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Tests for the existence of group effects and interactions for two-way models with dependent errors

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Abstract

In this paper, we propose tests for the existence of random effects and interactions for two-way models with dependent errors. We prove that the proposed tests are asymptotically distribution-free which have asymptotically size τ and are consistent. We elucidate the nontrivial power under the local alternative when a sample size tends to infinity and the number of groups is fixed. A simulation study is performed to investigate the finite-sample performance of the proposed tests. In the real data analysis, we apply our tests to the daily log-returns of 24 stock prices from six countries and four sectors. We find that there is no strong evidence to support the existence of substantial differences in the log-return across countries, nor to the existence of interactions between countries and sectors. However, there exists random effect differences in the daily log-return series across different sectors.

Keywords Interaction effects · Multivariate time series · Random effects · Spectral density · Two-way layout

1 Introduction

The importance of incorporating random effects and interactions in modeling has been recognized, especially in the fields of longitudinal data and panel data analysis. A one-way model is one of the most fundamental models to capture group effects for i.i.d. data, see e.g. Searle et al. (1992). The dynamic panel data model pioneered

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by Lillard and Willis (1978), which is a type of one-way models with group (individual) and time effects as well as time-dependent disturbances, has since been popular and extensively developed in theoretical and implementation fields, see Baltagi (2005). Gonçalves (2011) derived the asymptotic distribution of regression coefficients for panel data models with individual effects under cross-sectional dependence. You and Zhou (2013) considered semiparametric panel data partially linear additive models. Besides the individual effects, in many real-world problems, it is common to find the interactive effect as a generalization of additive individual and time effects. Bai and Li (2014) extended the panel data models taking into account the interactive effects. See also Li et al. (2016) and Ke et al. (2016) for more reference of interactive effects study, which are the cases that regression coefficients have structural breaks. Two-way models for i.i.d. data are well-developed as well, e.g., Clarke (2008, Sect. 5) and Akritas and Arnold (2000). Recently, González et al. (2021) proposed two-way models for spatial point processes. In contrast, few studies on two-way models for time series have examined. As an exception, two-way models with seasonal multiplicative ARIMA errors are studied by Sutradhar and MacNeill (1989).

Although several models with group (individual) effects have been investigated, little research has been done on testing problems for the existence of group effects and interactions. Nagahata and Taniguchi (2018) elaborated a test for the existence of the fixed effects in the one-way models in the framework of independent groups. Then Goto et al. (2022a) relaxed the restriction of independence among groups and proposed tests for the existence of fixed or random effects in the one-way models. As a related topic, Akharif et al. (2020) and Fihri et al. (2020) proposed optimal tests for the existence of random coefficients based on the locally asymptotic normality for the random coefficient regression models.

In this paper, we propose tests for the existence of random effects and interactions for two-way models with dependent errors. We allow the groups to be correlated, which makes the classical test based on the sum of squares statistic not asymptotically distribution-free (see Goto et al. 2022a, Sect. 7). Therefore, we propose a statistic naturally extended the classical statistic, and the corresponding test is asymptotically distribution-free for correlated groups. We prove that the proposed tests have asymptotically size τ under the null. Furthermore, we elucidate that the tests are also consistent under the alternative, and the nontrivial power is derived under the local alternative. It is worthy to mention that our tests are flexible which can be applied to models with fixed and mixed effects. A simulation study reveals that the proposed tests deliver good performance in size control under the null and have reasonable power under the alternative. We apply our tests to the daily log-return process of 24 stock prices which are from four sectors in six countries. We found that there is no evidence to support that substantial differences in the log-return among different countries exist, while the tests show that there does exist significant random effect differences in the log-return across various sectors.

The rest of this paper is organized as follows. In Sect. 3, we introduce the two-way models without interaction but with dependent errors, and present the proposed test for the existence of random effects. In Sect. 4, we define two-way models with both interactions and dependent errors and advocate a test for the existence of interactions. Sect. 5 investigates the finite-sample performance of the proposed test with simulation study. Sect. 6 conducts the tests to the log-return of stock prices in six countries and four sectors. In Sect. 7, we discuss the sum of square statistics and disturbances with a parametric spectral density matrix. The details of proofs of our theorems and additional simulation results are provided in supplementary material to this article.

