \left[\frac{(x-x_j,\,y-y_j)\,S_j\,(x-x_j,\,y-y_j)^t}{e^{\alpha(x,y)(M_j-M_c)}} + d\right]^{-q(x,y)}$$

Estimated from M>=5.0 for 1926-1995

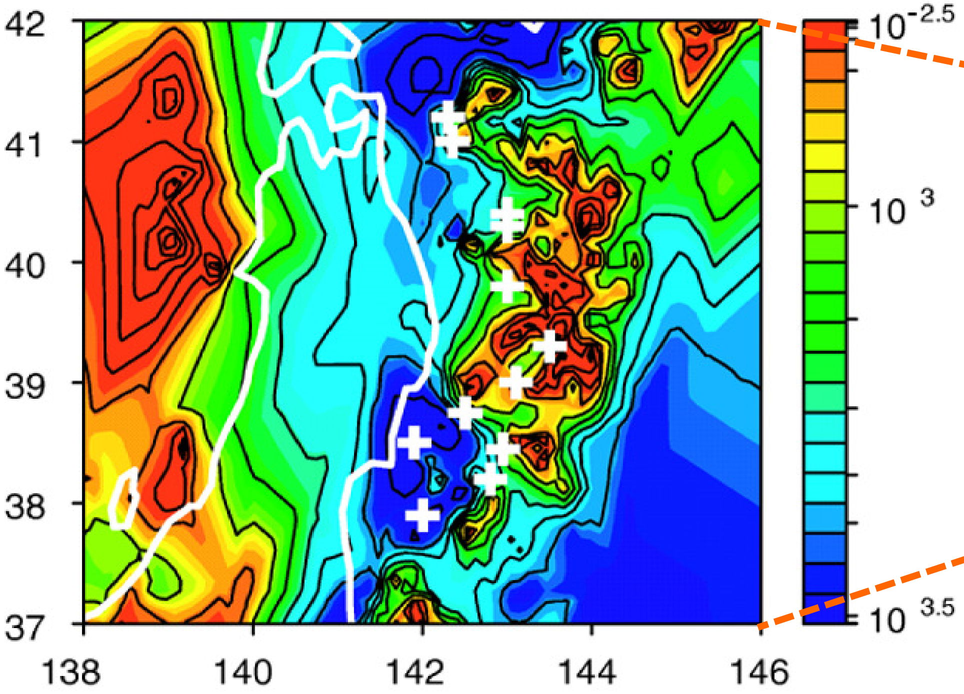
☆ = earthquakes of M>= 6.7 occurred during 1996 - 2009

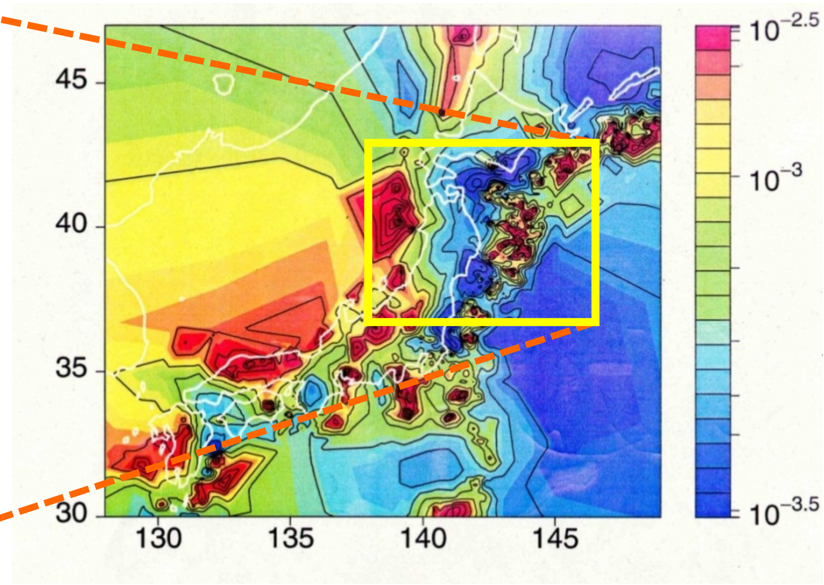


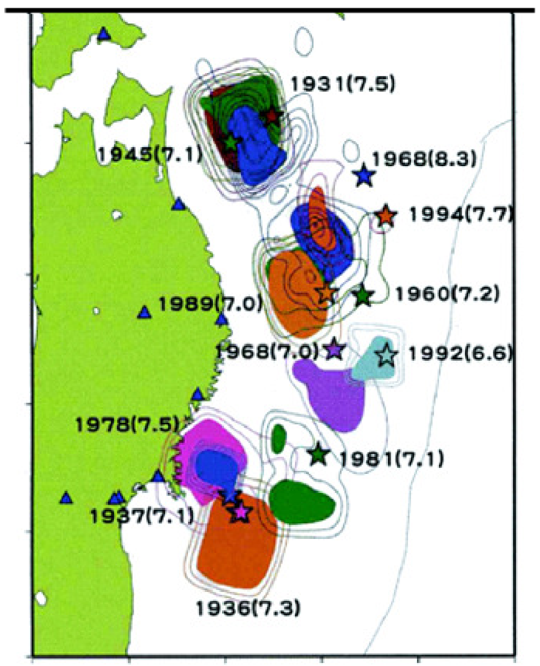
$K(x,y)$ **Aftershock productivity**

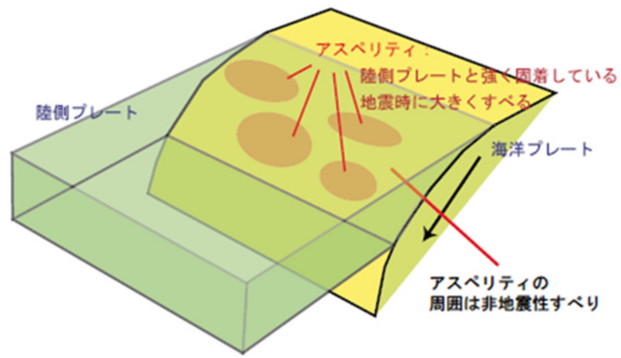


$$\lambda(t,x,y \mid H_t) = \mu(x,y) + \sum_{\{j;\, t_j<t\}} \frac{K(x,y)}{(t-t_j+c)^{p(x,y)}} \times \left[\frac{(x-x_j,\, y-y_j)\, S_j\, (x-x_j,\, y-y_j)^t}{e^{\alpha(x,y)(M_j-M_c)}} + d \right]^{-q(x,y)}$$



