Supplement to "A copula spectral test for pairwise time reversibility"

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This supplement contains a guidance of R programs implementing the proposed test, and additional numerical examples that were omitted for reason of space in the paper "A copula spectral test for pairwise time reversibility".

S1 A guidance of R programs

This section provides a guide for using the R programs that implement the proposed approach when applied to the simulation study of the paper.

The proposed test statistic is defined by (15) in the article. The R programs are available at

http://blog.sciencenet.cn/home.php?mod=space&uid=116301&do=blog&quickforward=1&id=1215670. The function `timerev.t` in "t.time.rev.R" is used to compute the observed value of s-tatistic $\mathcal{Z}_{n,M}^{(\tau_1,\tau_2)}$. To use the function, one needs to download the file "t.time.rev.R" and save it into a directory on a computer. It contains various supporting functions. The arguments of the function are listed as follows.

```
timerev.t(x,tau.v,bn,n,M)
```

In the function, `x` is the observed time series, `tau.v` is the probability pair $(\tau_1, \tau_2) \in [0,1]^2$, `bn` is the scaling parameter b_n, `n` is the sample size and `M` is the integer in (8). The other parameters are the same as those in the article.

The R program "experiments.R" is used to compute the empirical size and power for the simulation examples in Section 4. To use it, one should include the R scripts

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“t.time.rev.R” and ”simulation.model.R”. The program “simulation.model.R” is used to generate specific time series models, which are considered in Section 4 of the article.

S2 Additional simulation results

S2.1 Simulation results when using the Dianell kernel

With the same setting as in Section 4.2 but the weight function W replaced by the Dianell kernel, $W(x) = \frac{1}{2\pi}\mathbb{I}_{[-\pi,\pi]}(x)$, we calculated the rejection rate of our proposed test (16) from simulated data for each considered model and each sample size $n = 125, 250, 500, 1000, 2000$. Presented in Table S.1 are empirical rejection probabilities of the test for Models A, B and C, while those for Models A′, B′, D and E are in Table S.2.

Table S.1: Rejection probabilities of the test (16) from simulated data of Models A, B and C.

		n				
Model	(τ_1, τ_2)	125	250	500	1000	2000
A	(0.10,0.50)	0.048	0.045	0.053	0.040	0.055
	(0.10,0.90)	0.063	0.053	0.051	0.046	0.053
	(0.50,0.90)	0.040	0.037	0.040	0.037	0.047
B	(0.10,0.50)	0.036	0.049	0.051	0.058	0.047
	(0.10,0.90)	0.049	0.054	0.049	0.059	0.038
	(0.50,0.90)	0.055	0.041	0.057	0.039	0.053
C	(0.10,0.50)	0.058	0.058	0.061	0.050	0.044
	(0.10,0.90)	0.048	0.055	0.064	0.047	0.040
	(0.50,0.90)	0.048	0.054	0.062	0.058	0.055

S2.2 Simulation results for the RR, CCK, PP and BS tests

For comparison, we set the nominal size $\alpha = 0.05$ and computed the rejection rates of the RR test of Ramsey and Rothman (1996), the CCK test of Chen et al. (2000), the PP test of Paparoditis and Politis (2002) and the BS test of Beare and Seo (2014), from

Table S.2: Rejection probabilities of the test (16) from simulated data of Models A′, B′, D and E.

Model	(τ_1, τ_2)	n 125	250	500	1000	2000
$A'(\nu = 1)$	(0.10,0.50)	0.088	0.123	0.109	0.183	0.248
	(0.10,0.90)	0.090	0.084	0.058	0.066	0.058
	(0.50,0.90)	0.081	0.102	0.140	0.168	0.258
$A'(\nu = 5)$	(0.10,0.50)	0.037	0.059	0.043	0.064	0.072
	(0.10,0.90)	0.057	0.068	0.046	0.057	0.071
	(0.50,0.90)	0.050	0.046	0.037	0.055	0.058
$B'(\nu = 1)$	(0.10,0.50)	0.137	0.157	0.196	0.230	0.296
	(0.10,0.90)	0.284	0.211	0.172	0.133	0.123
	(0.50,0.90)	0.129	0.166	0.205	0.229	0.281
$B'(\nu = 5)$	(0.10,0.50)	0.055	0.048	0.058	0.058	0.050
	(0.10,0.90)	0.057	0.046	0.065	0.063	0.074
	(0.50,0.90)	0.044	0.048	0.045	0.055	0.051
D	(0.10,0.50)	0.055	0.056	0.054	0.089	0.121
	(0.10,0.90)	0.293	0.346	0.402	0.450	0.492
	(0.50,0.90)	0.042	0.054	0.076	0.078	0.117
E	(0.10,0.50)	0.126	0.197	0.271	0.409	0.597
	(0.10,0.90)	0.237	0.315	0.479	0.691	0.885
	(0.50,0.90)	0.139	0.170	0.249	0.371	0.594