2 Preliminary

2.1 Spectra density

The L^2-based spectral density has been considered as a pivotal index to describe time-dependence structures of stationary time series in the frequency domain. In this section, we recall the definition of the spectral density (matrix). Let $\mathbf{Y}_t = (Y_{t1}, \dots, Y_{tp})^\top$ denote a p-dimensional stationary process with zero mean and the autocovariance function at lag h, denoted by $\mathbf{\Gamma_Y}(h) = E(\mathbf{Y}_t\mathbf{Y}_{t+h}^\top)$, satisfying

$$\sum_{h=-\infty}^{\infty} ||\mathbf{\Gamma_Y}(h)|| < \infty,$$

where $||\mathbf{\Gamma_Y}(h)||$ is the square root of the largest eigenvalue of $\mathbf{\Gamma_Y}(h)\mathbf{\Gamma_Y}(h)^\top$. Then, the $p \times p$ spectral density matrix $\boldsymbol{f}_\mathbf{Y}(\lambda)$ for $\mathbf{Y}_t$ is given based on the Fourier transform (Fourier series) of $\mathbf{\Gamma_Y}(h)$ as

$$\boldsymbol{f}_\mathbf{Y}(\lambda) = \frac{1}{2\pi} \sum_{h=-\infty}^{\infty} \mathbf{\Gamma_Y}(h)e^{-ih\lambda},$$

for frequencies $\lambda \in [-\pi, \pi]$. In specific, each entry $f_{Y_{jk}}(\lambda)$ of $\boldsymbol{f}_\mathbf{Y}(\lambda)$ for $j, k = 1, \dots, p$ is given by

$$f_{Y_{jk}}(\lambda) = \frac{1}{2\pi} \sum_{h=-\infty}^{\infty} E(Y_{tj}Y_{t+h,k})e^{-ih\lambda}.$$

Note that

$$\mathbf{\Gamma_Y}(h) = \int_{-\pi}^{\pi} \boldsymbol{f}_\mathbf{Y}(\lambda)(\lambda)e^{ih\lambda} \text{ with } E(Y_{tj}Y_{t+h,k}) = \int_{-\pi}^{\pi} f_{Y_{jk}}(\lambda)e^{ih\lambda}.$$

Hence, it is obvious that the autospectrum $f_{Y_{jj}}(\lambda)$ measures the linear serial dependency in the frequency domain of component Y_{tj}, with the information of the spectrum $f_{Y_{jj}}(\lambda)$ equivalent to that of autocovariance functions $E(Y_{tj}Y_{t+h,j})$ for all lags h. The cross-spectrum $f_{Y_{jk}}(\lambda)$ represents the linear dependency at all lags and leads between Y_{tj} and Y_{tk}.

A typical example is the spectral density for Gaussian ARMA models of orders (p, q) defined by

$$f_{\mathrm{ARMA}}(\lambda) = \frac{\sigma^2}{2\pi} \frac{|1 + \sum_{j=1}^{q} \theta_j e^{-ij\lambda}|^2}{|1 + \sum_{j=1}^{p} \phi_j e^{-ij\lambda}|^2},$$

where σ, $\phi_1, \dots, \phi_p$, and $\theta_1, \dots, \theta_q$ are parameters, respectively. See e.g. Dette and Paparoditis (2009) and Fiecas and von Sachs (2014) for more examples and implementations of spectral density (matrix). We refer to Von Sachs (2020) and Taniguchi and Kakizawa (2000) for a comprehensive review and fundamental theorems of spectral density (matrix).

3 Two-way models with dependent errors

In this section, we introduce two-way random effects models and propose tests for the existence of random effects. Noted that our theory is also applicable to fixed effects models and mixed effects models (see Remarks 33 – 34).