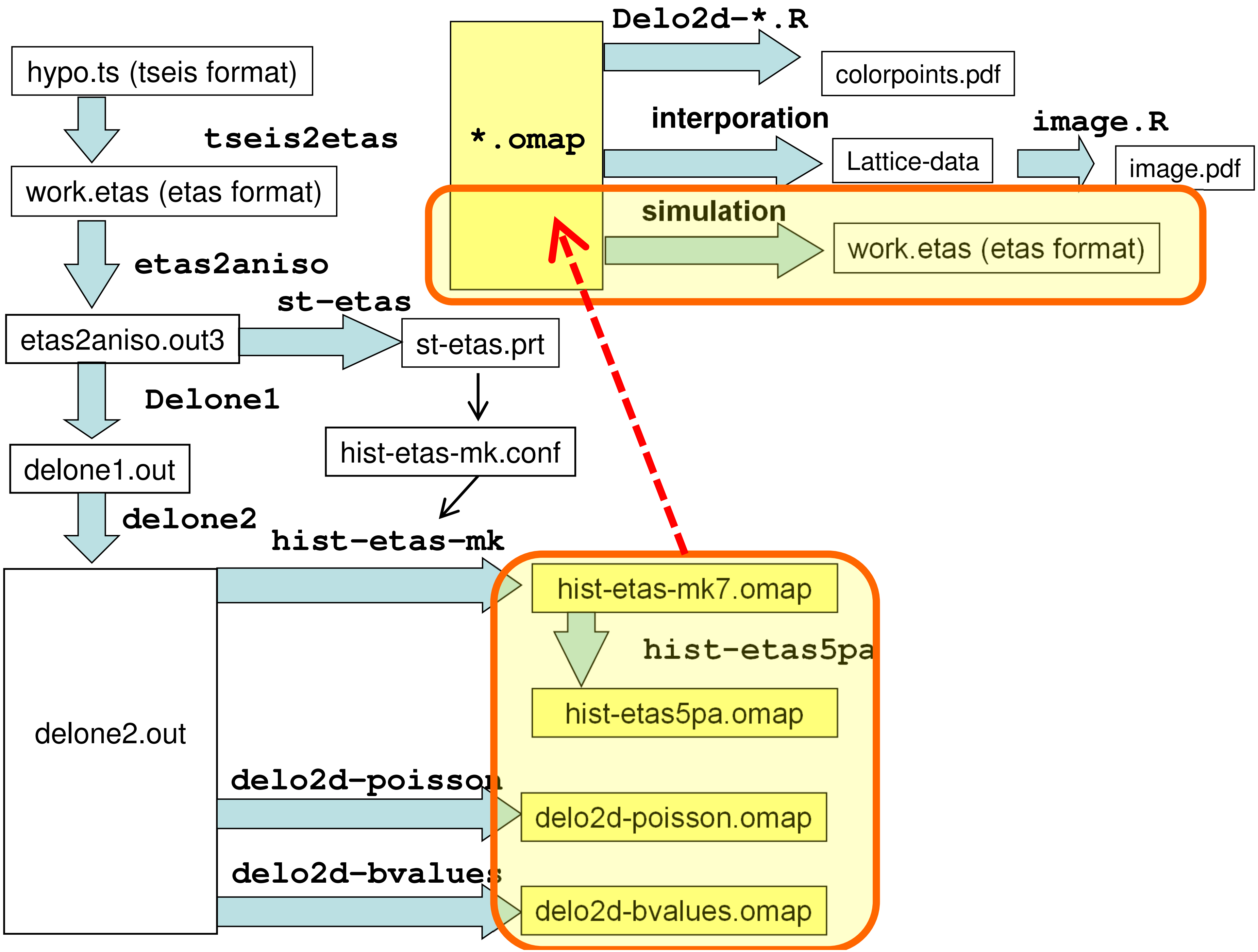




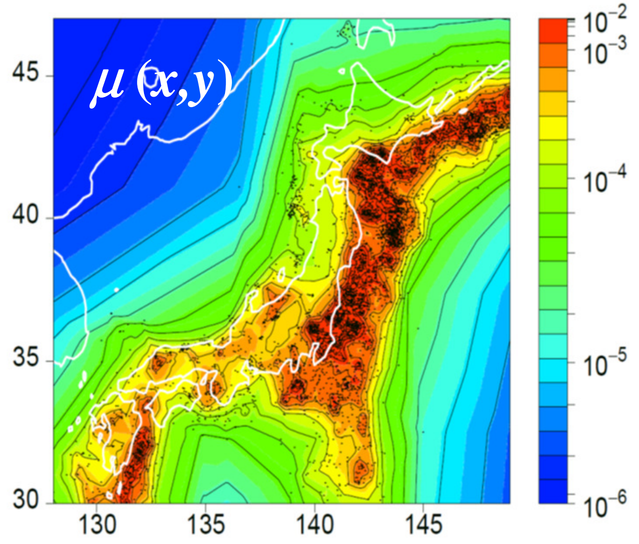


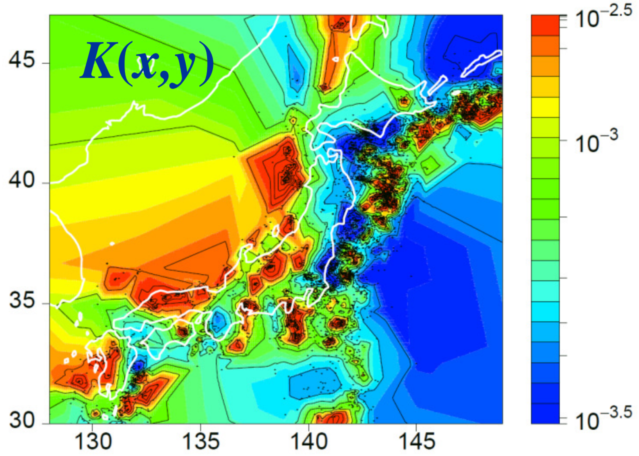
Space-Time ETAS model

$$\lambda(t, x, y) = \mu + \sum_{\{j:\, t_j < t\}} \frac{K}{(t - t_j + c)^{p}} \left\{ \frac{Q_j(x,\ y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

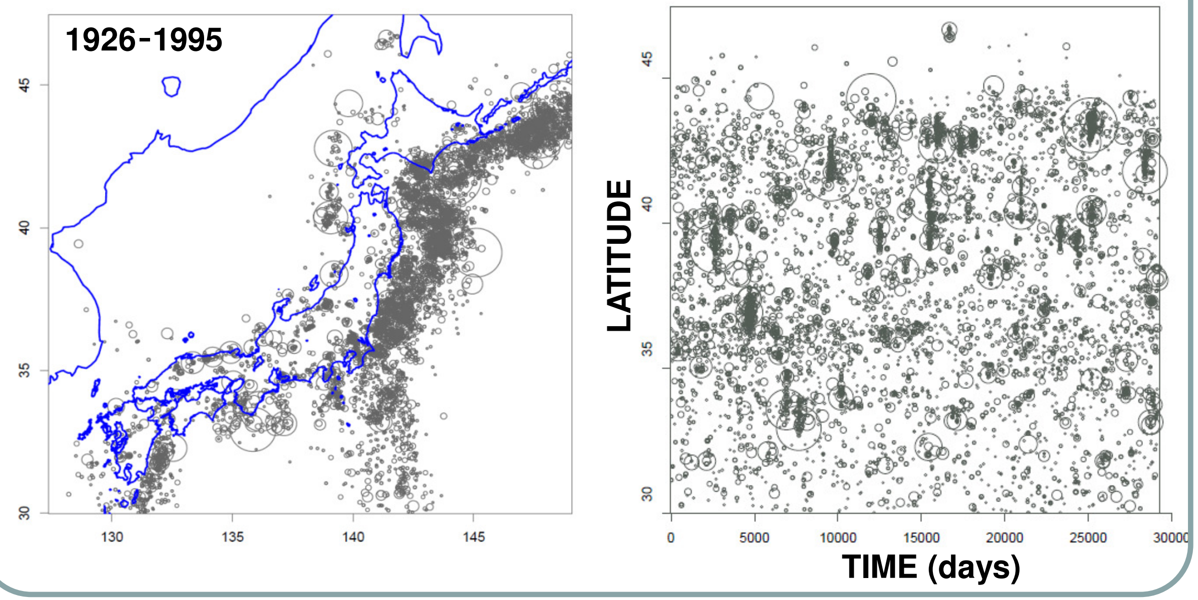


hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
hist-etas-mk.conf
Delone1
delone1.out
delone2
delone2.out
hist-etas-mk
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

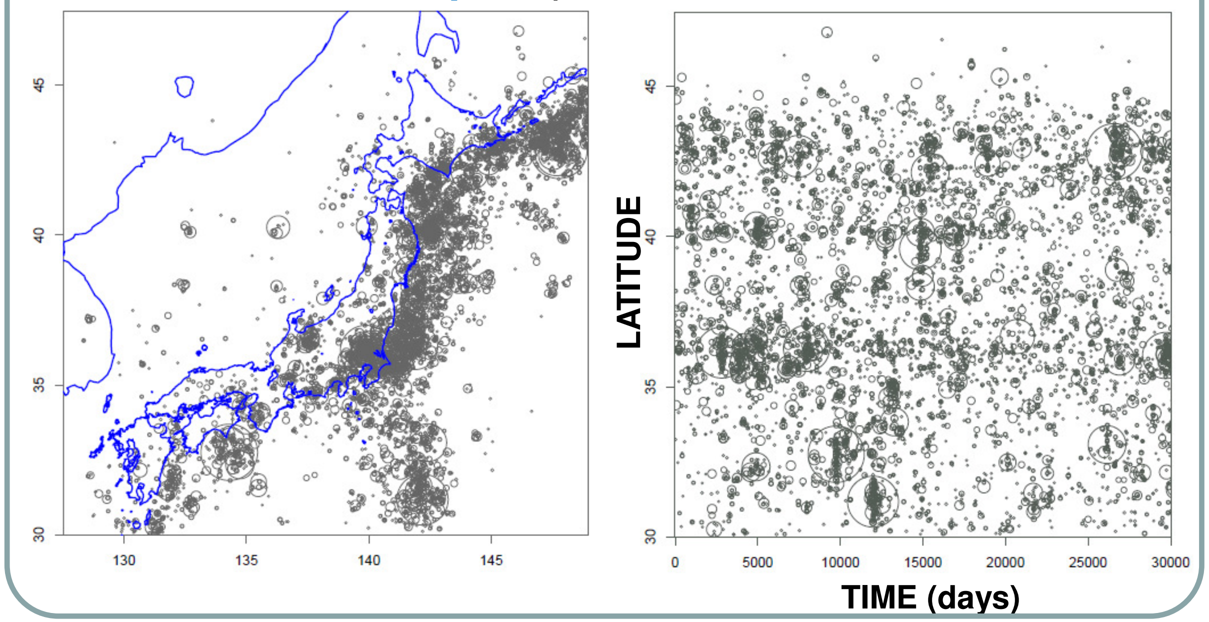


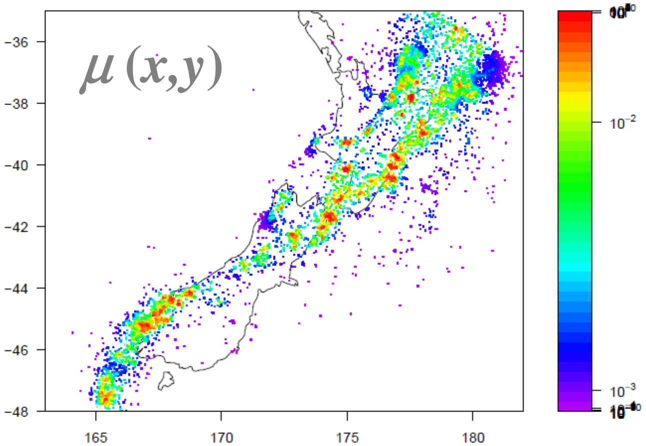


The real seismicity data

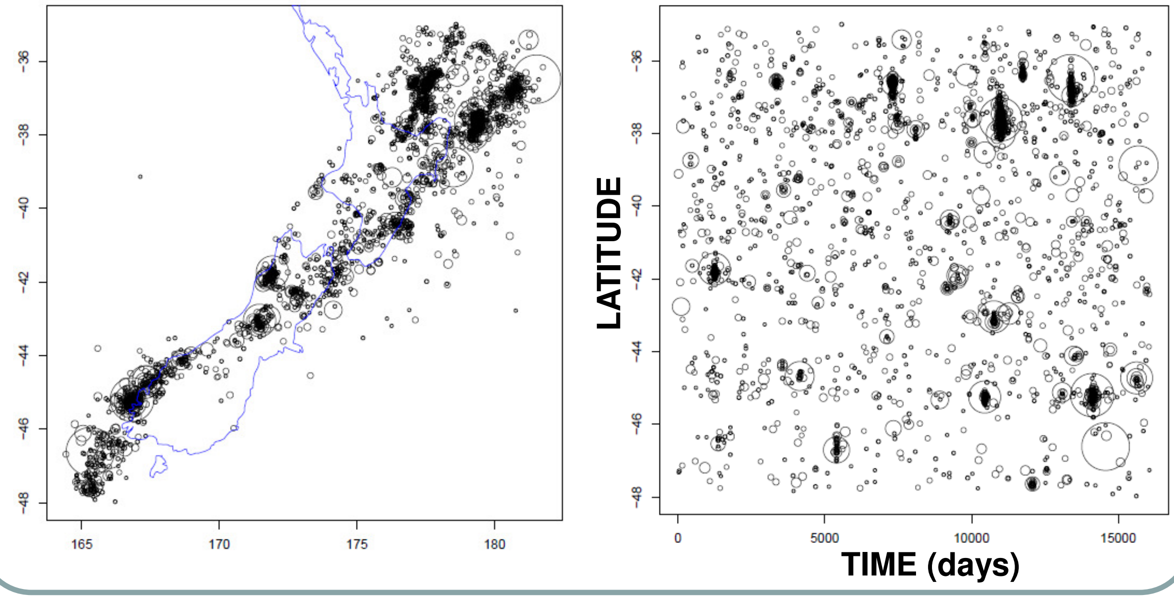


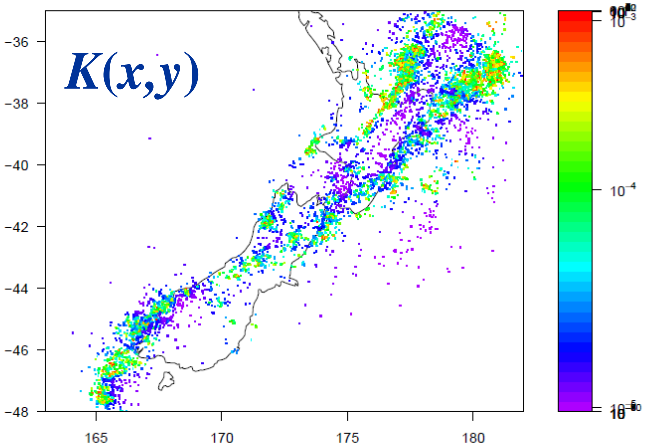
A **simulated example** by HIST-ETAS-mk Model



