

Supplementary Material: Automatic data-based bin width selection for rose diagram

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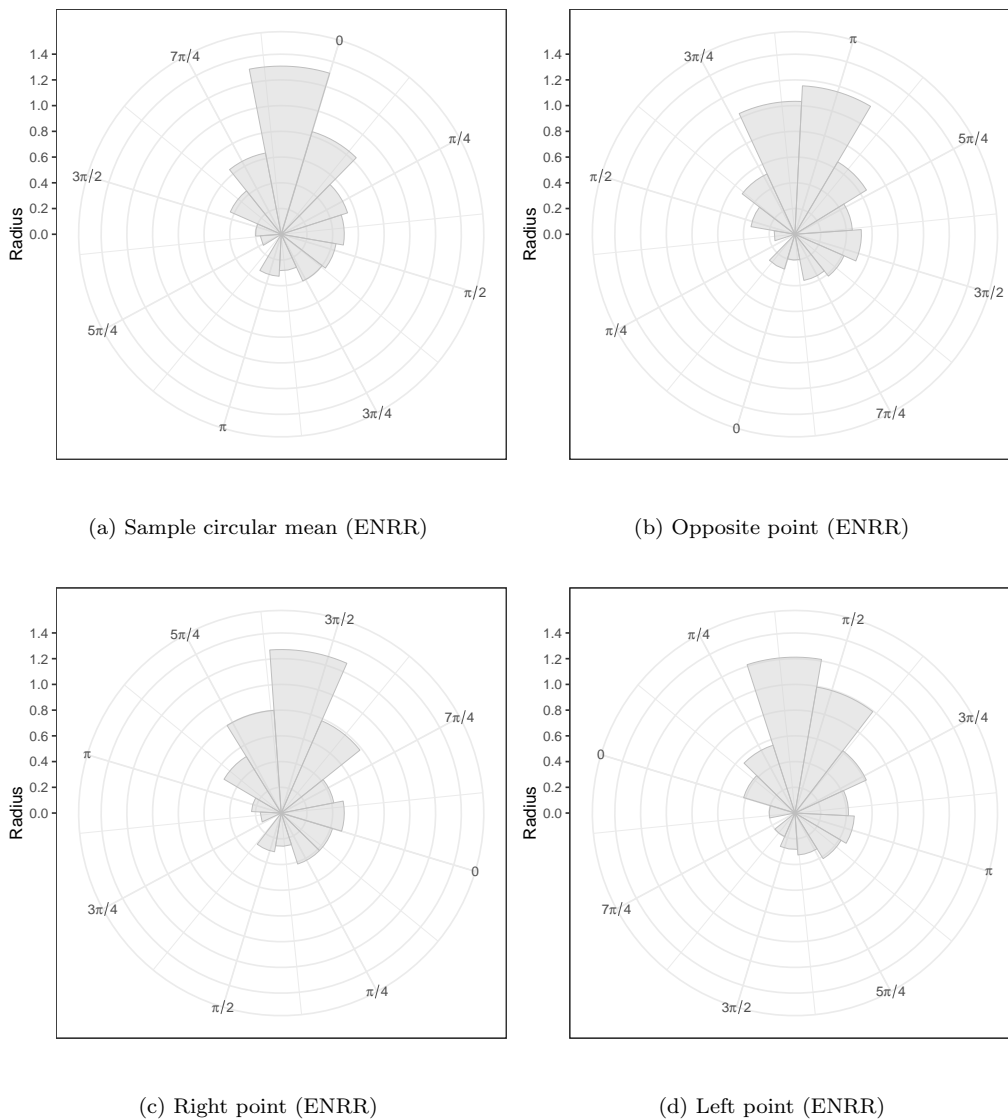
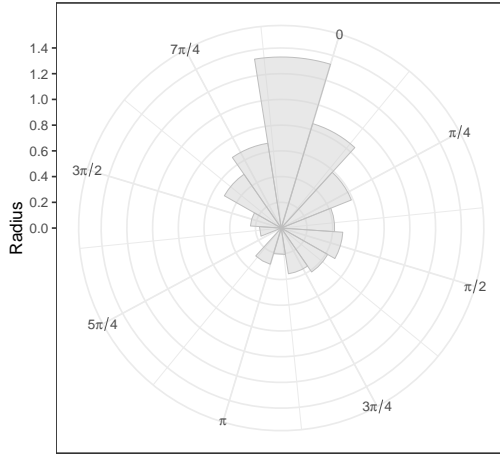


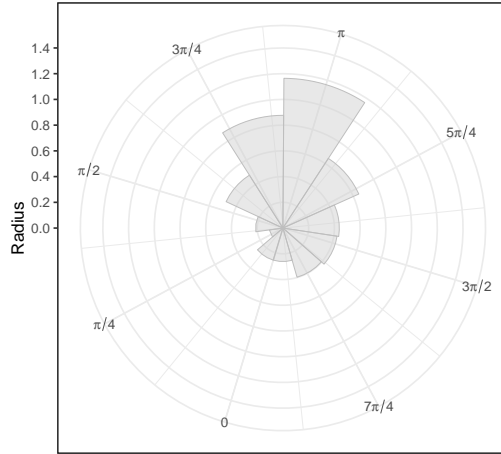
Figure 8: The four rose diagrams with bin width 0.4833 estimated by ENRR. Each starting point in (a)–(d) is denoted by zero. The values in the circle are measured clockwise from the starting point. (a) The starting point is 0.2922, which is the radian measured from north (upper direction). (b) The starting point is 3.4338. (c) The starting point is 1.863. (d) The starting point is 5.0046.

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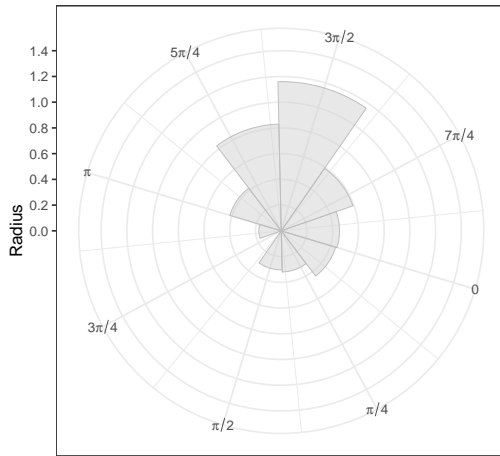
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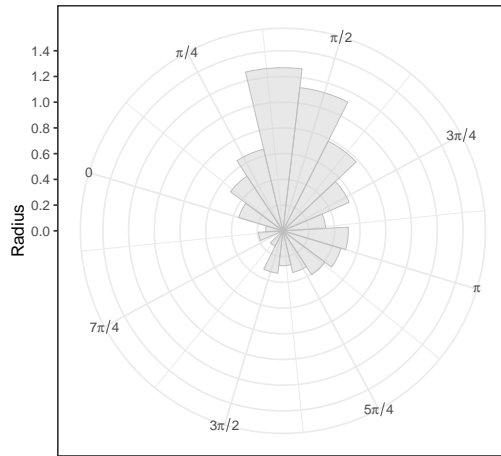
(a) Sample circular mean (EBCV)