simulated data for each sample size $n = 125, 250, 500, 1000, 2000$. The RR, CCK and PP tests rely the choice of lag parameter. For these three tests, we consider four choices of lag parameters, $k = 1, 2, 3, 4$. Since the closed-form of the asymptotic distributions of test statistics are unavailable, we constructed critical values of the PP and BS tests by using the local bootstrap of Paparoditis and Politis (2002). The local bootstrap was implemented using a Gaussian kernel, and the smoothing parameter determined using the data-dependent selection rule (cf. Paparoditis and Politis, 2002, pp. 315). In choosing the smoothing parameter, we fitted auxiliary Gaussian $AR(p)$ models to the observations with order $p = 5$ and 1, for the PP and BS tests, respectively. All results of the RR and

CCK tests are based on 1000 Monte Carlo replications. Note that the local bootstrap of Paparoditis and Politis (2002) is really time-consuming, especially when $n = 1000$ and 2000. To save time, for the PP and BS tests, we employed 200 bootstrap replications in computing each critical value and 200 experimental replications in computing the rejection probabilities.

Since the workability of the RR test relies on the existence of moments up to order 4 (Ramsey and Rothman, 1996), we only present empirical rejection probabilities of the RR test for Models $A'(\nu = 5)$, $B'(\nu = 5)$ and E in Table S.3. From Table S.3, we find that the empirical powers to detect the time irreversibility of Models $A'(\nu = 5)$ and $B'(\nu = 5)$ are relatively smaller, which is consistent to the counterpart simulation results in Table 2. For simulated data from Model E, the RR test performs well if taking a smaller lag parameter k, but performs poor with increasing k.

Table S.3: Rejection probabilities of the RR test of Ramsey and Rothman (1996) from simulated data of Models $A'(\nu = 5)$, $B'(\nu = 5)$ and E.

		n				
Model	k	125	250	500	1000	2000
$A'(\nu = 5)$	1	0.117	0.111	0.137	0.157	0.167
	2	0.044	0.032	0.037	0.033	0.049
	3	0.044	0.054	0.048	0.055	0.058
	4	0.047	0.051	0.043	0.048	0.056
$B'(\nu = 5)$	1	0.046	0.043	0.044	0.050	0.045
	2	0.104	0.102	0.126	0.145	0.160
	3	0.044	0.046	0.041	0.048	0.043
	4	0.080	0.068	0.082	0.085	0.101
E	1	0.961	1.000	1.000	1.000	1.000
	2	0.266	0.400	0.624	0.844	0.977
	3	0.095	0.126	0.163	0.244	0.361
	4	0.059	0.071	0.089	0.102	0.130

The CCK, PP and BS tests do not rely on any moment restriction of the model, so that we present empirical rejection probabilities of these three tests for all considered models (i.e., Models A', B', D and E) in investigating the power, in Tables S.4, S.5 and S.6, respectively. In computing the value of the CCK statistic, we still chose the exponential weight function $g(\omega) = \exp(-\omega)\mathbb{I}_{(0,\infty)}(\omega)$ as in Section 5. From Tables S.4 and S.5, we find that the CCK and PP tests perform poor for all considered models but Model E. For simulated data of Model E, both tests perform well only when taking the lag parameter $k = 1$. From the numerical results of the CCK and PP tests, it can be expectable that the rejection rates of the BS test are lower for Models A′ ($\nu = 5$), B′ ($\nu = 1$ and 5) and D while higher for Model E. But the rejection rates of the BS test is still very high for Model A′ ($\nu = 1$), which is quite out of expectation. One possible reason is that the BS test is specially designed for Markov models, which makes it outperform the other competitive approaches in detecting the time irreversibility of the data drawn from a Morkov model.