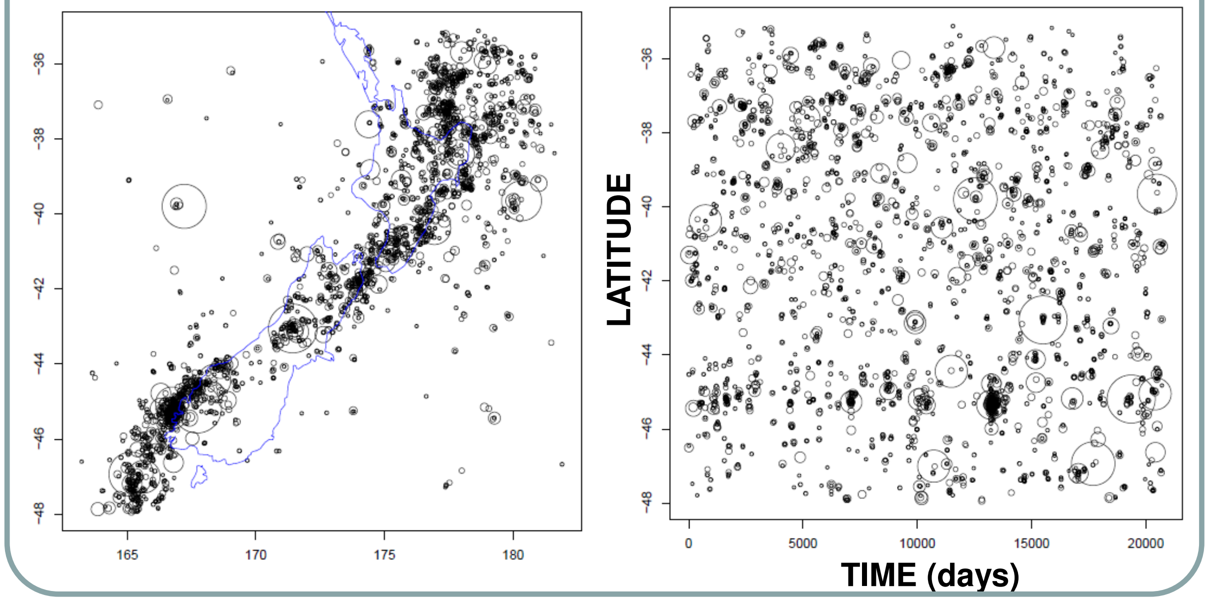


The real seismicity data

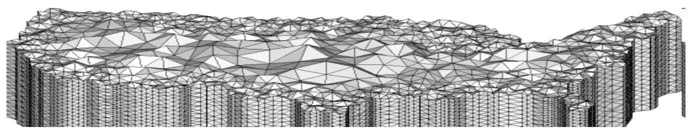




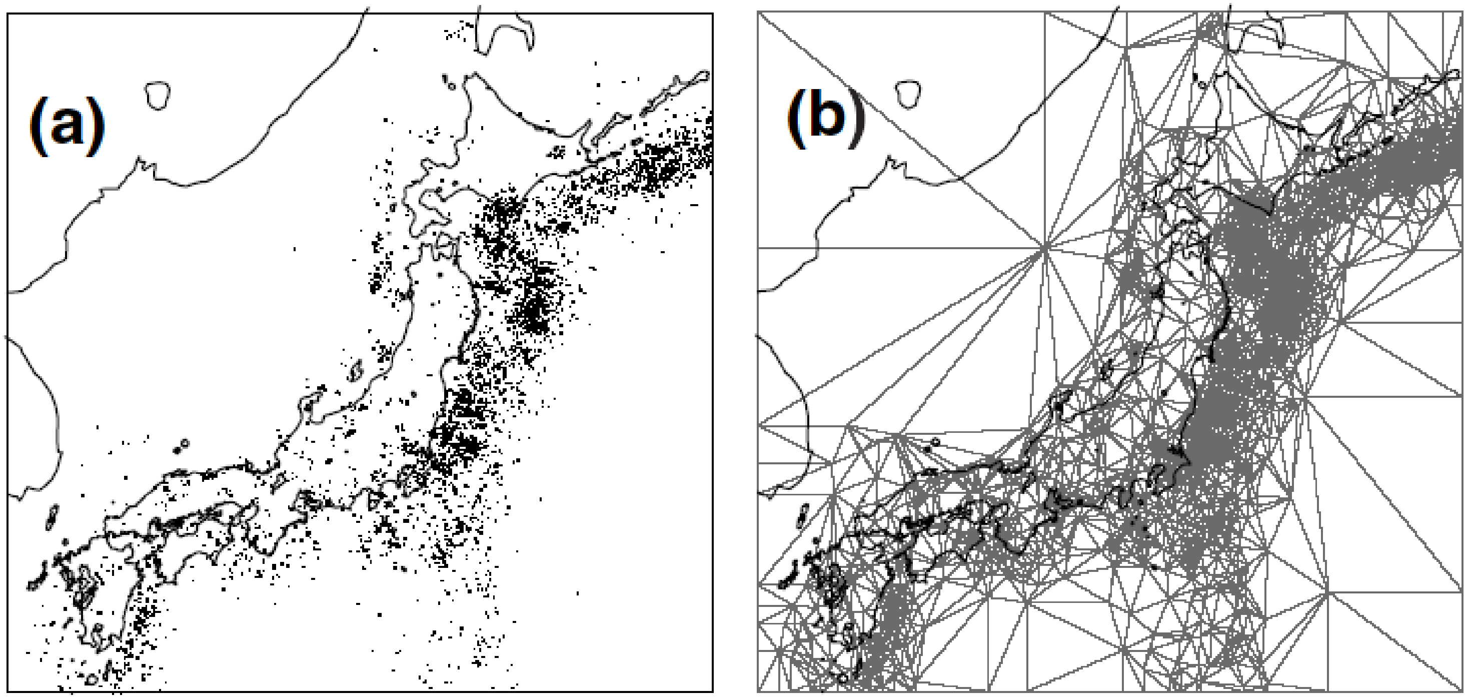
A **simulated example** by HIST-ETAS-mk Model



1926 - 1995,
M >= 5.0,
depth<100km

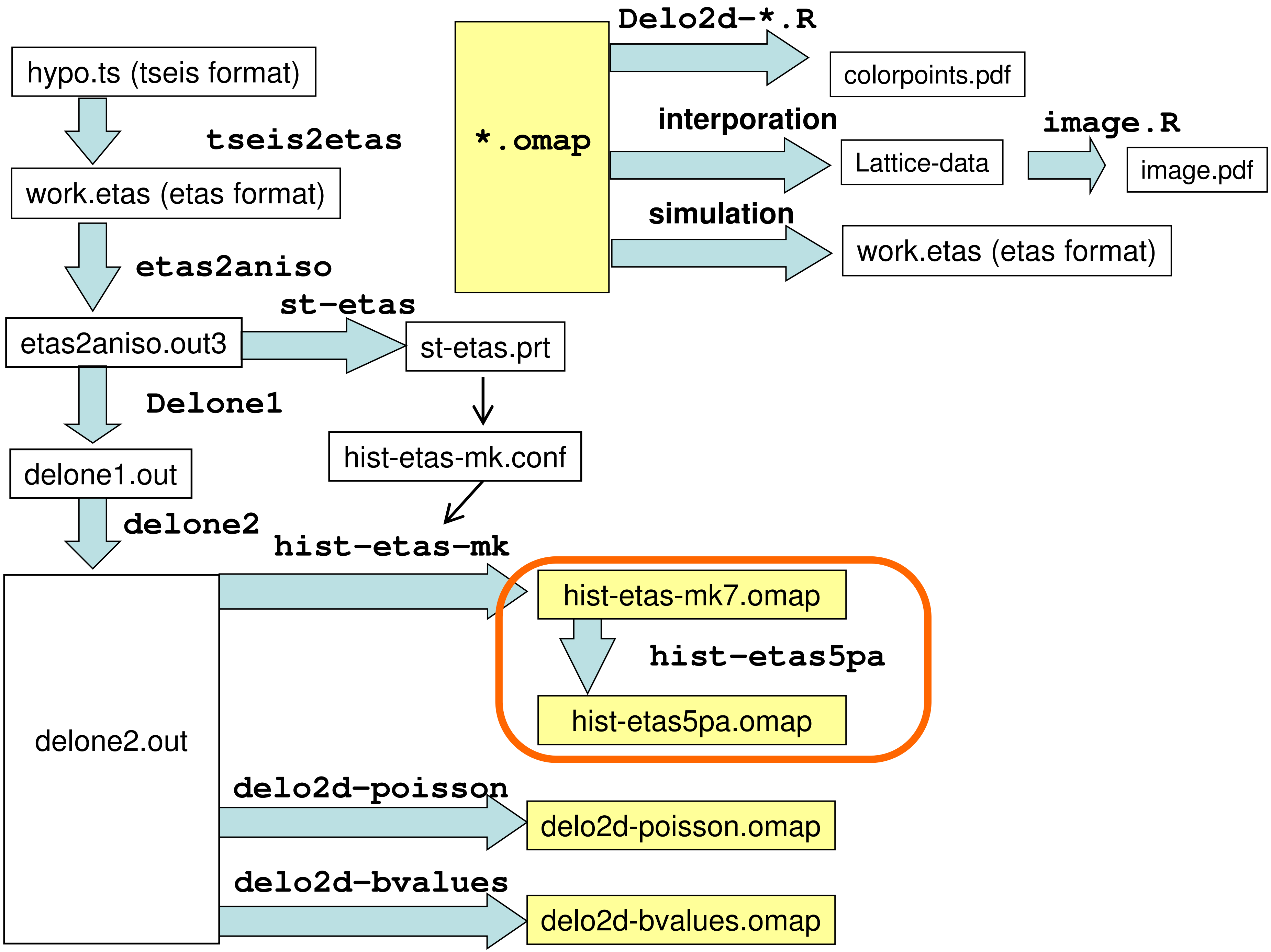


Delaunay-based function; an illustration



Space-Time ETAS model

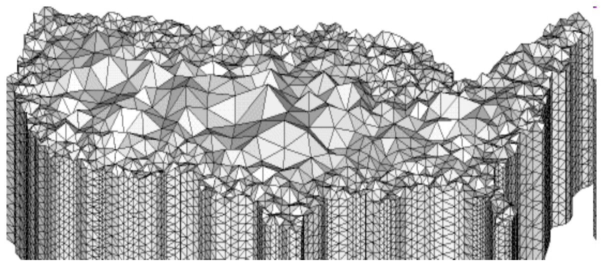
$$\lambda(t,x,y) = \mu + \sum_{\{j:\, t_j<t\}} \frac{K}{(t-t_j+c)^p} \left\{ \frac{Q_j(x,\, y)}{e^{\alpha M_j}} + d \right\}^{-q}$$


hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
Delone1
delone1.out
hist-etas-mk.conf
delone2
hist-etas-mk
delone2.out
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