(b) Opposite point (EBCV)

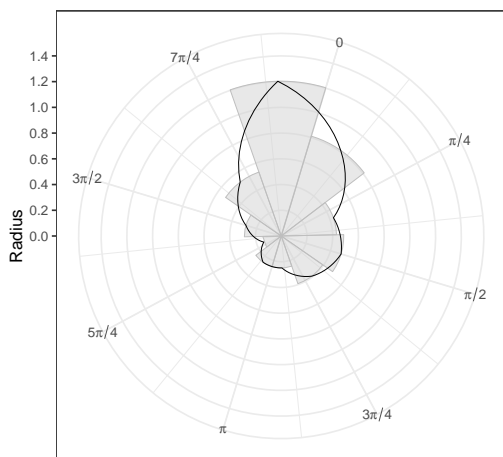


(c) Right point (EBCV)

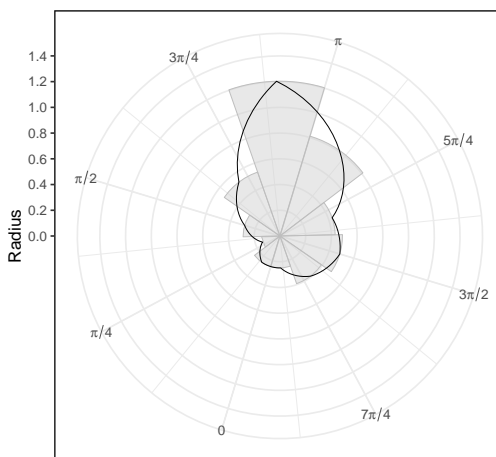


(d) Left point (EBCV)

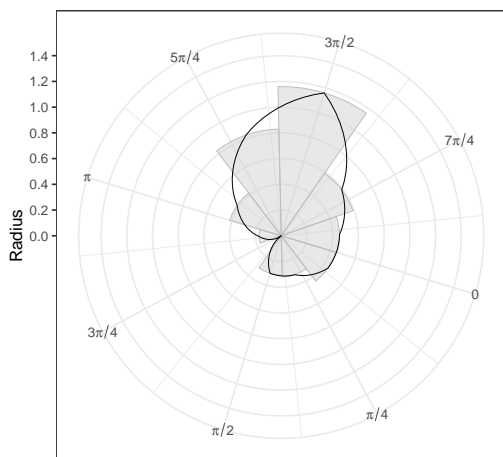
Figure 9: The four rose diagrams with the bin width estimated by EBCV. Each starting point in (a)–(d) is denoted by zero. The values in the circle are measured clockwise from the starting point. (a) The starting point is 0.2922, which is radian measured from north (upper direction). The bin width is 0.4488. (b) The starting point is 3.4338. The bin width is 0.5712. (c) The starting point is 1.863. The bin width is 0.6283. (d) The starting point is 5.0046. The bin width is 0.3491.



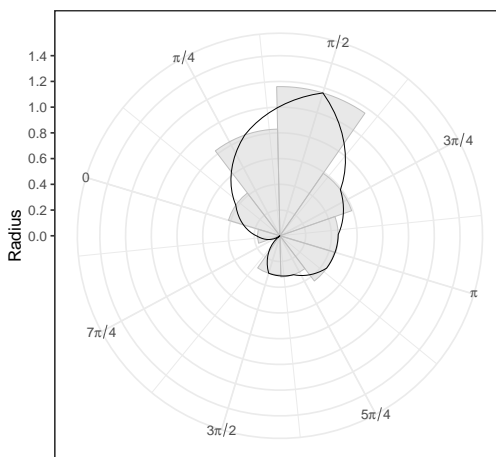
(a) Sample circular mean (ENRR)



(b) Opposite point (ENRR)

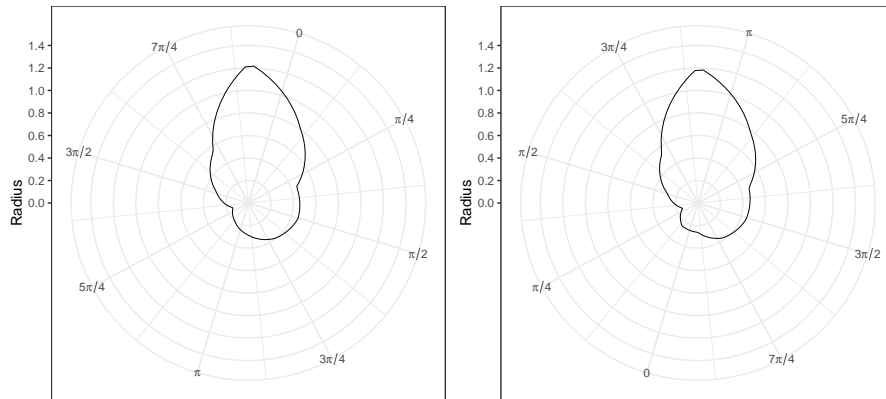


(c) Right point (ENRR)



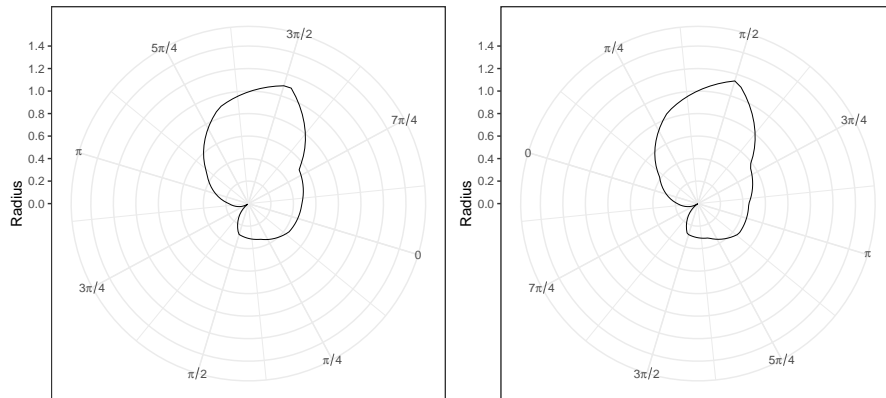
(d) Left point (ENRR)

Figure 10: The solid lines represents the polygon rose diagrams with bin width 0.6283 estimated by ENRR. Each starting point in (a)–(d) is denoted by zero. The values in the circle are measured clockwise from the starting point. (a) The starting point is 0.2922, which is the radian measured from north (upper direction). (b) The starting point is 3.4338. (c) The starting point is 1.863. (d) The starting point is 5.0046.



(a) Sample circular mean (NRR)

(b) Opposite point (NRR)



(c) Right point (NRR)

(d) Left point (NRR)

Figure 11: The slid lines represents the polygon rose diagrams with bin width 0.6345 estimated with NRR. Each starting point in (a)–(d) is denoted by zero. The values in the circle are measured clockwise from the starting point. (a) The stating point is 0.2922, which is the radian measured from north (upper direction). (b) The stating point is 3.4338. (c) The stating point is 1.863. (d) The stating point is 5.0046.