3.1 Settings

Two-way random effects models with dependent errors are defined as follows:

$$\boldsymbol{y}_{ijt} = \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\beta}_j + \boldsymbol{e}_{ijt}, \qquad i = 1, \dots, a; j = 1, \dots, b; t = 1, \dots, n_{ij}, \tag{1}$$

where $\boldsymbol{y}_{ijt} := (y_{ijt1}, \dots, y_{ijtp})^\top$ is a t-th p-dimensional observation in the (i, j)-th cell, $\boldsymbol{\mu} := (\mu_1, \dots, \mu_p)^\top$ is a grand mean, $\boldsymbol{\alpha}_i := (\alpha_{i1}, \dots, \alpha_{ip})^\top$ and $\boldsymbol{\beta}_j := (\beta_{j1}, \dots, \beta_{jp})^\top$ are random effects of the i-th level of factor A and the j-th level of factor B, respectively, and $\boldsymbol{e}_{ijt} := (e_{ijt1}, \dots, e_{ijtp})^\top$ is a centered stationary sequence. We assume that a finite realization $\{\boldsymbol{y}_{ijt}; i = 1, \dots, a; j = 1, \dots, b; t = 1, \dots, n_{ij}\}$ is available, there exists some constant $\rho_{ij} \in (0, 1)$ such that $n_{ij} = \rho_{ij} n$ with $n = \sum_{i=1}^{a} \sum_{j=1}^{b} n_{ij}$,

$\left(\boldsymbol{e}_{11t}^\top, \boldsymbol{e}_{21t}^\top, \dots, \boldsymbol{e}_{a1t}^\top, \boldsymbol{e}_{12t}^\top, \dots, \boldsymbol{e}_{a2t}^\top, \dots, \boldsymbol{e}_{1bt}^\top, \dots, \boldsymbol{e}_{abt}^\top\right)^\top$ has an abp-by-abp spectral density matrix $\boldsymbol{f}(\lambda) := \{\boldsymbol{f}_{j_1 j_2}(\lambda)\}_{j_1, j_2 = 1, \dots, b}$ with

$\boldsymbol{f}_{j_1 j_2}(\lambda) := \{\boldsymbol{f}_{j_1 j_2}^{i_1 i_2}(\lambda)\}_{i_1, i_2 = 1, \dots, a}$ and

$\boldsymbol{f}_{j_1 j_2}^{i_1 i_2}(\lambda) := \sum_{k \in \mathbb{Z}} \mathbb{E}(\boldsymbol{e}_{i_1 j_1 t+k} \boldsymbol{e}_{i_2 j_2 t}^\top) e^{\mathrm{i}k\lambda} / (2\pi)$, any two of $\{\boldsymbol{\alpha}_i\}$, $\{\boldsymbol{\beta}_j\}$, and $\{\boldsymbol{e}_{ijt}\}$ are independent,

$(\boldsymbol{\alpha}_1^\top, \dots, \boldsymbol{\alpha}_a^\top)^\top$ and $(\boldsymbol{\beta}_1^\top, \dots, \boldsymbol{\beta}_b^\top)^\top$ follow ap- and bp-dimensional centered normal distributions with variances ${}^{\alpha}\boldsymbol{\Sigma} := ({}^{\alpha}\boldsymbol{\Sigma}_{i_1 i_2})_{i_1, i_2 = 1, \dots, a}$ and ${}^{\beta}\boldsymbol{\Sigma} := ({}^{\beta}\boldsymbol{\Sigma}_{j_1 j_2})_{j_1, j_2 = 1, \dots, b}$, respectively, where ${}^{\alpha}\boldsymbol{\Sigma}_{i_1 i_2} := \mathbb{E}(\boldsymbol{\alpha}_{i_1} \boldsymbol{\alpha}_{i_2}^\top)$ and ${}^{\beta}\boldsymbol{\Sigma}_{j_1 j_2} := \mathbb{E}(\boldsymbol{\beta}_{j_1} \boldsymbol{\beta}_{j_2}^\top)$. Note that this setting allows to deal with correlated cells. The case of disturbances with parametric spectral densities is stated in Sect. 7.2. The Gaussian assumption on the random effects is not essential (see Remark 35).

In this paper, we use the following notation: $\overline{\boldsymbol{y}_{ij.}} := \sum_{t=1}^{n_{ij}} \boldsymbol{y}_{ijt} / n_{ij}$, $\overline{\boldsymbol{y}_{.j.}} := \sum_{i=1}^{a} \sum_{t=1}^{n_{ij}} \boldsymbol{y}_{ijt} / (a n_{ij})$, $\overline{\boldsymbol{y}_{i..}} := \sum_{j=1}^{b} \sum_{t=1}^{n_{ij}} \boldsymbol{y}_{ijt} / (b n_{ij})$, and $\overline{\boldsymbol{y}_{...}} := \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{t=1}^{n_{ij}} \boldsymbol{y}_{ijt} / (a b n_{ij})$.