Location Dependent Space-Time ETAS model and occurrence data $\{(t_i, x_i, y_i, M_i);\quad i = 1, \cdots, n\}$ in $[0, T] \times A$ are given. Then **Log Likelihood** is

$$\log L(\theta) = \log L\left(\mu_{\theta_1}, K_{\theta_2}, \alpha_{\theta_4}, p_{\theta_5}, q_{\theta_7}\right)$$
$$= \sum_{i=1}^{n} \log \lambda_\theta(t_i, x_i, y_i) - \int_0^T \iint_A \lambda_\theta(t, x, y)\,dt\,dx\,dy.$$

where $\theta = (\theta_1, \theta_2, \theta_4, \theta_5, \theta_7)$



Delaunay-based function: an illustration

Penalized Log Likelihood

$$Q(\theta|\ w_\mu, w_K, w_\alpha, w_p, w_q)$$
$$= \log L(\theta) - penalty(\theta|\ w_\mu, w_K, w_\alpha, w_p, w_q)$$

where the $penalty$ is

$$\iint_A dxdy \left\{w_1(\mu_x^2 + \mu_y^2) + w_2(K_x^2 + K_y^2) + w_3(\alpha_x^2 + \alpha_y^2) + w_4(p_x^2 + p_y^2) + w_5(q_x^2 + q_y^2)\right\}$$

Flatness constraints

$$\boldsymbol{\rho} = (w_\mu, w_K, w_\alpha, w_p, w_q)$$

$$posterior(\theta \mid \boldsymbol{\rho}) = \frac{L(\theta) \cdot prior(\theta \mid \boldsymbol{\rho})}{\Lambda(\boldsymbol{\rho})}$$

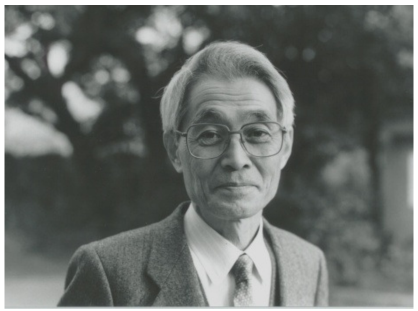
$$\Lambda(\boldsymbol{\rho}) = \int \cdots \int L(\theta) \cdot prior(\theta \mid \boldsymbol{\rho}) d\theta$$

(**Likelihood** of a Bayesian model; *Good* 1965)

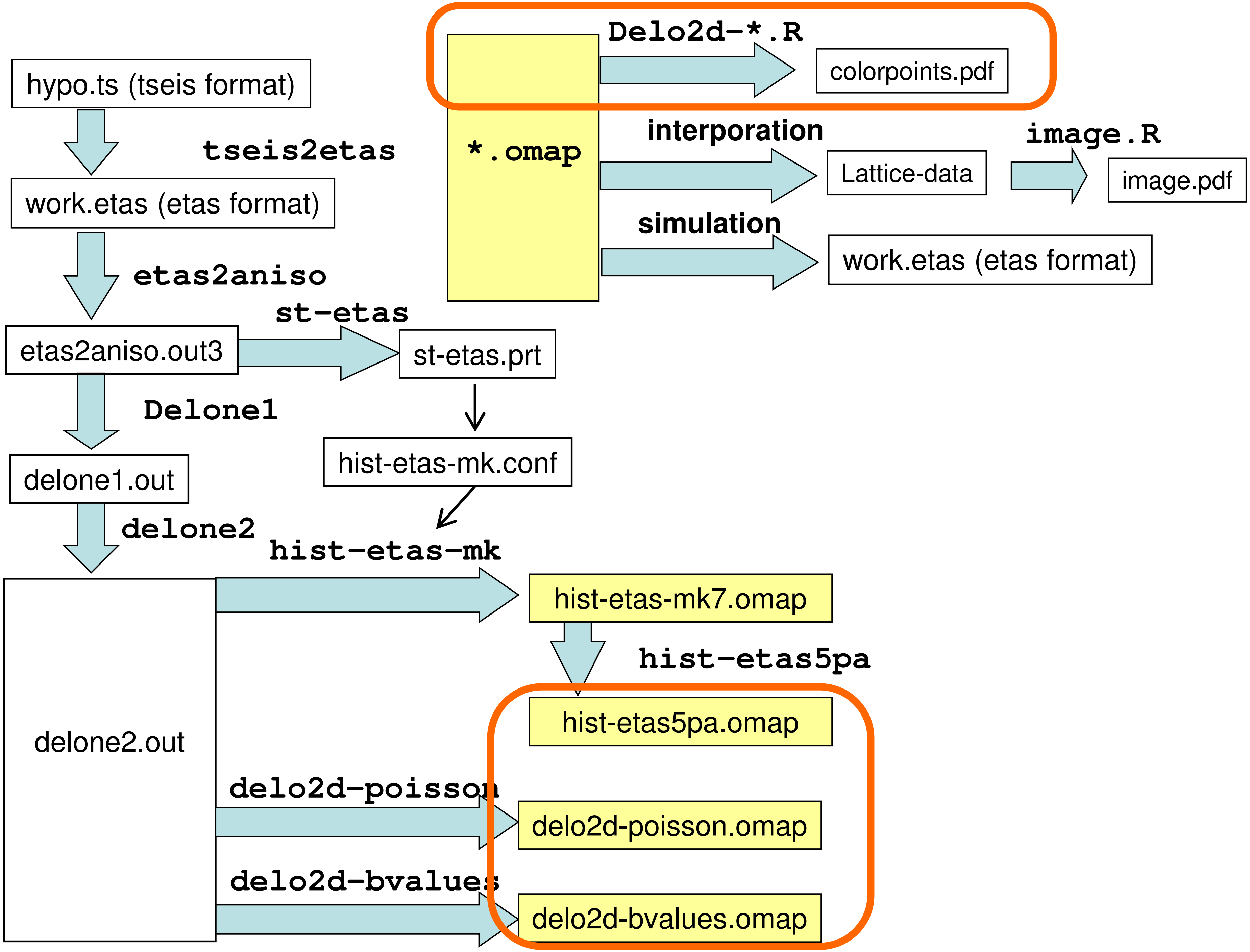
Choose $\boldsymbol{\rho}$ that maximize $\Lambda(\boldsymbol{\rho})$

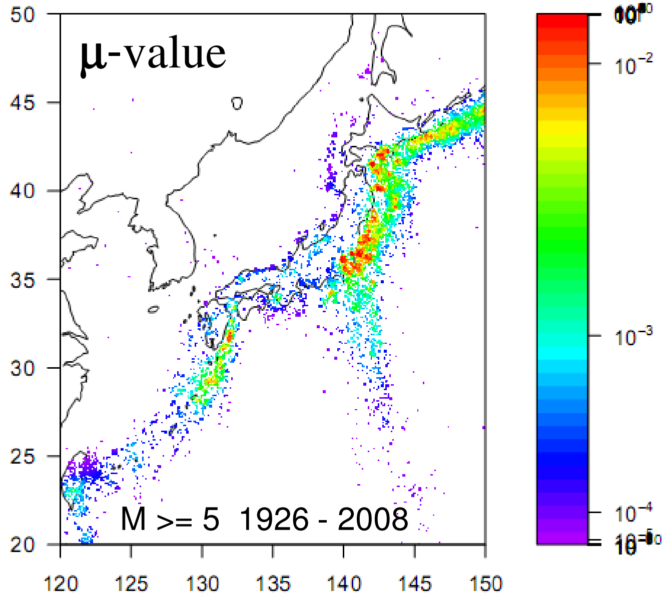
or minimize

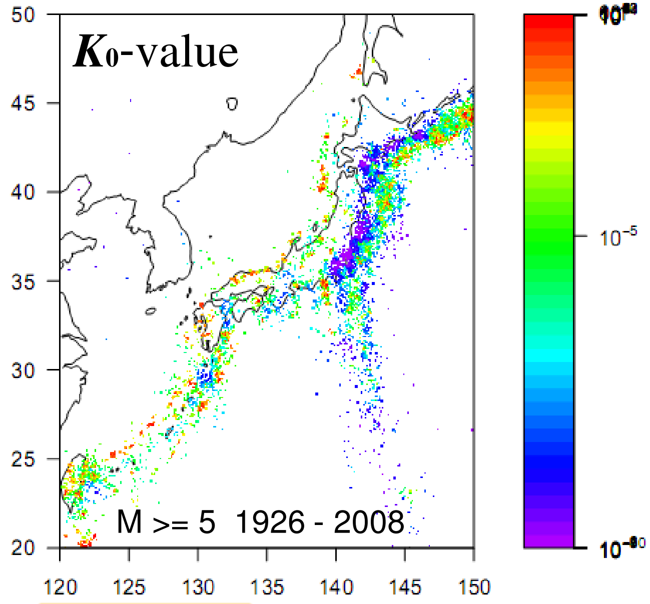
$$\boxed{ABIC = (-2)\max_{\boldsymbol{\rho}}\{\log \Lambda(\boldsymbol{\rho})\} + 2 \times \dim(\boldsymbol{\rho})}$$

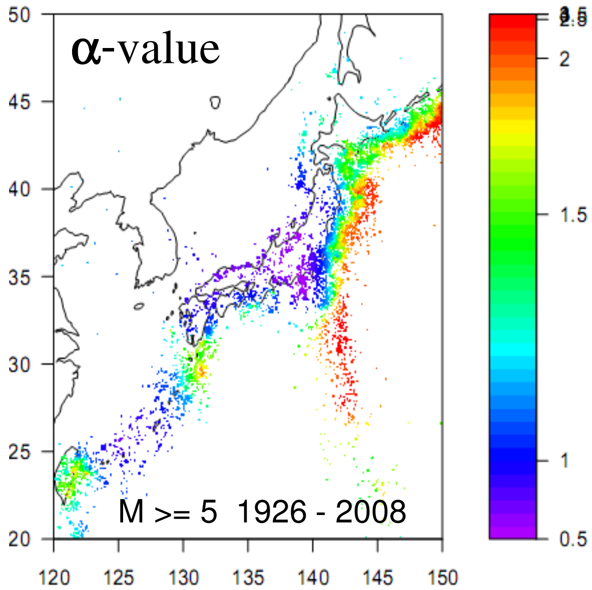


Akaike Bayesian information criterion (Akaike, 1980)