Table 4: The means (its standard error) of $100 \times$ integrated squared error of the rose diagrams (denoted by Rose) and the polygon rose diagrams (denoted by Poly) based on the number of repetitions $N=1000$ in Models 1–6 in the simulation in section 4. The sample size is $n = 50$. In the starting point t_0 , M denotes the sample circular mean, O denotes the opposite, R denotes the right point, and L denotes the left point.

\hat{r}_f	Selector	t_0	Model1	Model2	Model3	Model4	Model5	Model6
Rose	NRR	M	8.42(0.15)	7.33(0.12)	9.75(0.15)	10.06(0.17)	9.38(0.16)	11.25(0.17)
Rose	NRR	O	8.82(0.15)	7.72(0.13)	9.55(0.15)	10.29(0.16)	9.53(0.15)	11.56(0.17)
Rose	NRR	R	8.20(0.15)	7.65(0.13)	9.40(0.16)	10.02(0.17)	9.06(0.15)	11.18(0.17)
Rose	NRR	L	8.57(0.15)	7.33(0.13)	9.54(0.16)	10.54(0.16)	9.47(0.16)	10.82(0.16)
Rose	ENRR	M	7.92(0.14)	6.82(0.11)	9.82(0.15)	9.55(0.16)	8.81(0.15)	11.40(0.16)
Rose	ENRR	O	7.97(0.14)	7.13(0.12)	8.90(0.15)	9.54(0.16)	8.93(0.16)	10.54(0.16)
Rose	ENRR	R	7.89(0.14)	7.10(0.12)	9.06(0.15)	9.62(0.16)	8.72(0.15)	10.38(0.17)
Rose	ENRR	L	7.74(0.14)	6.94(0.11)	8.70(0.15)	9.67(0.16)	8.87(0.15)	10.39(0.17)
Rose	BCV	M	8.55(0.13)	7.43(0.11)	11.21(0.15)	10.47(0.15)	9.08(0.15)	11.93(0.16)
Rose	BCV	O	8.43(0.12)	7.93(0.11)	10.18(0.15)	9.93(0.14)	9.10(0.12)	11.31(0.14)
Rose	BCV	R	7.64(0.11)	7.29(0.08)	9.78(0.12)	9.99(0.16)	8.12(0.12)	13.43(0.22)
Rose	BCV	L	8.60(0.12)	6.93(0.11)	10.57(0.11)	11.20(0.14)	10.71(0.15)	13.21(0.15)
Rose	EBCV	M	12.07(0.23)	10.52(0.19)	15.24(0.24)	14.17(0.29)	12.62(0.27)	15.64(0.25)
Rose	EBCV	O	11.69(0.23)	11.02(0.19)	13.29(0.24)	12.94(0.28)	11.60(0.24)	14.91(0.25)
Rose	EBCV	R	7.23(0.09)	6.95(0.08)	9.21(0.09)	9.33(0.11)	7.83(0.08)	11.68(0.13)
Rose	EBCV	L	8.44(0.12)	6.48(0.10)	10.63(0.11)	10.94(0.13)	10.19(0.14)	13.02(0.15)
Rose	BCR	M	12.91(0.21)	12.31(0.19)	13.83(0.22)	13.88(0.20)	13.40(0.19)	14.60(0.21)
Rose	BCR	O	12.91(0.21)	12.31(0.19)	13.83(0.22)	13.88(0.20)	13.40(0.19)	14.60(0.21)
Rose	BCR	R	12.98(0.21)	12.17(0.19)	13.48(0.22)	13.85(0.20)	13.73(0.21)	14.38(0.20)
Rose	BCR	L	12.98(0.21)	12.17(0.19)	13.48(0.22)	13.85(0.20)	13.73(0.21)	14.38(0.20)
Rose	AR	M	12.01(0.21)	10.72(0.19)	13.15(0.23)	14.69(0.24)	13.57(0.21)	14.90(0.24)
Rose	AR	O	12.11(0.21)	10.76(0.19)	13.58(0.23)	14.47(0.22)	13.21(0.21)	15.32(0.23)
Rose	AR	R	12.37(0.22)	10.43(0.18)	13.34(0.24)	14.52(0.23)	13.72(0.22)	15.35(0.23)
Rose	AR	L	12.48(0.22)	10.78(0.19)	13.3(0.23)	14.43(0.22)	13.81(0.22)	14.92(0.22)
Rose	SRR	M	10.72(0.18)	10.08(0.16)	11.83(0.19)	12.02(0.18)	11.45(0.18)	12.83(0.18)
Rose	SRR	O	9.63(0.16)	8.99(0.15)	10.27(0.17)	10.75(0.18)	10.19(0.16)	11.28(0.16)
Rose	SRR	R	10.38(0.17)	10.11(0.16)	10.78(0.17)	11.46(0.17)	11.08(0.16)	12.33(0.18)
Rose	SRR	L	10.17(0.17)	9.80(0.15)	10.89(0.17)	11.04(0.17)	10.72(0.16)	11.67(0.17)
Rose	SPR	M	10.03(0.18)	9.50(0.17)	10.78(0.18)	11.05(0.18)	10.50(0.17)	11.81(0.18)
Rose	SPR	O	10.59(0.18)	9.87(0.15)	11.18(0.17)	11.79(0.17)	11.14(0.16)	12.29(0.18)
Rose	SPR	R	10.33(0.19)	9.71(0.18)	10.95(0.19)	11.26(0.20)	10.63(0.18)	12.10(0.19)
Rose	SPR	L	10.61(0.18)	9.90(0.17)	11.55(0.19)	11.88(0.17)	11.31(0.17)	12.42(0.18)
Rose	EAR	M	12.14(0.22)	10.65(0.19)	13.32(0.24)	14.42(0.23)	13.27(0.21)	15.19(0.23)
Rose	EAR	O	12.40(0.22)	10.68(0.19)	13.35(0.23)	14.60(0.23)	13.50(0.22)	15.02(0.23)
Rose	EAR	R	12.24(0.22)	10.45(0.19)	13.29(0.24)	14.65(0.22)	13.49(0.22)	15.19(0.23)
Rose	EAR	L	12.27(0.22)	10.48(0.19)	13.52(0.23)	14.53(0.23)	13.48(0.22)	15.17(0.23)
Rose	ESRR	M	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Rose	ESRR	O	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Rose	ESRR	R	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Rose	ESRR	L	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Rose	ESPR	M	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Rose	ESPR	O	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Rose	ESPR	R	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Rose	ESPR	L	10.16(0.18)	9.56(0.16)	10.79(0.19)	11.39(0.18)	10.97(0.18)	12.05(0.18)
Poly	NRR	M	6.29(0.25)	5.56(0.21)	6.94(0.25)	6.65(0.23)	6.28(0.21)	7.67(0.28)
Poly	NRR	O	5.36(0.11)	4.70(0.11)	6.30(0.12)	6.21(0.12)	5.41(0.10)	7.26(0.15)
Poly	NRR	R	6.02(0.18)	5.61(0.18)	6.46(0.15)	7.01(0.25)	6.44(0.18)	7.41(0.17)
Poly	NRR	L	6.06(0.17)	5.40(0.17)	6.40(0.21)	6.41(0.13)	6.05(0.14)	6.45(0.15)
Poly	ENRR	M	4.05(0.09)	3.56(0.08)	5.47(0.10)	4.92(0.11)	4.49(0.10)	6.09(0.11)
Poly	ENRR	O	4.04(0.09)	3.75(0.09)	4.91(0.10)	4.91(0.11)	4.44(0.10)	5.61(0.10)
Poly	ENRR	R	4.03(0.09)	3.65(0.09)	4.97(0.11)	4.96(0.11)	4.38(0.09)	5.49(0.11)
Poly	ENRR	L	4.02(0.09)	3.69(0.08)	4.69(0.10)	4.94(0.10)	4.38(0.10)	5.51(0.11)

