# Supplemental material for "Large-sample properties of multiple imputation estimators for parameters of logistic regression with covariates missing at random separately or simultaneously"

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**Abstract** This web appendix consists of Appendixes A-C. Appendix A provides two tables and one figure from the simulation study in Section 4.1. A brief description of the actual data and the R code used to process the data in Section 4.2 are presented in Appendixes B and C, respectively.

## Appendix

### Appendix A. Tables and Figure

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Table S1 Simulation results of Study 3 ($n = 1,000$); $\boldsymbol{\beta} = (1.2, 1, 1, 1)^\intercal$; $\boldsymbol{\alpha} = (1.4, 0.6, 0.6)^\intercal$; $\boldsymbol{\gamma} = (0.7, -0.2, 0.1, -1.2)^\intercal$; observed selection probabilities: $(0.45, 0.20, 0.20, 0.15)$; $M = (10, 20, 30)$

| | | | | | | Estimation Method | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $M = 10$ | | | | | $M = 20$ | | | | | $M = 30$ | | | | |
| Parameter | | FML | CC | SIPW | SAEM | MI1 | MI2 | MI1n | MI2n | RFMI | MI1 | MI2 | MI1n | MI2n | RFMI | MI1 | MI2 | MI1n | MI2n | RFMI |
| $\beta_0$ | Bias | 0.0109 | 0.0827 | 0.0189 | 0.0134 | 0.0145 | 0.0130 | 0.0145 | 0.0130 | -0.0466 | 0.0143 | 0.0126 | 0.0143 | 0.0126 | -0.0465 | 0.0142 | 0.0125 | 0.0142 | 0.0125 | -0.0466 |
| | SD | 0.1521 | 0.2257 | 0.1661 | 0.1712 | 0.1585 | 0.1567 | 0.1585 | 0.1567 | 0.1398 | 0.1583 | 0.1565 | 0.1583 | 0.1565 | 0.1394 | 0.1584 | 0.1565 | 0.1584 | 0.1565 | 0.1384 |
| | ASE | 0.1479 | 0.2223 | 0.1544 | 0.1673 | 0.1494 | 0.1495 | 0.1484 | 0.1511 | 0.1586 | 0.1493 | 0.1494 | 0.1484 | 0.1511 | 0.1585 | 0.1492 | 0.1494 | 0.1483 | 0.1510 | 0.1581 |
| | MSE | 0.0233 | 0.0578 | 0.0279 | 0.0295 | 0.0253 | 0.0247 | 0.0253 | 0.0247 | 0.0217 | 0.0253 | 0.0246 | 0.0253 | 0.0246 | 0.0216 | 0.0253 | 0.0246 | 0.0253 | 0.0246 | 0.0213 |
| | CP | 0.9490 | 0.9430 | 0.9350 | 0.9530 | 0.9400 | 0.9380 | 0.9360 | 0.9410 | 0.9580 | 0.9400 | 0.9370 | 0.9360 | 0.9400 | 0.9590 | 0.9390 | 0.9400 | 0.9350 | 0.9410 | 0.9590 |
| $\beta_1$ | Bias | 0.0016 | 0.0310 | 0.0112 | 0.0263 | 0.0056 | 0.0059 | 0.0056 | 0.0059 | -0.2149 | 0.0058 | 0.0058 | 0.0058 | 0.0058 | -0.2103 | 0.0057 | 0.0054 | 0.0057 | 0.0054 | -0.2118 |
| | SD | 0.2131 | 0.3394 | 0.2331 | 0.2753 | 0.2232 | 0.2211 | 0.2232 | 0.2211 | 0.2132 | 0.2232 | 0.2213 | 0.2232 | 0.2213 | 0.2122 | 0.2232 | 0.2212 | 0.2232 | 0.2212 | 0.2100 |