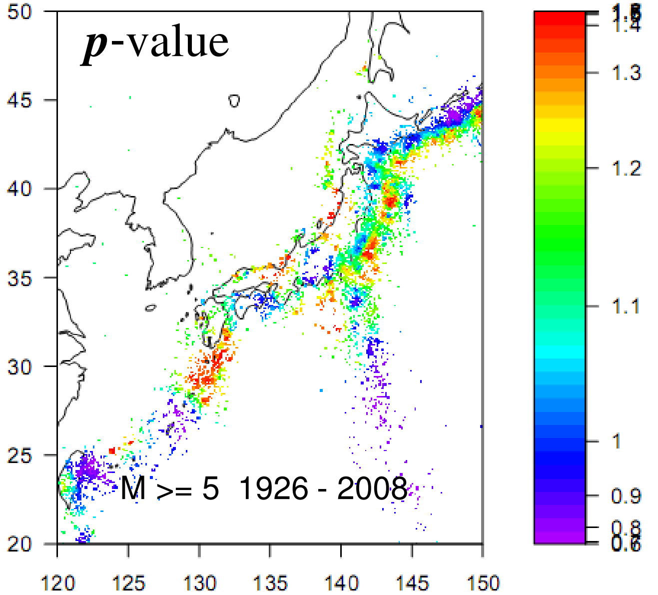

hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
hist-etas-mk.conf
Delone1
delone1.out
delone2
hist-etas-mk
delone2.out
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)

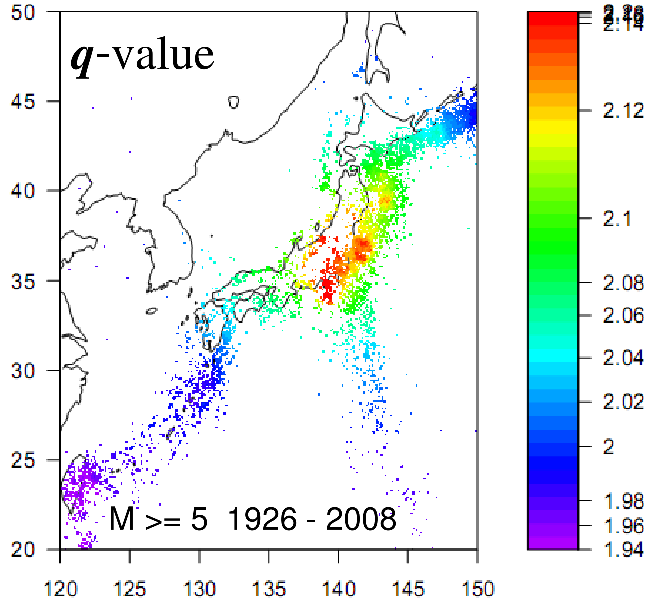


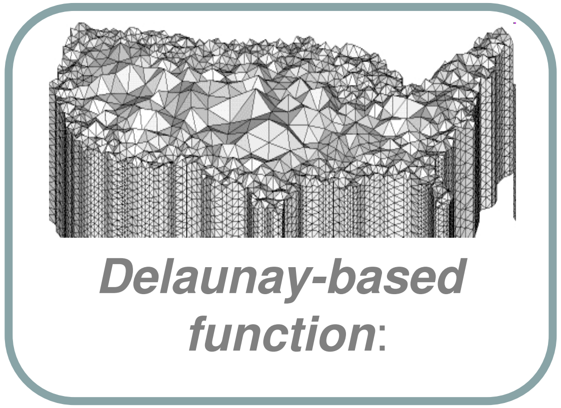




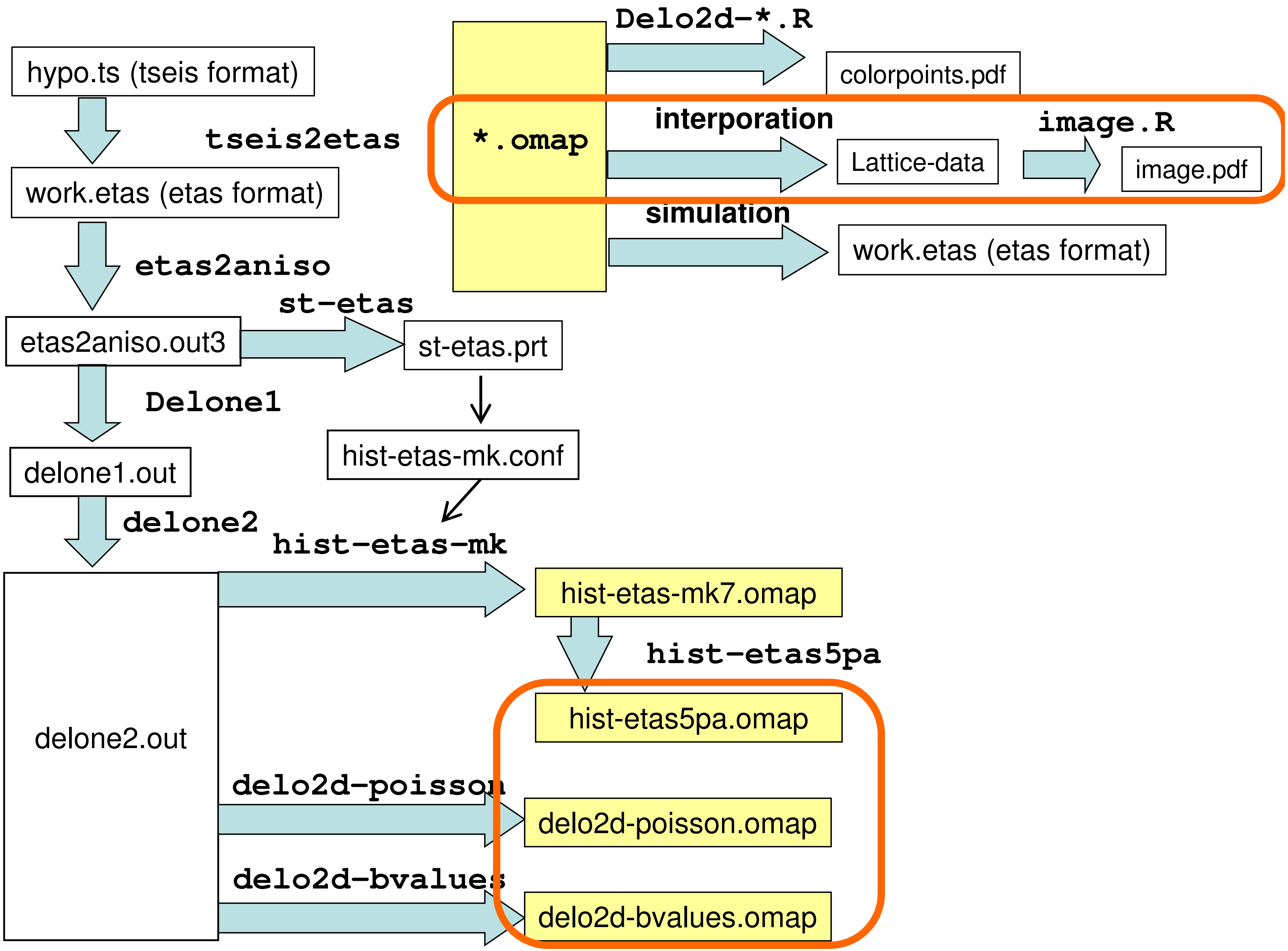
$$\lambda(t, x, y \mid H_t) = \mu(x, y) + \sum_{\{j;\, t_j < t\}} \frac{K_0(x_j, y_j)}{(t - t_j + c)^{p(x_j, y_j)}} \left[\frac{(x - x_j,\, y - y_j)\, S_j\, (x - x_j,\, y - y_j)^t}{e^{\alpha(x_j, y_j)(M_j - M_c)}} + d \right]^{-q(x_j, y_j)}$$

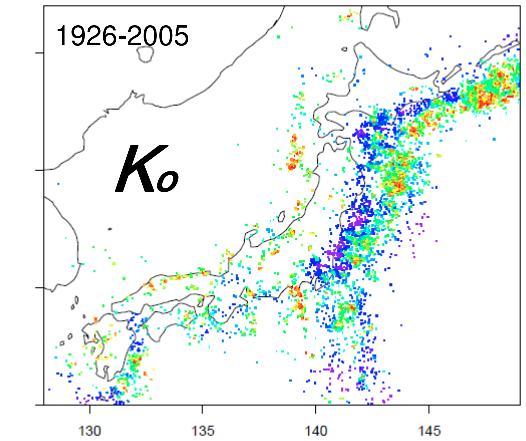


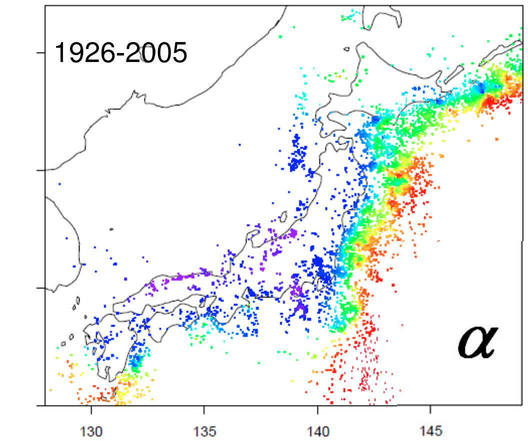


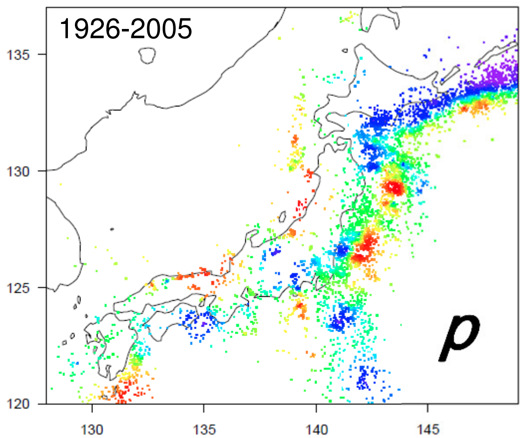


Precursory period
1885 - 1925

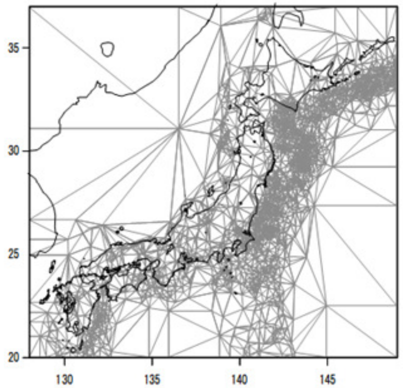

hypo.ts (tseis format)
tseis2etas
work.etas (etas format)
etas2aniso
etas2aniso.out3
st-etas
st-etas.prt
Delone1
delone1.out
hist-etas-mk.conf
delone2
hist-etas-mk
delone2.out
hist-etas-mk7.omap
hist-etas5pa
hist-etas5pa.omap
delo2d-poisson
delo2d-poisson.omap
delo2d-bvalues
delo2d-bvalues.omap
*.omap
Delo2d-*.R
colorpoints.pdf
interporation
Lattice-data
image.R
image.pdf
simulation
work.etas (etas format)