Remark 31 The model defined in (1) seems only applicable to the data which have a single observation in each cell, but also the model can be applicable to the data with multiple observations in each cell as follows: Set p as pq, where q is the number of observations in each cell,

$\boldsymbol{y}_{ijt} := (y_{ijt11}, \ldots y_{ijt1q}, y_{ijt21}, \ldots, y_{ijtp1}, \ldots, y_{ijtpq})^\top$,
$\boldsymbol{\mu} := (\mu_1 \mathbf{1}_q^\top, \ldots, \mu_p \mathbf{1}_q^\top)^\top$, $\boldsymbol{\alpha}_i := (\alpha_{i1}\mathbf{1}_q^\top, \ldots, \alpha_{ip}\mathbf{1}_q^\top)^\top$,
$\boldsymbol{\beta}_j := (\beta_{j1}\mathbf{1}_q^\top, \ldots, \beta_{jp}\mathbf{1}_q^\top)^\top$, and
$\boldsymbol{e}_{ijt} := (e_{ijt11}, \ldots, e_{ijt1q}, e_{ijt21} \ldots, e_{ijtp1}, \ldots, , e_{ijtpq})^\top$, where $\mathbf{1}_q^\top$ is a q-dimensional vector with all elements equal to one. Furthermore, p and q can depend on i.

3.2 Test for the existence of random effects

In this subsection, we develop testing theory for the existence of random effects of the model (1). We consider the following hypothesis:

$$H_{\boldsymbol{\alpha}} \; : \; {}^{\boldsymbol{\alpha}}\boldsymbol{\Sigma} = \boldsymbol{O}_{ap} \quad \text{vs} \quad K_{\boldsymbol{\alpha}} \; : \; {}^{\boldsymbol{\alpha}}\boldsymbol{\Sigma} \neq \boldsymbol{O}_{ap}, \tag{2}$$

where $\boldsymbol{O}_{ap}$ is an ap-by-ap matrix all of whose entries are zero. From the symmetry of the model with respect to factors A and B, we can construct a test and establish theoretical justification for the hypothesis

$$H_{\boldsymbol{\beta}} \; : \; {}^{\boldsymbol{\beta}}\boldsymbol{\Sigma} = \boldsymbol{O}_{bp} \text{ vs } K_{\boldsymbol{\beta}} \; : \; {}^{\boldsymbol{\beta}}\boldsymbol{\Sigma} \neq \boldsymbol{O}_{bp} \tag{3}$$

in the same way.

Let $\hat{\boldsymbol{f}}_n(\lambda) := (\hat{\boldsymbol{f}}_{j_1 j_2}(\lambda))_{j_1, j_2 = 1, \ldots, b}$ be the nonparametric spectral density matrix estimator defined as $\hat{\boldsymbol{f}}_{j_1 j_2}(\lambda) := \{\hat{\boldsymbol{f}}^{i_1 i_2}_{j_1 j_2}(\lambda)\}_{i_1, i_2 = 1, \ldots, a}$, where, for $\lambda \in [-\pi, \pi]$,

$$\hat{\boldsymbol{f}}^{i_1 i_2}_{j_1 j_2}(\lambda) := \frac{1}{2\pi} \sum_{\{h \in \mathbb{Z}; |h| \leq \min\{n_{i_1 j_1}, n_{i_2 j_2}\} - 1\}} \omega\left(\frac{h}{M_n}\right) \hat{\boldsymbol{\Gamma}}^{i_1 i_2}_{j_1 j_2}(h) e^{-\mathrm{i} h \lambda},$$