| | ASE | 0.2143 | 0.3291 | 0.2280 | 0.2728 | 0.2174 | 0.2178 | 0.2192 | 0.2220 | 0.2588 | 0.2173 | 0.2177 | 0.2192 | 0.2220 | 0.2569 | 0.2172 | 0.2176 | 0.2192 | 0.2219 | 0.2566 |
| | MSE | 0.0454 | 0.1161 | 0.0544 | 0.0765 | 0.0499 | 0.0489 | 0.0499 | 0.0489 | 0.0916 | 0.0498 | 0.0490 | 0.0498 | 0.0490 | 0.0893 | 0.0498 | 0.0490 | 0.0498 | 0.0490 | 0.0889 |
| | CP | 0.9530 | 0.9500 | 0.9510 | 0.9480 | 0.9460 | 0.9460 | 0.9470 | 0.9510 | 0.8960 | 0.9470 | 0.9490 | 0.9470 | 0.9530 | 0.8950 | 0.9440 | 0.9480 | 0.9470 | 0.9510 | 0.9030 |
| $\beta_2$ | Bias | 0.0090 | 0.0206 | 0.0281 | 0.0170 | 0.0176 | 0.0149 | 0.0176 | 0.0149 | -0.2615 | 0.0172 | 0.0141 | 0.0172 | 0.0141 | -0.2587 | 0.0170 | 0.0138 | 0.0170 | 0.0138 | -0.2594 |
| | SD | 0.2053 | 0.3019 | 0.2406 | 0.2591 | 0.2241 | 0.2211 | 0.2241 | 0.2211 | 0.1759 | 0.2236 | 0.2208 | 0.2236 | 0.2208 | 0.1782 | 0.2236 | 0.2209 | 0.2236 | 0.2209 | 0.1747 |
| | ASE | 0.1964 | 0.3015 | 0.2125 | 0.2486 | 0.1996 | 0.2003 | 0.1982 | 0.2048 | 0.2316 | 0.1993 | 0.2001 | 0.1981 | 0.2047 | 0.2316 | 0.1993 | 0.2001 | 0.1981 | 0.2046 | 0.2307 |
| | MSE | 0.0422 | 0.0916 | 0.0587 | 0.0674 | 0.0505 | 0.0491 | 0.0505 | 0.0491 | 0.0993 | 0.0503 | 0.0489 | 0.0503 | 0.0489 | 0.0987 | 0.0503 | 0.0490 | 0.0503 | 0.0490 | 0.0978 |
| | CP | 0.9460 | 0.9530 | 0.9200 | 0.9480 | 0.9200 | 0.9240 | 0.9170 | 0.9290 | 0.8180 | 0.9210 | 0.9260 | 0.9190 | 0.9310 | 0.8180 | 0.9200 | 0.9250 | 0.9210 | 0.9300 | 0.8280 |
| $\beta_3$ | Bias | 0.0112 | 0.1394 | 0.0287 | 0.0229 | 0.0126 | 0.0125 | 0.0126 | 0.0125 | -0.0130 | 0.0127 | 0.0125 | 0.0127 | 0.0125 | -0.0127 | 0.0127 | 0.0125 | 0.0127 | 0.0125 | -0.0130 |
| | SD | 0.1952 | 0.2982 | 0.2052 | 0.1988 | 0.1959 | 0.1960 | 0.1959 | 0.1960 | 0.1923 | 0.1959 | 0.1960 | 0.1959 | 0.1960 | 0.1921 | 0.1960 | 0.1960 | 0.1960 | 0.1960 | 0.1923 |
| | ASE | 0.1824 | 0.2930 | 0.1837 | 0.1862 | 0.1825 | 0.1825 | 0.1804 | 0.1822 | 0.1819 | 0.1825 | 0.1825 | 0.1804 | 0.1822 | 0.1819 | 0.1825 | 0.1825 | 0.1804 | 0.1822 | 0.1819 |
| | MSE | 0.0382 | 0.1083 | 0.0429 | 0.0401 | 0.0386 | 0.0386 | 0.0386 | 0.0386 | 0.0371 | 0.0385 | 0.0386 | 0.0385 | 0.0386 | 0.0371 | 0.0386 | 0.0386 | 0.0386 | 0.0386 | 0.0372 |
| | CP | 0.9310 | 0.9350 | 0.9170 | 0.9390 | 0.9310 | 0.9310 | 0.9280 | 0.9310 | 0.9340 | 0.9310 | 0.9320 | 0.9300 | 0.9310 | 0.9310 | 0.9310 | 0.9320 | 0.9300 | 0.9300 | 0.9310 |