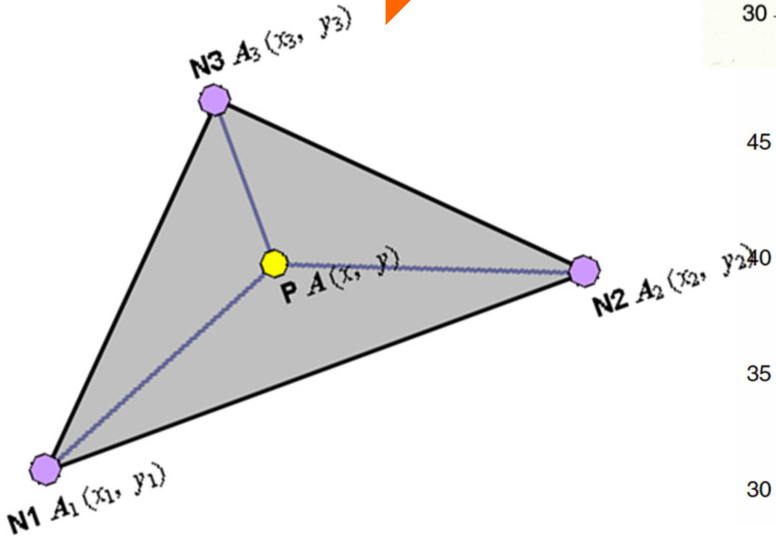


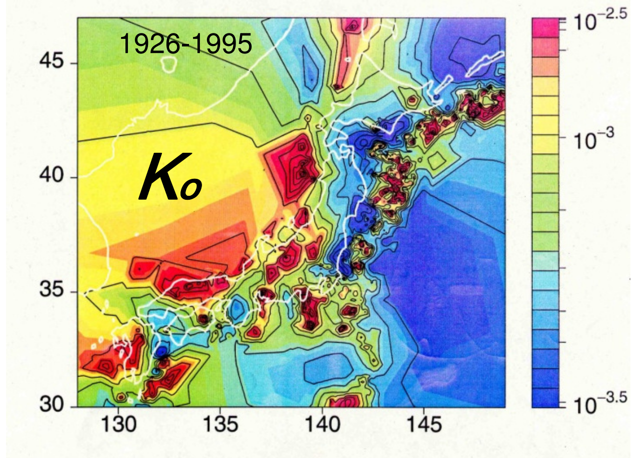


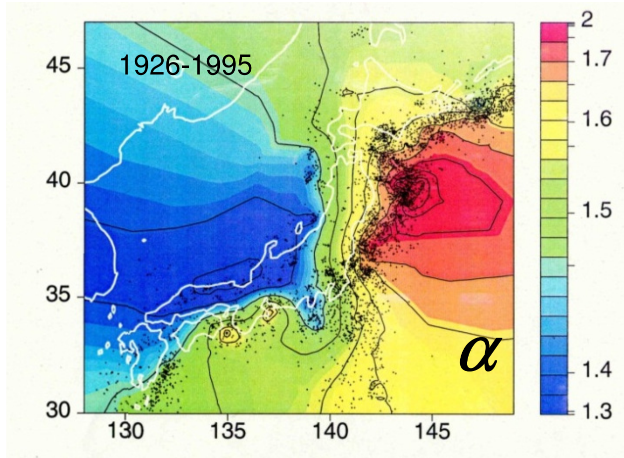
Delaunay-based function:
defined on the vertices

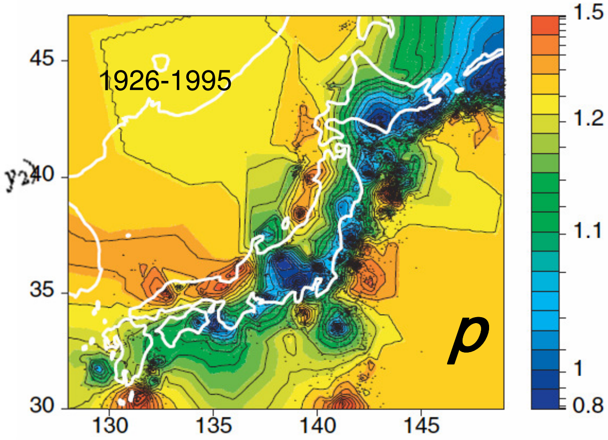


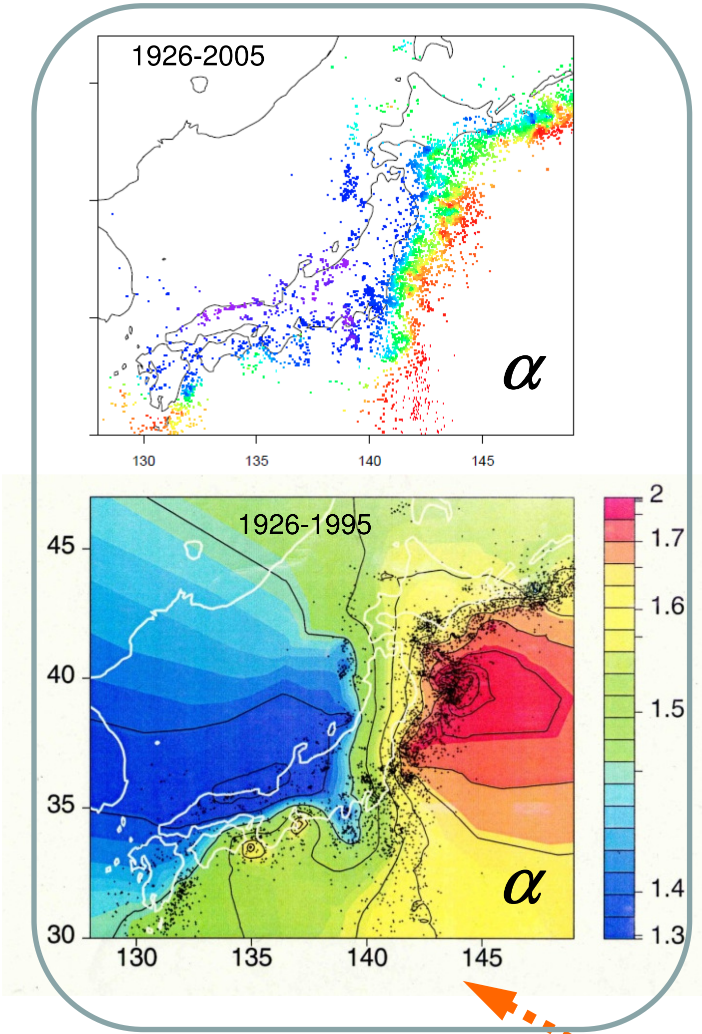
Linear interpolation
to the lattice points