Table 5: The means (its standard error) of $100 \times$ integrated squared error of the rose diagrams (denoted by Rose) and the polygon rose diagrams (denoted by Poly) based on the number of repetitions $N=1000$ in Models 1–6 in the simulation in section 4. The sample size is $n = 100$. In the starting point t_0 , M denotes the sample circular mean, O denotes the opposite, R denotes the right point, and L denotes the left point.

\hat{r}_f	Selector	t_0	Model1	Model2	Model3	Model4	Model5	Model6
Rose	NRR	M	4.95(0.15)	4.56(0.12)	5.70(0.15)	5.89(0.17)	5.53(0.16)	6.75(0.17)
Rose	NRR	O	5.11(0.15)	4.63(0.13)	5.91(0.15)	6.14(0.16)	5.67(0.15)	6.80(0.17)
Rose	NRR	R	4.86(0.15)	4.50(0.13)	5.66(0.16)	5.91(0.17)	5.45(0.15)	6.69(0.17)
Rose	NRR	L	5.17(0.15)	4.44(0.13)	5.90(0.16)	6.19(0.16)	5.59(0.16)	6.66(0.16)
Rose	ENRR	M	4.79(0.14)	4.26(0.11)	5.75(0.15)	5.71(0.16)	5.32(0.15)	6.46(0.16)
Rose	ENRR	O	4.66(0.14)	4.31(0.12)	5.55(0.15)	5.77(0.16)	5.29(0.16)	6.42(0.16)
Rose	ENRR	R	4.69(0.14)	4.20(0.12)	5.33(0.15)	5.66(0.16)	5.28(0.15)	6.42(0.17)
Rose	ENRR	L	4.78(0.14)	4.27(0.11)	5.55(0.15)	5.69(0.16)	5.31(0.15)	6.15(0.17)
Rose	BCV	M	5.09(0.13)	4.95(0.11)	6.84(0.15)	5.55(0.15)	5.22(0.15)	6.70(0.16)
Rose	BCV	O	5.30(0.12)	5.43(0.11)	6.92(0.15)	6.18(0.14)	5.83(0.12)	7.35(0.14)
Rose	BCV	R	4.75(0.11)	5.33(0.08)	6.43(0.12)	5.68(0.16)	4.98(0.12)	8.12(0.22)
Rose	BCV	L	5.45(0.12)	4.49(0.11)	7.86(0.11)	6.68(0.14)	6.38(0.15)	7.90(0.15)
Rose	EBCV	M	6.41(0.23)	6.08(0.19)	8.94(0.24)	6.56(0.29)	6.25(0.27)	7.46(0.25)
Rose	EBCV	O	7.11(0.23)	8.33(0.19)	8.54(0.24)	7.69(0.28)	7.88(0.24)	8.97(0.25)
Rose	EBCV	R	5.00(0.09)	5.46(0.08)	6.35(0.09)	5.77(0.11)	5.28(0.08)	7.00(0.13)
Rose	EBCV	L	5.54(0.12)	4.52(0.10)	8.04(0.11)	6.78(0.13)	6.32(0.14)	8.09(0.15)
Rose	BCR	M	13.15(0.21)	12.50(0.19)	13.31(0.22)	13.64(0.20)	13.43(0.19)	14.05(0.21)
Rose	BCR	O	13.15(0.21)	12.50(0.19)	13.31(0.22)	13.64(0.20)	13.43(0.19)	14.05(0.21)
Rose	BCR	R	13.15(0.21)	12.50(0.19)	13.31(0.22)	13.64(0.20)	13.43(0.21)	14.05(0.20)
Rose	BCR	L	13.15(0.21)	12.50(0.19)	13.31(0.22)	13.64(0.20)	13.43(0.21)	14.05(0.20)
Rose	AR	M	7.89(0.21)	7.08(0.19)	8.38(0.23)	9.49(0.24)	8.99(0.21)	9.87(0.24)
Rose	AR	O	7.93(0.21)	6.93(0.19)	8.53(0.23)	9.59(0.22)	8.94(0.21)	9.96(0.23)
Rose	AR	R	7.95(0.22)	7.07(0.18)	8.37(0.24)	9.62(0.23)	8.97(0.22)	9.97(0.23)
Rose	AR	L	8.19(0.22)	7.07(0.19)	8.45(0.23)	9.55(0.22)	8.88(0.22)	9.93(0.22)
Rose	SRR	M	5.80(0.18)	5.67(0.16)	6.25(0.19)	6.51(0.18)	6.29(0.18)	6.93(0.18)
Rose	SRR	O	5.80(0.16)	5.67(0.15)	6.25(0.17)	6.51(0.18)	6.29(0.16)	6.93(0.16)
Rose	SRR	R	5.99(0.17)	5.64(0.16)	6.25(0.17)	6.53(0.17)	6.31(0.16)	7.01(0.18)
Rose	SRR	L	5.99(0.17)	5.64(0.15)	6.25(0.17)	6.53(0.17)	6.31(0.16)	7.01(0.17)
Rose	SPR	M	6.01(0.18)	5.68(0.17)	6.34(0.18)	6.57(0.18)	6.32(0.17)	7.08(0.18)
Rose	SPR	O	6.25(0.18)	5.98(0.15)	6.77(0.17)	6.83(0.17)	6.58(0.16)	7.31(0.18)
Rose	SPR	R	6.05(0.19)	5.68(0.18)	6.63(0.19)	6.73(0.20)	6.33(0.18)	7.30(0.19)
Rose	SPR	L	6.25(0.18)	5.96(0.17)	6.54(0.19)	7.00(0.17)	6.58(0.17)	7.47(0.18)
Rose	EAR	M	7.92(0.22)	6.91(0.19)	8.41(0.24)	9.61(0.23)	8.90(0.21)	9.78(0.23)
Rose	EAR	O	8.11(0.22)	6.97(0.19)	8.34(0.23)	9.66(0.23)	8.83(0.22)	9.81(0.23)
Rose	EAR	R	8.01(0.22)	7.06(0.19)	8.33(0.24)	9.59(0.22)	8.94(0.22)	9.79(0.23)
Rose	EAR	L	7.96(0.22)	6.92(0.19)	8.38(0.23)	9.60(0.23)	8.88(0.22)	9.87(0.23)
Rose	ESRR	M	5.80(0.18)	5.67(0.16)	6.25(0.19)	6.51(0.18)	6.29(0.18)	6.93(0.18)
Rose	ESRR	O	5.80(0.18)	5.67(0.16)	6.25(0.19)	6.51(0.18)	6.29(0.18)	6.93(0.18)
Rose	ESRR	R	5.99(0.18)	5.64(0.16)	6.25(0.19)	6.53(0.18)	6.31(0.18)	7.01(0.18)
Rose	ESRR	L	5.99(0.18)	5.64(0.16)	6.25(0.19)	6.53(0.18)	6.31(0.18)	7.01(0.18)
Rose	ESPR	M	5.80(0.18)	5.67(0.16)	6.25(0.19)	6.51(0.18)	6.29(0.18)	6.93(0.18)
Rose	ESPR	O	5.80(0.18)	5.67(0.16)	6.25(0.19)	6.51(0.18)	6.29(0.18)	6.93(0.18)
Rose	ESPR	R	5.99(0.18)	5.64(0.16)	6.25(0.19)	6.53(0.18)	6.31(0.18)	7.01(0.18)
Rose	ESPR	L	5.99(0.18)	5.64(0.16)	6.25(0.19)	6.53(0.18)	6.31(0.18)	7.01(0.18)
Poly	NRR	M	3.59(0.25)	3.31(0.21)	3.62(0.25)	3.34(0.23)	3.59(0.21)	4.00(0.28)
Poly	NRR	O	2.93(0.11)	2.60(0.11)	3.71(0.12)	3.44(0.12)	3.12(0.10)	4.09(0.15)
Poly	NRR	R	3.34(0.18)	3.15(0.18)	3.72(0.15)	3.56(0.25)	3.45(0.18)	4.18(0.17)
Poly	NRR	L	3.31(0.17)	3.10(0.17)	3.68(0.21)	3.38(0.13)	3.37(0.14)	3.64(0.15)
Poly	ENRR	M	2.17(0.09)	2.01(0.08)	3.04(0.10)	2.55(0.11)	2.37(0.10)	3.22(0.11)
Poly	ENRR	O	2.16(0.09)	2.04(0.09)	2.90(0.10)	2.56(0.11)	2.40(0.10)	3.10(0.10)
Poly	ENRR	R	2.17(0.09)	1.97(0.09)	2.69(0.11)	2.56(0.11)	2.38(0.09)	3.16(0.11)
Poly	ENRR	L	2.22(0.09)	2.01(0.08)	2.73(0.10)	2.59(0.10)	2.42(0.10)	2.93(0.11)