FML: full-data maximum likelihood estimation method
CC: complete-case estimation method
SIPW: semiparametric inverse probability weighting estimation method
MI1 (MI1 estimation method): MI1 estimator whose value is the solution of $\boldsymbol{U}_{M1}(\boldsymbol{\beta}) = \mathbf{0}$ in Equation (5) with Rubin's estimated variance in Equation (12)
MI2 (MI2 estimation method): MI2 estimator whose value is the solution of $\boldsymbol{U}_{M2}(\boldsymbol{\beta}) = \mathbf{0}$ in Equation (6) with Rubin's estimated variance in Equation (13)
MI1n (MI1n estimation method): MI1 estimator whose value is the solution of $\boldsymbol{U}_{M1}(\boldsymbol{\beta}) = \mathbf{0}$ in Equation (5) with proposed estimated variance in Equation (10)
MI2n (MI2 estimation method): MI2 estimator whose value is the solution of $\boldsymbol{U}_{M2}(\boldsymbol{\beta}) = \mathbf{0}$ in Equation (6) with proposed estimated variance in Equation (11)
RFMI: random forest multiple imputation method
SAEM: stochastic approximation of expectation-maximization method

Table S2 Simulation results of Study 4 ($M = 15$; $n = 1,500$); $\boldsymbol{\beta} = (-2.7, 1.0, 0.6, 0.5)^{\mathsf{T}}$; $\boldsymbol{\gamma} = (0.7, -0.2, 0.1, -1.2)^{\mathsf{T}}$; three sets of $\boldsymbol{\alpha}$ are $(2.6, 0.6, 0.6)^{\mathsf{T}}$, $(1.6, 0.6, 0.6)^{\mathsf{T}}$, and $(1.0, 0.6, 0.6)^{\mathsf{T}}$ to create observed selection probabilities: $(0.66, 0.11, 0.11, 0.12)$, $(0.47, 0.17, 0.17, 0.19)$, and $(0.33, 0.22, 0.22, 0.23)$, respectively