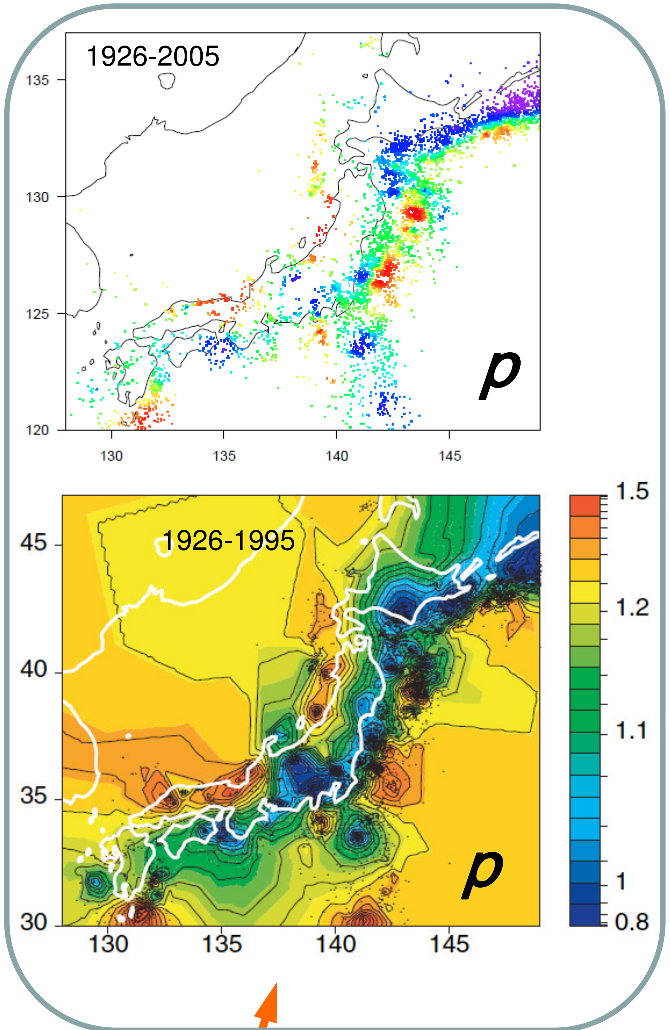


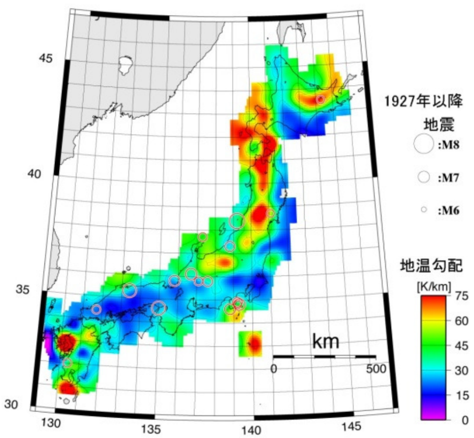




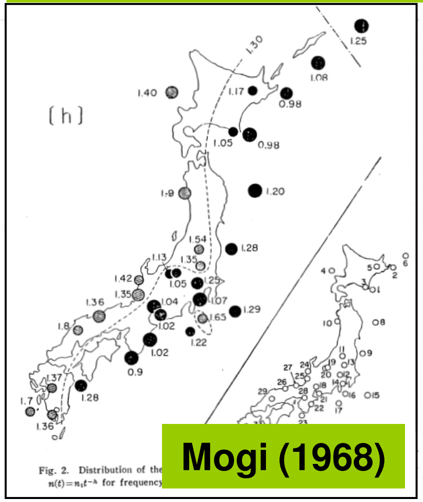


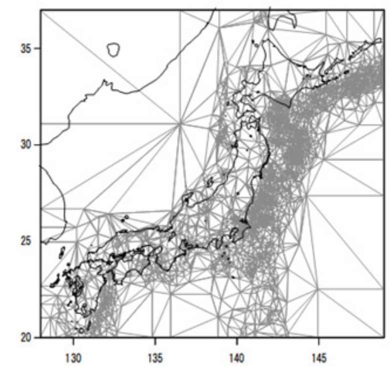






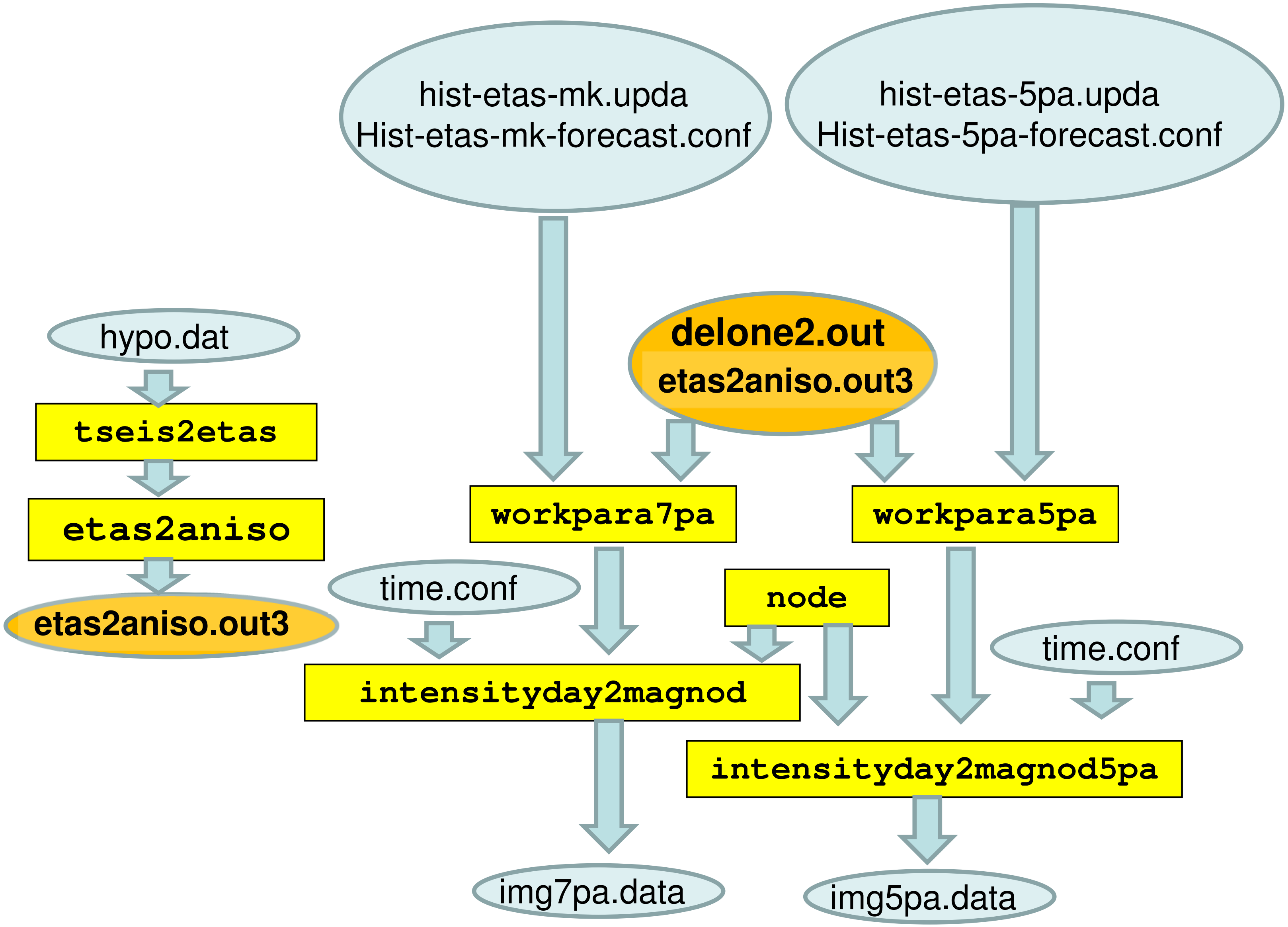
Crust temperature gradient

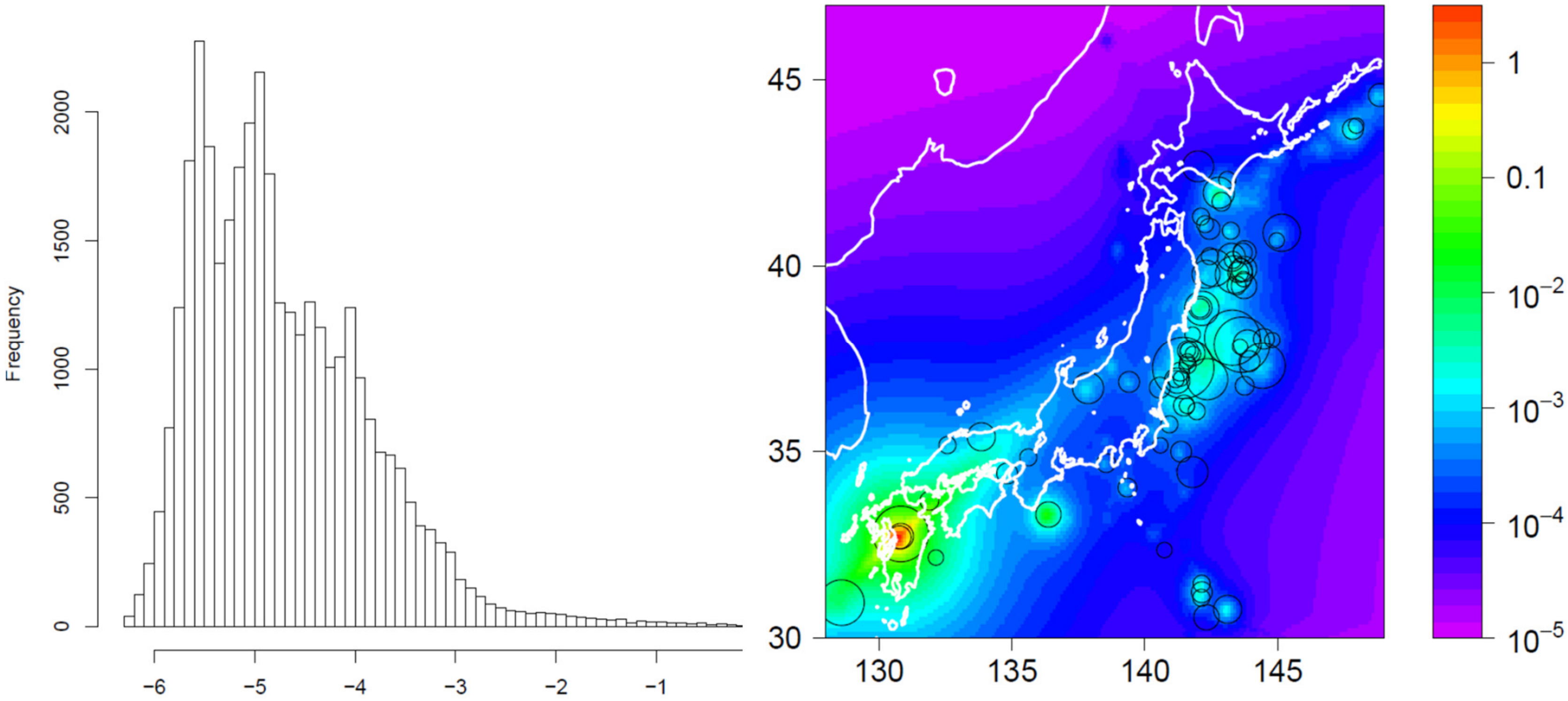




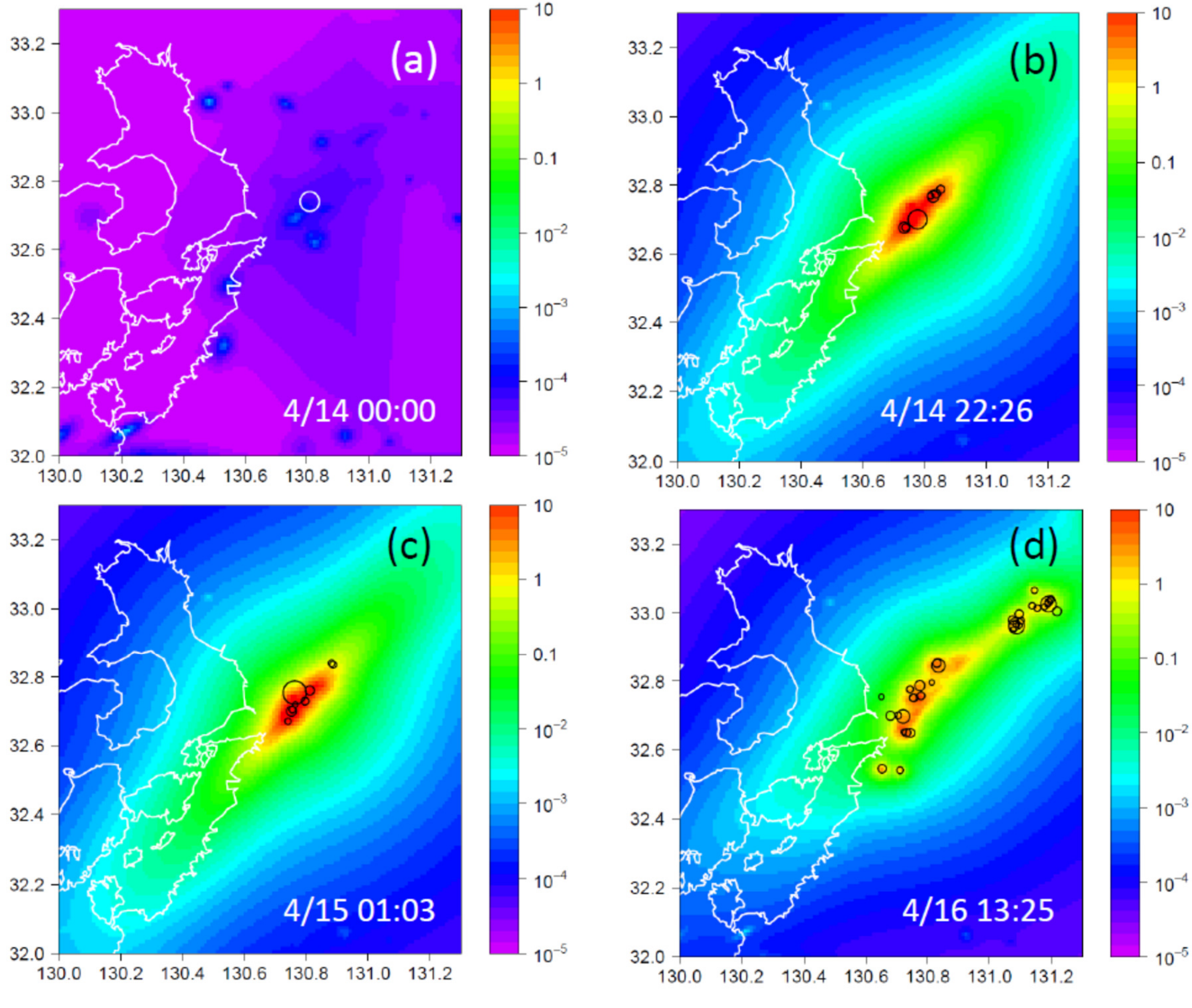
$$\lambda(t, x, y) = \mu + \sum_{\{j:\, t_j < t\}} \frac{K}{(t - t_j + c)^{p}} \left\{ \frac{Q_j(x, y)}{e^{\alpha M_j}} + d \right\}^{-q}$$