References

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Table S.4: Rejection probabilities of the CCK test of Chen et al. (2000) from simulated data of Models A′, B′, D and E.

		n				
Model	k	125	250	500	1000	2000
$A'(\nu = 1)$	1	0.074	0.053	0.058	0.049	0.037
	2	0.059	0.065	0.055	0.045	0.060
	3	0.053	0.044	0.052	0.059	0.051
	4	0.056	0.044	0.057	0.041	0.057
$A'(\nu = 5)$	1	0.059	0.047	0.056	0.054	0.052
	2	0.051	0.049	0.039	0.041	0.045
	3	0.030	0.040	0.043	0.039	0.054
	4	0.030	0.041	0.037	0.036	0.037
$B'(\nu = 1)$	1	0.054	0.050	0.057	0.057	0.051
	2	0.072	0.068	0.053	0.050	0.048
	3	0.056	0.058	0.055	0.043	0.036
	4	0.061	0.058	0.049	0.045	0.058
$B'(\nu = 5)$	1	0.052	0.044	0.048	0.058	0.039
	2	0.054	0.048	0.051	0.040	0.039
	3	0.029	0.044	0.039	0.045	0.049
	4	0.032	0.034	0.034	0.034	0.047
D	1	0.020	0.024	0.020	0.028	0.032
	2	0.051	0.055	0.054	0.045	0.048
	3	0.031	0.022	0.024	0.024	0.028
	4	0.049	0.041	0.051	0.050	0.036
E	1	0.003	0.061	0.638	1.000	1.000
	2	0.000	0.000	0.000	0.000	0.004
	3	0.000	0.000	0.000	0.000	0.000
	4	0.000	0.000	0.000	0.000	0.000

Table S.5: Rejection probabilities of the PP test of Paparoditis and Politis (2002) from simulated data of Models A$'$, B$'$, D and E.

		n				
Model	k	125	250	500	1000	2000
$\mathrm{A}'(\nu = 1)$	1	0.010	0.030	0.025	0.040	0.045
	2	0.050	0.045	0.050	0.050	0.030
	3	0.065	0.040	0.040	0.040	0.045
	4	0.040	0.040	0.090	0.070	0.050
$\mathrm{A}'(\nu = 5)$	1	0.025	0.065	0.070	0.020	0.090
	2	0.050	0.040	0.060	0.040	0.050
	3	0.060	0.055	0.080	0.060	0.030
	4	0.030	0.075	0.030	0.060	0.060
$\mathrm{B}'(\nu = 1)$	1	0.025	0.055	0.015	0.005	0.010
	2	0.035	0.065	0.030	0.070	0.050
	3	0.025	0.040	0.025	0.030	0.010
	4	0.040	0.050	0.075	0.015	0.065
$\mathrm{B}'(\nu = 5)$	1	0.055	0.075	0.040	0.055	0.055
	2	0.055	0.055	0.050	0.040	0.095
	3	0.075	0.075	0.050	0.070	0.060
	4	0.040	0.070	0.065	0.055	0.050
D	1	0.000	0.000	0.000	0.000	0.000
	2	0.085	0.080	0.090	0.100	0.090
	3	0.005	0.005	0.005	0.000	0.000
	4	0.125	0.080	0.105	0.090	0.055
E	1	0.830	0.985	1.000	1.000	1.000
	2	0.090	0.115	0.085	0.160	0.140
	3	0.080	0.060	0.080	0.080	0.110
	4	0.045	0.030	0.070	0.085	0.070

Table S.6: Rejection probabilities of the BS test of Beare and Seo (2014) from simulated data of Models A′, B′, D and E.

Model	n 125	250	500	1000	2000
$\mathrm{A}'(\nu=1)$	0.140	0.335	0.690	0.995	1.000
$\mathrm{A}'(\nu=5)$	0.070	0.095	0.080	0.095	0.100
$\mathrm{B}'(\nu=1)$	0.010	0.005	0.005	0.005	0.005
$\mathrm{B}'(\nu=5)$	0.040	0.045	0.030	0.065	0.060
D	0.005	0.000	0.000	0.000	0.000
E	0.895	1.000	1.000	1.000	1.000