| | | Estimation Method | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parameter | | FML | CC | SIPW | MI1 | MI2 | MI1n | MI2n | RFMI | SAEM |
| Observed selection probabilities: $(0.66, 0.11, 0.11, 0.12)$; $\boldsymbol{\alpha} = (2.6, 0.6, 0.6)^{\mathsf{T}}$ | | | | | | | | | | |
| $\beta_0$ | Bias | -0.0078 | 0.0238 | -0.0122 | -0.0114 | -0.0077 | -0.0114 | -0.0077 | 0.1172 | -0.0142 |
| | SD | 0.1712 | 0.2027 | 0.1926 | 0.1843 | 0.1822 | 0.1843 | 0.1822 | 0.1626 | 0.1855 |
| | ASE | 0.1660 | 0.1989 | 0.1857 | 0.1751 | 0.1751 | 0.1777 | 0.1814 | 0.1725 | 0.1782 |
| | MSE | 0.0294 | 0.0416 | 0.0372 | 0.0341 | 0.0332 | 0.0341 | 0.0332 | 0.0402 | 0.0346 |
| | CP | 0.9480 | 0.9400 | 0.9400 | 0.9350 | 0.9360 | 0.9370 | 0.9440 | 0.8940 | 0.9320 |
| $\beta_1$ | Bias | -0.0029 | 0.0058 | 0.0022 | 0.0009 | -0.0022 | 0.0009 | -0.0022 | -0.1300 | 0.0084 |
| | SD | 0.1453 | 0.1749 | 0.1741 | 0.1642 | 0.1637 | 0.1642 | 0.1637 | 0.1430 | 0.1667 |
| | ASE | 0.1448 | 0.1756 | 0.1731 | 0.1578 | 0.1581 | 0.1615 | 0.1678 | 0.1612 | 0.1629 |
| | MSE | 0.0211 | 0.0306 | 0.0303 | 0.0270 | 0.0268 | 0.0270 | 0.0268 | 0.0373 | 0.0279 |
| | CP | 0.9480 | 0.9500 | 0.9470 | 0.9380 | 0.9370 | 0.9430 | 0.9530 | 0.8960 | 0.9440 |
| $\beta_2$ | Bias | 0.0045 | 0.0012 | 0.0041 | 0.0051 | 0.0015 | 0.0051 | 0.0015 | -0.0816 | 0.0046 |
| | SD | 0.1465 | 0.1803 | 0.1805 | 0.1672 | 0.1652 | 0.1672 | 0.1652 | 0.1418 | 0.1662 |
| | ASE | 0.1464 | 0.1776 | 0.1758 | 0.1601 | 0.1602 | 0.1640 | 0.1704 | 0.1618 | 0.1645 |
| | MSE | 0.0215 | 0.0325 | 0.0326 | 0.0280 | 0.0273 | 0.0280 | 0.0273 | 0.0268 | 0.0276 |
| | CP | 0.9430 | 0.9510 | 0.9400 | 0.9370 | 0.9340 | 0.9470 | 0.9520 | 0.9390 | 0.9390 |
| $\beta_3$ | Bias | -0.0011 | 0.0688 | -0.0001 | -0.0001 | -0.0006 | -0.0001 | -0.0006 | -0.0067 | 0.0017 |
| | SD | 0.1461 | 0.1784 | 0.1487 | 0.1485 | 0.1481 | 0.1485 | 0.1481 | 0.1454 | 0.1487 |
| | ASE | 0.1455 | 0.1754 | 0.1474 | 0.1464 | 0.1464 | 0.1467 | 0.1470 | 0.1454 | 0.1464 |
| | MSE | 0.0213 | 0.0366 | 0.0221 | 0.0221 | 0.0219 | 0.0221 | 0.0219 | 0.0212 | 0.0221 |
| | CP | 0.9490 | 0.9420 | 0.9460 | 0.9440 | 0.9470 | 0.9440 | 0.9480 | 0.9500 | 0.9460 |
| Observed selection probabilities: $(0.47, 0.17, 0.17, 0.19)$; $\boldsymbol{\alpha} = (1.6, 0.6, 0.6)^{\mathsf{T}}$ | | | | | | | | | | |
| $\beta_0$ | Bias | -0.0078 | 0.0423 | -0.0168 | -0.0159 | -0.0088 | -0.0159 | -0.0088 | 0.1829 | -0.0226 |
| | SD | 0.1712 | 0.2408 | 0.2167 | 0.1972 | 0.1908 | 0.1972 | 0.1908 | 0.1571 | 0.1945 |
| | ASE | 0.1660 | 0.2332 | 0.2070 | 0.1799 | 0.1799 | 0.1887 | 0.1925 | 0.1762 | 0.1885 |
| | MSE | 0.0294 | 0.0598 | 0.0472 | 0.0391 | 0.0365 | 0.0391 | 0.0365 | 0.0581 | 0.0383 |
| | CP | 0.9480 | 0.9300 | 0.9420 | 0.9310 | 0.9360 | 0.9430 | 0.9500 | 0.8390 | 0.9440 |
| $\beta_1$ | Bias | -0.0029 | 0.0115 | 0.0085 | 0.0082 | 0.0025 | 0.0082 | 0.0025 | -0.1983 | 0.0193 |
| | SD | 0.1453 | 0.2111 | 0.2076 | 0.1830 | 0.1769 | 0.1830 | 0.1769 | 0.1411 | 0.1807 |
| | ASE | 0.1448 | 0.2072 | 0.2022 | 0.1645 | 0.1646 | 0.1769 | 0.1839 | 0.1721 | 0.1773 |