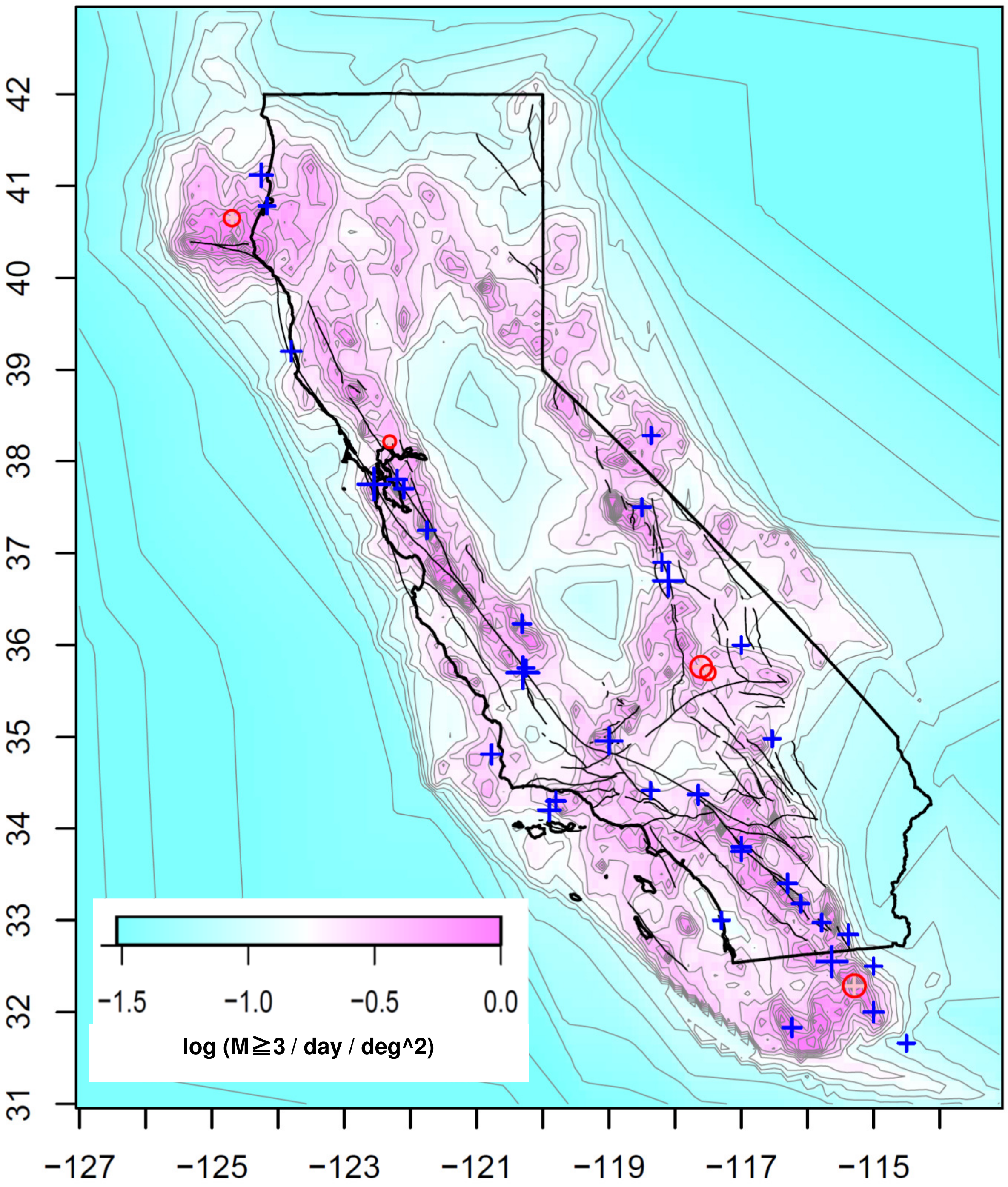
Forecasting flow chart



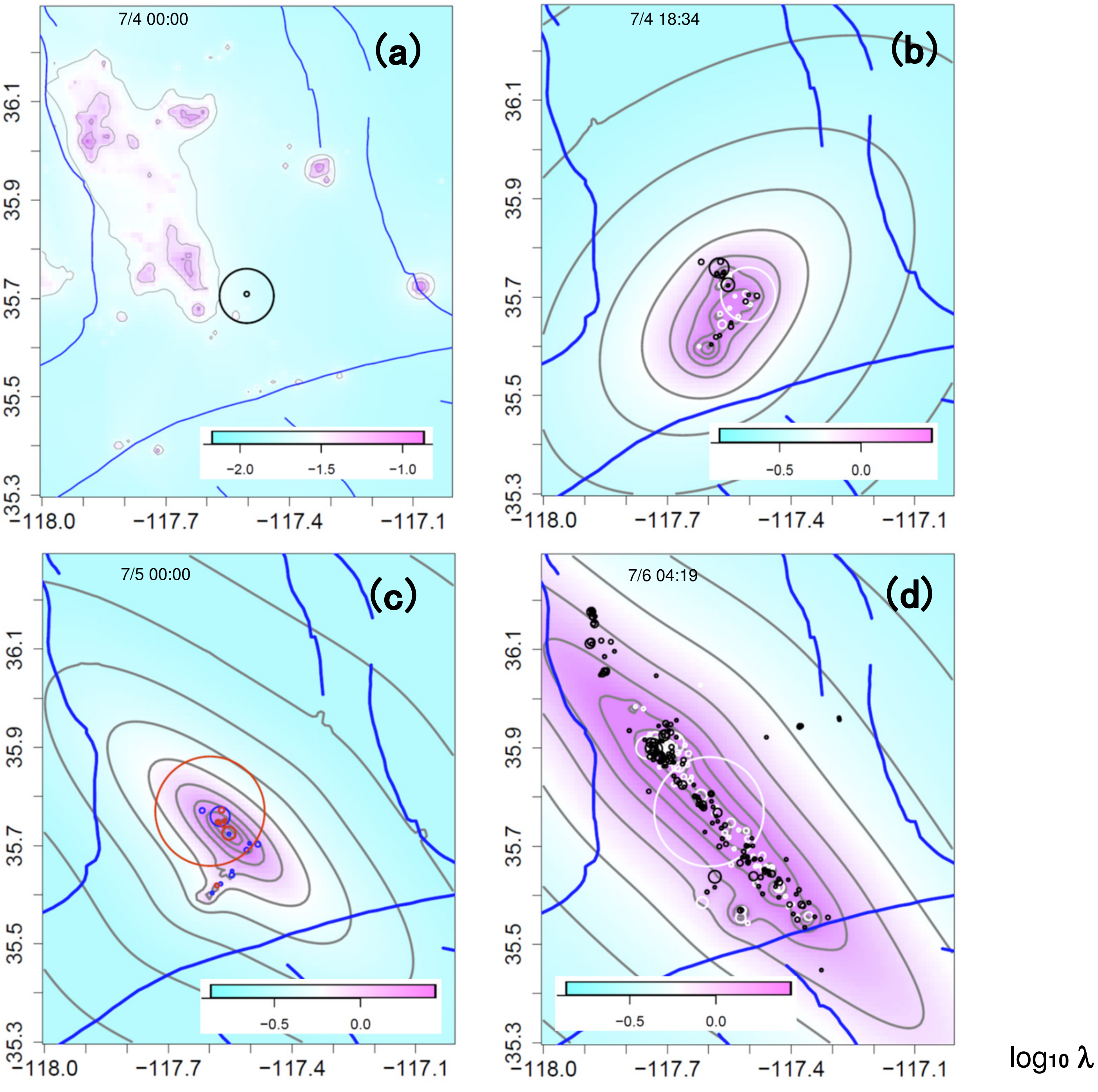


Frequency
0
500
1000
1500
2000
−6
−5
−4
−3
−2
−1
45
40
35
30
130
135
140
145
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}


(a)
4/14 00:00
(b)
4/14 22:26
(c)
4/15 01:03
(d)
4/16 13:25
33.2
33.0
32.8
32.6
32.4
32.2
32.0
130.0
130.2
130.4
130.6
130.8
131.0
131.2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}



42
41
40
39
38
37
36
35
34
33
32
31
−127
−125
−123
−121
−119
−117
−115
−1.5
−1.0
−0.5
0.0
log (M≧3 / day / deg^2)



7/4 00:00
(a)
7/4 18:34
(b)
7/5 00:00
(c)
7/6 04:19
(d)
36.1
35.9
35.7
35.5
35.3
-118.0
-117.7
-117.4
-117.1
-2.0
-1.5
-1.0
-0.5
0.0
log10 λ

For the detail, see

Ogata, Y. (2010). Significant improvements of the space-time ETAS model for forecasting of accurate baseline seismicity, *Earth, Planets and Space*, doi:10.5047eps.2010.09.001.

Also, softwares already available for the time domain ETAS fitting, diagnostic analysis and its manual. Please visit

WEB home page of our project team:

http:www.ism.ac.jp~ogataSsgssgE.html