Table 6: The means (its standard error) of $100 \times$ integrated squared error of the rose diagrams (denoted by Rose) and the polygon rose diagrams (denoted by Poly) based on the number of repetitions $N=1000$ in Models 1–6 in the simulation in section 4. The sample size is $n = 200$. In the starting point t_0 , M denotes the sample circular mean, O denotes the opposite, R denotes the right point, and L denotes the left point.

\hat{r}_f	Selector	t_0	Model1	Model2	Model3	Model4	Model5	Model6
Rose	NRR	M	3.18(0.04)	2.79(0.03)	3.55(0.03)	3.63(0.04)	3.38(0.04)	4.07(0.04)
Rose	NRR	O	3.25(0.04)	2.86(0.03)	3.63(0.04)	3.67(0.04)	3.44(0.04)	4.04(0.04)
Rose	NRR	R	3.06(0.03)	2.77(0.03)	3.55(0.04)	3.58(0.04)	3.28(0.04)	4.09(0.04)
Rose	NRR	L	3.13(0.04)	2.78(0.03)	3.55(0.03)	3.65(0.04)	3.38(0.04)	4.11(0.04)
Rose	ENRR	M	3.02(0.03)	2.69(0.03)	3.51(0.03)	3.48(0.04)	3.25(0.04)	3.87(0.04)
Rose	ENRR	O	3.05(0.04)	2.69(0.03)	3.44(0.03)	3.50(0.04)	3.26(0.04)	3.90(0.04)
Rose	ENRR	R	2.98(0.03)	2.70(0.03)	3.48(0.03)	3.48(0.04)	3.23(0.04)	3.89(0.04)
Rose	ENRR	L	3.02(0.03)	2.71(0.03)	3.45(0.03)	3.50(0.04)	3.24(0.04)	3.96(0.04)
Rose	BCV	M	2.78(0.05)	2.73(0.05)	3.46(0.06)	3.05(0.04)	2.94(0.05)	3.67(0.05)
Rose	BCV	O	2.92(0.05)	2.93(0.06)	3.52(0.06)	3.17(0.04)	3.01(0.04)	3.92(0.07)
Rose	BCV	R	2.69(0.04)	3.30(0.06)	3.62(0.05)	3.15(0.06)	2.69(0.03)	4.15(0.10)
Rose	BCV	L	2.78(0.04)	2.55(0.04)	4.33(0.08)	3.11(0.05)	3.02(0.05)	3.78(0.06)
Rose	EBCV	M	3.11(0.07)	3.02(0.07)	3.83(0.09)	3.28(0.05)	3.21(0.07)	3.76(0.06)
Rose	EBCV	O	3.54(0.11)	3.90(0.13)	3.84(0.09)	3.61(0.10)	3.48(0.10)	4.46(0.13)
Rose	EBCV	R	2.96(0.04)	3.47(0.06)	3.61(0.04)	3.41(0.05)	3.05(0.03)	4.07(0.07)
Rose	EBCV	L	3.04(0.04)	2.77(0.03)	4.42(0.08)	3.39(0.05)	3.32(0.06)	4.05(0.06)
Rose	BCR	M	13.35(0.11)	12.83(0.10)	13.24(0.11)	13.96(0.10)	13.69(0.10)	14.09(0.11)
Rose	BCR	O	13.35(0.11)	12.83(0.10)	13.24(0.11)	13.96(0.10)	13.69(0.10)	14.09(0.11)
Rose	BCR	R	13.35(0.11)	12.83(0.10)	13.24(0.11)	13.96(0.10)	13.69(0.10)	14.09(0.11)
Rose	BCR	L	13.35(0.11)	12.83(0.10)	13.24(0.11)	13.96(0.10)	13.69(0.10)	14.09(0.11)
Rose	AR	M	5.16(0.07)	4.58(0.06)	5.34(0.07)	6.22(0.07)	5.81(0.07)	6.21(0.07)
Rose	AR	O	5.23(0.07)	4.67(0.05)	5.27(0.06)	6.21(0.07)	5.87(0.07)	6.25(0.08)
Rose	AR	R	5.21(0.07)	4.54(0.06)	5.35(0.07)	6.41(0.08)	5.79(0.07)	6.27(0.07)
Rose	AR	L	5.20(0.07)	4.60(0.06)	5.37(0.07)	6.30(0.08)	5.85(0.07)	6.33(0.08)
Rose	SRR	M	4.15(0.05)	4.03(0.05)	4.27(0.05)	4.53(0.06)	4.36(0.05)	4.70(0.06)