| | MSE | 0.0211 | 0.0447 | 0.0432 | 0.0335 | 0.0313 | 0.0335 | 0.0313 | 0.0592 | 0.0330 |
| | CP | 0.9480 | 0.9560 | 0.9530 | 0.9250 | 0.9380 | 0.9500 | 0.9660 | 0.8230 | 0.9490 |
| $\beta_2$ | Bias | 0.0045 | 0.0009 | 0.0004 | 0.0032 | -0.0034 | 0.0032 | -0.0034 | -0.1290 | 0.0068 |
| | SD | 0.1465 | 0.2133 | 0.2149 | 0.1886 | 0.1807 | 0.1886 | 0.1807 | 0.1414 | 0.1824 |
| | ASE | 0.1464 | 0.2095 | 0.2059 | 0.1669 | 0.1675 | 0.1802 | 0.1870 | 0.1701 | 0.1791 |
| | MSE | 0.0215 | 0.0455 | 0.0462 | 0.0356 | 0.0327 | 0.0356 | 0.0327 | 0.0366 | 0.0333 |
| | CP | 0.9430 | 0.9450 | 0.9350 | 0.9220 | 0.9340 | 0.9380 | 0.9530 | 0.9150 | 0.9430 |
| $\beta_3$ | Bias | -0.0011 | 0.1008 | 0.0023 | 0.0009 | 0.0004 | 0.0009 | 0.0004 | -0.0103 | 0.0029 |
| | SD | 0.1461 | 0.2159 | 0.1516 | 0.1498 | 0.1493 | 0.1498 | 0.1493 | 0.1446 | 0.1495 |
| | ASE | 0.1455 | 0.2065 | 0.1499 | 0.1470 | 0.1469 | 0.1478 | 0.1484 | 0.1453 | 0.1471 |
| | MSE | 0.0213 | 0.0568 | 0.0230 | 0.0224 | 0.0223 | 0.0224 | 0.0223 | 0.0210 | 0.0224 |
| | CP | 0.9490 | 0.9150 | 0.9450 | 0.9420 | 0.9450 | 0.9460 | 0.9460 | 0.9510 | 0.9440 |
| Observed selection probabilities: $(0.33, 0.22, 0.22, 0.23)$; $\boldsymbol{\alpha} = (1, 0.6, 0.6)^{\mathsf{T}}$ | | | | | | | | | | |
| $\beta_0$ | Bias | -0.0078 | 0.0503 | -0.0235 | -0.0160 | -0.0055 | -0.0160 | -0.0055 | 0.2392 | -0.0231 |
| | SD | 0.1712 | 0.2868 | 0.2559 | 0.2216 | 0.2074 | 0.2216 | 0.2074 | 0.1549 | 0.2079 |
| | ASE | 0.1660 | 0.2768 | 0.2346 | 0.1827 | 0.1834 | 0.2017 | 0.2022 | 0.1796 | 0.1984 |
| | MSE | 0.0294 | 0.0848 | 0.0660 | 0.0494 | 0.0430 | 0.0494 | 0.0430 | 0.0812 | 0.0438 |
| | CP | 0.9480 | 0.9320 | 0.9280 | 0.9090 | 0.9220 | 0.9340 | 0.9440 | 0.7440 | 0.9420 |
| $\beta_1$ | Bias | -0.0029 | 0.0184 | 0.0139 | 0.0090 | -0.0001 | 0.0090 | -0.0001 | -0.2599 | 0.0211 |
| | SD | 0.1453 | 0.2489 | 0.2539 | 0.2105 | 0.1945 | 0.2105 | 0.1945 | 0.1391 | 0.1952 |
| | ASE | 0.1448 | 0.2474 | 0.2386 | 0.1686 | 0.1702 | 0.1953 | 0.1981 | 0.1811 | 0.1911 |
| | MSE | 0.0211 | 0.0623 | 0.0647 | 0.0444 | 0.0378 | 0.0444 | 0.0378 | 0.0869 | 0.0386 |
| | CP | 0.9480 | 0.9530 | 0.9330 | 0.8860 | 0.9100 | 0.9250 | 0.9510 | 0.7350 | 0.9440 |
| $\beta_2$ | Bias | 0.0045 | -0.0009 | 0.0028 | 0.0027 | -0.0064 | 0.0027 | -0.0064 | -0.1668 | 0.0077 |
| | SD | 0.1465 | 0.2448 | 0.2509 | 0.2130 | 0.1975 | 0.2130 | 0.1975 | 0.1404 | 0.1976 |
| | ASE | 0.1464 | 0.2500 | 0.2435 | 0.1712 | 0.1729 | 0.1996 | 0.2018 | 0.1775 | 0.1931 |
| | MSE | 0.0215 | 0.0599 | 0.0629 | 0.0454 | 0.0390 | 0.0454 | 0.0390 | 0.0475 | 0.0391 |
| | CP | 0.9430 | 0.9610 | 0.9520 | 0.8830 | 0.9110 | 0.9360 | 0.9550 | 0.9040 | 0.9470 |
| $\beta_3$ | Bias | -0.0011 | 0.1248 | 0.0024 | -0.0001 | -0.0008 | -0.0001 | -0.0008 | -0.0133 | 0.0022 |
| | SD | 0.1461 | 0.2447 | 0.1569 | 0.1519 | 0.1499 | 0.1519 | 0.1499 | 0.1437 | 0.1493 |
| | ASE | 0.1455 | 0.2462 | 0.1538 | 0.1475 | 0.1473 | 0.1492 | 0.1497 | 0.1451 | 0.1476 |
| | MSE | 0.0213 | 0.0754 | 0.0246 | 0.0231 | 0.0225 | 0.0231 | 0.0225 | 0.0208 | 0.0223 |
| | CP | 0.9490 | 0.9310 | 0.9510 | 0.9450 | 0.9460 | 0.9480 | 0.9490 | 0.9460 | 0.9450 |