Rose	SRR	O	4.27(0.05)	4.15(0.05)	4.45(0.05)	4.57(0.05)	4.44(0.05)	4.80(0.06)
Rose	SRR	R	4.40(0.05)	4.21(0.05)	4.41(0.05)	4.72(0.06)	4.53(0.06)	4.83(0.06)
Rose	SRR	L	4.41(0.05)	4.19(0.05)	4.43(0.05)	4.72(0.06)	4.54(0.06)	4.84(0.06)
Rose	SPR	M	3.47(0.04)	3.33(0.04)	3.66(0.04)	3.80(0.05)	3.58(0.04)	4.12(0.05)
Rose	SPR	O	3.57(0.05)	3.33(0.04)	3.74(0.04)	3.85(0.05)	3.64(0.05)	4.13(0.05)
Rose	SPR	R	3.46(0.04)	3.31(0.04)	3.66(0.04)	3.77(0.05)	3.52(0.05)	4.11(0.05)
Rose	SPR	L	3.48(0.04)	3.28(0.04)	3.72(0.04)	3.78(0.05)	3.60(0.05)	4.12(0.05)
Rose	EAR	M	5.19(0.07)	4.54(0.06)	5.26(0.07)	6.27(0.07)	5.81(0.07)	6.29(0.08)
Rose	EAR	O	5.20(0.07)	4.55(0.06)	5.20(0.07)	6.25(0.08)	5.82(0.07)	6.23(0.08)
Rose	EAR	R	5.22(0.07)	4.58(0.06)	5.30(0.07)	6.29(0.07)	5.79(0.07)	6.36(0.08)
Rose	EAR	L	5.14(0.07)	4.56(0.06)	5.30(0.06)	6.29(0.08)	5.75(0.07)	6.33(0.08)
Rose	ESRR	M	4.20(0.05)	4.11(0.05)	4.24(0.05)	4.46(0.06)	4.35(0.06)	4.64(0.06)
Rose	ESRR	O	4.16(0.05)	4.05(0.05)	4.22(0.05)	4.54(0.06)	4.34(0.06)	4.70(0.06)
Rose	ESRR	R	4.23(0.05)	4.02(0.05)	4.20(0.05)	4.52(0.06)	4.30(0.05)	4.69(0.06)
Rose	ESRR	L	4.20(0.05)	4.04(0.05)	4.23(0.05)	4.48(0.06)	4.36(0.06)	4.58(0.06)
Rose	ESPR	M	3.51(0.04)	3.29(0.04)	3.63(0.04)	3.83(0.05)	3.55(0.04)	4.03(0.05)
Rose	ESPR	O	3.51(0.04)	3.29(0.04)	3.63(0.04)	3.83(0.05)	3.55(0.04)	4.03(0.05)
Rose	ESPR	R	3.51(0.04)	3.29(0.04)	3.63(0.04)	3.83(0.05)	3.55(0.04)	4.03(0.05)
Rose	ESPR	L	3.51(0.04)	3.29(0.04)	3.63(0.04)	3.83(0.05)	3.55(0.04)	4.03(0.05)
Poly	NRR	M	1.84(0.10)	1.73(0.09)	2.44(0.36)	2.04(0.20)	1.87(0.09)	2.10(0.11)
Poly	NRR	O	1.75(0.04)	1.44(0.05)	2.19(0.08)	1.82(0.04)	1.81(0.04)	2.21(0.05)
Poly	NRR	R	1.90(0.07)	1.74(0.08)	2.16(0.06)	1.88(0.06)	1.89(0.06)	4.66(2.33)
Poly	NRR	L	1.82(0.06)	2.93(1.26)	2.23(0.06)	1.81(0.05)	1.91(0.06)	2.13(0.07)
Poly	ENRR	M	1.24(0.02)	1.18(0.02)	1.76(0.03)	1.41(0.03)	1.30(0.02)	1.72(0.03)
Poly	ENRR	O	1.23(0.02)	1.16(0.02)	1.56(0.03)	1.41(0.03)	1.33(0.02)	1.76(0.03)
Poly	ENRR	R	1.24(0.02)	1.15(0.02)	1.60(0.02)	1.44(0.03)	1.30(0.02)	1.80(0.03)
Poly	ENRR	L	1.25(0.02)	1.16(0.02)	1.68(0.02)	1.41(0.03)	1.31(0.02)	1.69(0.03)

Table 7: The means (its standard error) of $100 \times$ integrated squared error of the rose diagrams (denoted by Rose) and the polygon rose diagrams (denoted by Poly) based on the number of repetitions $N=1000$ in Models 1–6 in the simulation in section 4. The sample size is $n = 500$. In the starting point t_0 , M denotes the sample circular mean, O denotes the opposite, R denotes the right point, and L denotes the left point.

\hat{r}_f	Selector	t_0	Model1	Model2	Model3	Model4	Model5	Model6
Rose	NRR	M	1.68(0.02)	1.54(0.01)	1.92(0.01)	1.95(0.02)	1.79(0.02)	2.16(0.02)
Rose	NRR	O	1.71(0.02)	1.54(0.02)	1.97(0.02)	1.96(0.02)	1.80(0.02)	2.18(0.02)
Rose	NRR	R	1.67(0.02)	1.51(0.01)	1.94(0.02)	1.92(0.02)	1.75(0.02)	2.18(0.02)
Rose	NRR	L	1.67(0.02)	1.53(0.02)	1.94(0.02)	1.93(0.02)	1.77(0.02)	2.15(0.02)
Rose	ENRR	M	1.64(0.02)	1.50(0.01)	1.91(0.01)	1.89(0.02)	1.72(0.02)	2.12(0.02)
Rose	ENRR	O	1.64(0.02)	1.49(0.01)	1.90(0.01)	1.89(0.02)	1.73(0.02)	2.11(0.02)
Rose	ENRR	R	1.63(0.02)	1.47(0.01)	1.89(0.01)	1.88(0.02)	1.72(0.02)	2.12(0.02)
Rose	ENRR	L	1.64(0.02)	1.48(0.01)	1.90(0.01)	1.89(0.02)	1.72(0.02)	2.10(0.02)
Rose	BCV	M	1.40(0.01)	1.30(0.01)	1.64(0.01)	1.61(0.01)	1.48(0.01)	1.83(0.01)
Rose	BCV	O	1.41(0.01)	1.33(0.02)	1.66(0.01)	1.62(0.02)	1.47(0.01)	1.86(0.02)
Rose	BCV	R	1.37(0.01)	1.29(0.02)	1.68(0.01)	1.56(0.01)	1.41(0.01)	1.83(0.02)
Rose	BCV	L	1.37(0.01)	1.26(0.01)	1.79(0.04)	1.54(0.01)	1.45(0.01)	1.82(0.02)
Rose	EBCV	M	1.51(0.01)	1.41(0.01)	1.77(0.01)	1.74(0.01)	1.62(0.01)	1.95(0.01)
Rose	EBCV	O	1.53(0.01)	1.46(0.03)	1.80(0.02)	1.80(0.03)	1.61(0.01)	1.99(0.02)
Rose	EBCV	R	1.54(0.02)	1.44(0.02)	1.80(0.01)	1.73(0.01)	1.60(0.01)	1.96(0.02)
Rose	EBCV	L	1.53(0.01)	1.41(0.01)	1.93(0.03)	1.73(0.01)	1.61(0.01)	1.99(0.02)
Rose	BCR	M	13.35(0.07)	13.03(0.07)	13.54(0.07)	13.83(0.07)	13.76(0.07)	13.89(0.07)
Rose	BCR	O	13.35(0.07)	13.03(0.07)	13.54(0.07)	13.83(0.07)	13.76(0.07)	13.89(0.07)
Rose	BCR	R	13.35(0.07)	13.03(0.07)	13.54(0.07)	13.83(0.07)	13.76(0.07)	13.89(0.07)
Rose	BCR	L	13.35(0.07)	13.03(0.07)	13.54(0.07)	13.83(0.07)	13.76(0.07)	13.89(0.07)
Rose	AR	M	3.03(0.03)	2.73(0.03)	3.14(0.03)	3.55(0.03)	3.30(0.03)	3.54(0.03)
Rose	AR	O	3.08(0.03)	2.80(0.03)	3.13(0.03)	3.58(0.03)	3.31(0.03)	3.57(0.03)
Rose	AR	R	3.04(0.03)	2.76(0.03)	3.11(0.03)	3.56(0.03)	3.27(0.03)	3.56(0.03)
Rose	AR	L	3.03(0.03)	2.78(0.03)	3.15(0.03)	3.52(0.03)	3.30(0.03)	3.55(0.03)
Rose	SRR	M	2.44(0.02)	2.42(0.02)	2.5(0.02)	2.55(0.02)	2.52(0.02)	2.60(0.02)
Rose	SRR	O	2.53(0.02)	2.49(0.02)	2.52(0.02)	2.61(0.03)	2.54(0.02)	2.67(0.03)
Rose	SRR	R	2.42(0.02)	2.38(0.02)	2.53(0.02)	2.58(0.03)	2.48(0.02)	2.66(0.03)
Rose	SRR	L	2.42(0.02)	2.41(0.02)	2.51(0.02)	2.59(0.03)	2.50(0.02)	2.66(0.03)
Rose	SPR	M	1.85(0.02)	1.78(0.02)	1.95(0.02)	2.01(0.02)	1.88(0.02)	2.15(0.02)
Rose	SPR	O	1.84(0.02)	1.77(0.02)	1.97(0.02)	2.00(0.02)	1.88(0.02)	2.14(0.02)
Rose	SPR	R	1.83(0.02)	1.78(0.02)	1.96(0.02)	2.00(0.02)	1.86(0.02)	2.14(0.02)
Rose	SPR	L	1.83(0.02)	1.77(0.02)	1.96(0.02)	1.98(0.02)	1.88(0.02)	2.14(0.02)
Rose	EAR	M	3.05(0.03)	2.75(0.03)	3.13(0.03)	3.53(0.03)	3.31(0.03)	3.52(0.03)
Rose	EAR	O	3.05(0.03)	2.75(0.03)	3.12(0.03)	3.56(0.03)	3.30(0.03)	3.53(0.03)
Rose	EAR	R	3.06(0.03)	2.77(0.03)	3.12(0.03)	3.55(0.03)	3.30(0.03)	3.53(0.03)
Rose	EAR	L	3.08(0.03)	2.78(0.03)	3.12(0.03)	3.55(0.03)	3.27(0.03)	3.54(0.03)
Rose	ESRR	M	2.42(0.02)	2.38(0.02)	2.51(0.02)	2.58(0.03)	2.50(0.02)	2.63(0.03)
Rose	ESRR	O	2.42(0.02)	2.42(0.02)	2.49(0.02)	2.55(0.02)	2.47(0.02)	2.59(0.02)
Rose	ESRR	R	2.45(0.02)	2.40(0.02)	2.46(0.02)	2.56(0.03)	2.45(0.02)	2.61(0.02)
Rose	ESRR	L	2.44(0.02)	2.39(0.02)	2.50(0.02)	2.55(0.02)	2.52(0.02)	2.60(0.02)
Rose	ESPR	M	1.84(0.02)	1.78(0.02)	1.97(0.02)	2.00(0.02)	1.88(0.02)	2.14(0.02)
Rose	ESPR	O	1.84(0.02)	1.78(0.02)	1.97(0.02)	2.00(0.02)	1.88(0.02)	2.14(0.02)
Rose	ESPR	R	1.84(0.02)	1.78(0.02)	1.97(0.02)	2.00(0.02)	1.88(0.02)	2.14(0.02)
Rose	ESPR	L	1.84(0.02)	1.78(0.02)	1.97(0.02)	2.00(0.02)	1.88(0.02)	2.14(0.02)
Poly	NRR	M	0.71(0.03)	0.89(0.06)	0.93(0.06)	0.95(0.07)	0.95(0.06)	1.05(0.06)
Poly	NRR	O	0.75(0.02)	0.75(0.02)	1.02(0.05)	0.99(0.06)	0.89(0.02)	1.19(0.02)
Poly	NRR	R	0.74(0.02)	1.17(0.36)	0.96(0.03)	0.99(0.05)	1.00(0.06)	1.18(0.04)
Poly	NRR	L	0.75(0.03)	0.84(0.04)	1.03(0.03)	0.94(0.03)	0.94(0.04)	1.08(0.03)
Poly	ENRR	M	0.61(0.01)	0.56(0.01)	0.87(0.01)	0.69(0.01)	0.62(0.01)	0.87(0.01)
Poly	ENRR	O	0.61(0.01)	0.56(0.01)	0.83(0.01)	0.70(0.01)	0.62(0.01)	0.95(0.01)
Poly	ENRR	R	0.61(0.01)	0.56(0.01)	0.87(0.01)	0.69(0.01)	0.62(0.01)	0.93(0.01)
Poly	ENRR	L	0.61(0.01)	0.57(0.01)	0.83(0.01)	0.69(0.01)	0.62(0.01)	0.91(0.01)

Table 8: The means (its standard error) of $100 \times$ integrated squared error of the rose diagrams (denoted by Rose) and the polygon rose diagrams (denoted by Poly) based on the number of repetitions $N=1000$ in Models 1–6 in the simulation in section 4. The sample size is $n = 1000$. In the starting point t_0 , M denotes the sample circular mean, O denotes the opposite, R denotes the right point, and L denotes the left point.