FML: full-data maximum likelihood estimation method
CC: complete-case estimation method
SIPW: semiparametric inverse probability weighting estimation method
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MI1n (MI1 estimation method): MI1 estimator whose value is the solution of $\boldsymbol{U}_{M1}(\boldsymbol{\beta}) = \mathbf{0}$ in Equation (5) with proposed estimated variance in Equation (10)
MI2n (MI2 estimation method): MI2 estimator whose value is the solution of $\boldsymbol{U}_{M2}(\boldsymbol{\beta}) = \mathbf{0}$ in Equation (6) with proposed estimated variance in Equation (11)
RFMI: random forest multiple imputation method
SAEM: stochastic approximation of expectation-maximization method

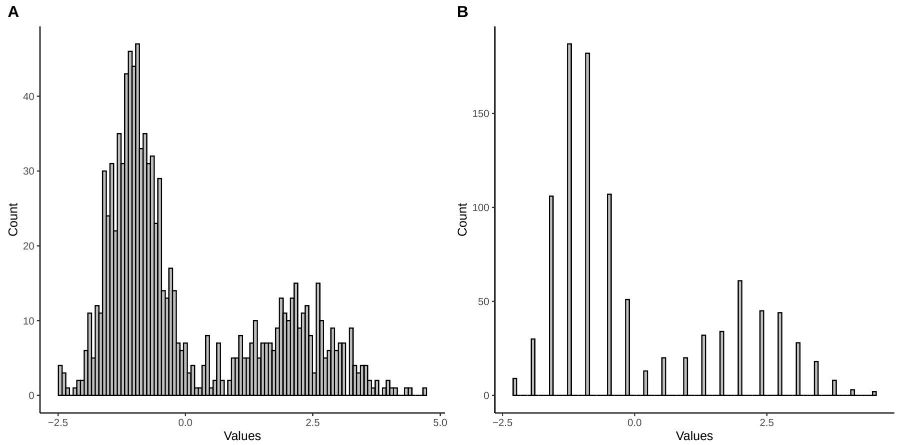


**Fig. S1** Histograms of data from a mixture of two normal distributions (left) and data from a discretized mixture of two normal distributions (right)

## Appendix B. Introduction to the Real Data Example

The FCNM Data were derived from a survey of 1,634 respondents who visited the Feng Chia Night Market (FCNM) in Taichung City, Taiwan, conducted between November 17 and 21, 2022. The variables are defined as follows:

1. $Y$ ($1 =$ Yes, $0 =$ No): Binary outcome, response to the question "Did you stay in a nearby hotel or daily rental suite during your visit to the FCNM?".
2. $\mathsf{X}_1$ ($1 = \{1, 2, 3\}$ times; $0 =$ more than three times; $5 =$ missing values): Response to the question "What is the number of times you went shopping at the FCNM in the last half year?".
3. $\mathsf{X}_2$ ($1 = \{1, 2, 3\}$ times; $0 =$ more than three times; $5 =$ missing values): Response to the question "How many times have you visited a night market for shopping in the past two months (including this visit)?"
4. $Z$ ($Z = \{0.1, 0.35, 0.75, 1.25, 1.75, 3\}$ after taking the median and being divided by $1,000$): Response to question "How much do you spend each time you visit the FCNM?".
5. $W_1$ ($1 =$ within 15 minutes; $2 =$ between 15 and 60 minutes; $3 =$ exceeding 60 minutes): Response to question "How much time did you spend traveling to the FCNM?".
6. $W_2$ ($1 =$ Taichung City or a nearby neighborhood of Taichung City; $0 =$ a neighborhood far away from Taichung City): Indicate the respondent's current city of residence.
7. $d_j$ is $\delta_j$, with $j = 1, 2, 3, 4$, indicating the missing status of $\mathsf{X}_1$ and $\mathsf{X}_2$.

## Appendix C. Introduction to the R Code for Analyzing the FCNM Data

The R code, Rcode_RealExample_FCNM.txt, performs a logistic regression analysis on the FCNM data set described above. It estimates the model parameters by using various methods, including complete case (CC), semiparametric inverse probability weighting (SIPW), multiple imputation (MI1 and MI2), the proposed extensions (MI1n and MI2n), random forest multiple imputation (RFMI), and stochastic approximation of expectation-maximization (SAEM), implemented in the `misaem` package in `R`. The code also computes the asymptotic standard errors (ASEs) and $p$-values.