\hat{r}_f	Selector	t_0	Model1	Model2	Model3	Model4	Model5	Model6
Rose	NRR	M	1.04(0.01)	0.97(0.01)	1.22(0.01)	1.20(0.01)	1.14(0.01)	1.35(0.01)
Rose	NRR	O	1.04(0.01)	0.98(0.01)	1.24(0.01)	1.22(0.01)	1.15(0.01)	1.36(0.01)
Rose	NRR	R	1.03(0.01)	0.95(0.01)	1.22(0.01)	1.19(0.01)	1.13(0.01)	1.35(0.01)
Rose	NRR	L	1.03(0.01)	0.96(0.01)	1.23(0.01)	1.20(0.01)	1.12(0.01)	1.35(0.01)
Rose	ENRR	M	1.02(0.01)	0.94(0.01)	1.20(0.01)	1.18(0.01)	1.11(0.01)	1.33(0.01)
Rose	ENRR	O	1.02(0.01)	0.94(0.01)	1.21(0.01)	1.17(0.01)	1.11(0.01)	1.32(0.01)
Rose	ENRR	R	1.01(0.01)	0.94(0.01)	1.20(0.01)	1.17(0.01)	1.12(0.01)	1.32(0.01)
Rose	ENRR	L	1.02(0.01)	0.94(0.01)	1.20(0.01)	1.18(0.01)	1.10(0.01)	1.33(0.01)
Rose	BCV	M	0.86(0.01)	0.79(0.01)	1.05(0.01)	1.03(0.01)	0.95(0.01)	1.17(0.01)
Rose	BCV	O	0.86(0.01)	0.80(0.01)	1.06(0.01)	1.03(0.01)	0.96(0.01)	1.18(0.01)
Rose	BCV	R	0.84(0.01)	0.77(0.01)	1.06(0.01)	1.02(0.01)	0.92(0.01)	1.19(0.01)
Rose	BCV	L	0.85(0.01)	0.78(0.01)	1.06(0.01)	1.01(0.01)	0.94(0.01)	1.17(0.01)
Rose	EBCV	M	0.94(0.01)	0.87(0.01)	1.12(0.01)	1.12(0.01)	1.03(0.01)	1.26(0.01)
Rose	EBCV	O	0.94(0.01)	0.87(0.01)	1.14(0.01)	1.12(0.01)	1.04(0.01)	1.27(0.01)
Rose	EBCV	R	0.93(0.01)	0.86(0.01)	1.14(0.01)	1.11(0.01)	1.04(0.01)	1.27(0.01)
Rose	EBCV	L	0.94(0.01)	0.87(0.01)	1.14(0.01)	1.11(0.01)	1.04(0.01)	1.27(0.01)
Rose	BCR	M	13.33(0.05)	12.98(0.05)	13.47(0.05)	13.94(0.05)	13.7(0.05)	13.87(0.05)
Rose	BCR	O	13.33(0.05)	12.98(0.05)	13.47(0.05)	13.94(0.05)	13.70(0.05)	13.87(0.05)
Rose	BCR	R	13.33(0.05)	12.98(0.05)	13.47(0.05)	13.94(0.05)	13.70(0.05)	13.87(0.05)
Rose	BCR	L	13.33(0.05)	12.98(0.05)	13.47(0.05)	13.94(0.05)	13.70(0.05)	13.87(0.05)
Rose	AR	M	2.07(0.02)	1.89(0.01)	2.15(0.02)	2.37(0.02)	2.23(0.02)	2.33(0.02)
Rose	AR	O	2.08(0.02)	1.90(0.02)	2.14(0.02)	2.40(0.02)	2.24(0.02)	2.35(0.02)
Rose	AR	R	2.08(0.02)	1.87(0.01)	2.15(0.02)	2.38(0.02)	2.24(0.02)	2.34(0.02)
Rose	AR	L	2.07(0.02)	1.87(0.01)	2.15(0.01)	2.37(0.02)	2.24(0.02)	2.35(0.02)
Rose	SRR	M	1.65(0.01)	1.62(0.01)	1.69(0.01)	1.71(0.01)	1.69(0.01)	1.74(0.01)
Rose	SRR	O	1.65(0.01)	1.64(0.01)	1.69(0.01)	1.73(0.01)	1.70(0.01)	1.72(0.01)
Rose	SRR	R	1.65(0.01)	1.63(0.01)	1.69(0.01)	1.71(0.01)	1.69(0.01)	1.72(0.01)
Rose	SRR	L	1.64(0.01)	1.62(0.01)	1.69(0.01)	1.72(0.01)	1.71(0.01)	1.73(0.01)
Rose	SPR	M	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Rose	SPR	O	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Rose	SPR	R	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Rose	SPR	L	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Rose	EAR	M	2.07(0.02)	1.88(0.01)	2.15(0.02)	2.38(0.02)	2.24(0.02)	2.34(0.02)
Rose	EAR	O	2.07(0.02)	1.89(0.02)	2.15(0.02)	2.39(0.02)	2.23(0.02)	2.35(0.02)
Rose	EAR	R	2.06(0.02)	1.89(0.01)	2.14(0.02)	2.37(0.02)	2.23(0.02)	2.36(0.02)
Rose	EAR	L	2.07(0.02)	1.88(0.01)	2.13(0.02)	2.37(0.02)	2.24(0.02)	2.36(0.02)
Rose	ESRR	M	1.64(0.01)	1.63(0.01)	1.69(0.01)	1.72(0.01)	1.69(0.01)	1.72(0.01)
Rose	ESRR	O	1.64(0.01)	1.63(0.01)	1.69(0.01)	1.72(0.01)	1.69(0.01)	1.72(0.01)
Rose	ESRR	R	1.64(0.01)	1.63(0.01)	1.69(0.01)	1.72(0.01)	1.69(0.01)	1.72(0.01)
Rose	ESRR	L	1.64(0.01)	1.63(0.01)	1.69(0.01)	1.72(0.01)	1.69(0.01)	1.72(0.01)
Rose	ESPR	M	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Rose	ESPR	O	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Rose	ESPR	R	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Rose	ESPR	L	1.13(0.01)	1.10(0.01)	1.24(0.01)	1.23(0.01)	1.19(0.01)	1.32(0.01)
Poly	NRR	M	0.40(0.02)	0.80(0.21)	0.62(0.04)	0.47(0.03)	0.65(0.05)	0.53(0.02)
Poly	NRR	O	0.42(0.01)	0.48(0.02)	0.70(0.03)	0.46(0.01)	0.56(0.02)	0.59(0.01)
Poly	NRR	R	0.42(0.02)	0.54(0.03)	0.67(0.03)	0.46(0.01)	0.56(0.02)	0.63(0.02)
Poly	NRR	L	0.43(0.02)	0.55(0.04)	0.67(0.03)	0.45(0.01)	0.61(0.03)	0.59(0.01)
Poly	ENRR	M	0.35(0.01)	0.32(0.01)	0.52(0.01)	0.41(0.01)	0.38(0.01)	0.55(0.01)
Poly	ENRR	O	0.35(0.01)	0.33(0.01)	0.52(0.01)	0.41(0.01)	0.38(0.01)	0.54(0.01)
Poly	ENRR	R	0.34(0.01)	0.33(0.01)	0.49(0.01)	0.40(0.01)	0.37(0.01)	0.54(0.01)
Poly	ENRR	L	0.34(0.01)	0.32(0.01)	0.49(0.01)	0.40(0.01)	0.37(0.01)	0.56(0.01)