$\omega(x) = \int_{-\infty}^{\infty} W(t) e^{\mathrm{i}xt}\, \mathrm{d}t$, the function $W(\cdot)$ satisfies (A2) in Assumption 31 below, M_n is a positive sequence such that $M_n \to \infty$ and $M_n / n \to 0$ as $\min\limits_{\substack{i=1,\ldots,a \\ j=1,\ldots,b}} n_{ij} \to \infty$, and $\hat{\boldsymbol{\Gamma}}^{i_1 i_2}_{j_1 j_2}(h)$ is defined, for $h \in \{0, \ldots, \min\{n_{i_1 j_1}, n_{i_2 j_2}\} - 1\}$, as

$$\frac{1}{\min\{n_{i_1 j_1}, n_{i_2 j_2}\} - |h|} \sum_{t=1}^{\min\{n_{i_1 j_1}, n_{i_2 j_2}\} - |h|} (\boldsymbol{y}_{i_1 j_1 t+h} - \overline{\boldsymbol{y}_{i_1 j_1 \cdot}})(\boldsymbol{y}_{i_2 j_2 t} - \overline{\boldsymbol{y}_{i_2 j_2 \cdot}})^\top,$$

and, for $h \in \{-\min\{n_{i_1 j_1}, n_{i_2 j_2}\} + 1, \ldots, 0\}$, as

$$\frac{1}{\min\{n_{i_1 j_1}, n_{i_2 j_2}\} - |h|} \sum_{t=-h+1}^{\min\{n_{i_1 j_1}, n_{i_2 j_2}\}} (\boldsymbol{y}_{i_1 j_1 t+h} - \overline{\boldsymbol{y}_{i_1 j_1 \cdot}})(\boldsymbol{y}_{i_2 j_2 t} - \overline{\boldsymbol{y}_{i_2 j_2 \cdot}})^\top.$$

For notational simplicity, we define the quantities:

$$\zeta^{i_1 i_2}_{j_1 j_2} := \frac{2\pi \min\{\rho_{i_1 j_1}, \rho_{i_2 j_2}\}}{\rho_{i_1 j_1}\rho_{i_2 j_2}} \boldsymbol{f}^{i_1 i_2}_{j_1 j_2}(0), \text{ and}$$
$$\hat{\zeta}^{i_1 i_2}_{j_1 j_2} := \frac{2\pi \min\{\rho_{i_1 j_1}, \rho_{i_2 j_2}\}}{\rho_{i_1 j_1}\rho_{i_2 j_2}} \hat{\boldsymbol{f}}^{i_1 i_2}_{j_1 j_2}(0).$$

A proposed test statistic for (2) is defined as

$$T_{n,\boldsymbol{\alpha}} := \left(\overline{\boldsymbol{y}_{1..}}^\top - \overline{\boldsymbol{y}_{...}}^\top \ \dots \ \overline{\boldsymbol{y}_{a..}}^\top - \overline{\boldsymbol{y}_{...}}^\top\right) \hat{\boldsymbol{V}}^-_{n,\boldsymbol{\alpha}} \left(\overline{\boldsymbol{y}_{1..}}^\top - \overline{\boldsymbol{y}_{...}}^\top \ \dots \ \overline{\boldsymbol{y}_{a..}}^\top - \overline{\boldsymbol{y}_{...}}^\top\right)^\top, \tag{4}$$

where $\hat{\boldsymbol{V}}^-_{n,\boldsymbol{\alpha}}$ is the Moore–Penrose inverse matrix of $\hat{\boldsymbol{V}}_{n,\boldsymbol{\alpha}} := ({}^{\boldsymbol{\alpha}}\hat{\boldsymbol{V}}_{i_1 i_2})_{i_1, i_2 = 1,\dots,a}$,

$${}^{\boldsymbol{\alpha}}\hat{\boldsymbol{V}}_{i_1 i_2} := \hat{\zeta}^{i_1 i_2}_{..} - \hat{\zeta}^{i_1 \cdot}_{..} - \hat{\zeta}^{\cdot i_2}_{..} + \hat{\zeta}^{\cdot\cdot}_{..},$$

and a subscript dot denotes taking the average with respect to the corresponding element, for example, $\hat{\zeta}^{i_1 i_2}_{..} := \sum_{j_1, j_2 = 1}^{b} \hat{\zeta}^{i_1 i_2}_{j_1 j_2} / b^2$, $\hat{\zeta}^{i_1 \cdot}_{..} := \sum_{s=1}^{a} \sum_{j_1, j_2 = 1}^{b} \hat{\zeta}^{i_1, s}_{j_1 j_2} / (ab^2)$, and so on. Since $\left(\overline{\boldsymbol{y}_{1..}}^\top - \overline{\boldsymbol{y}_{...}}^\top, \dots, \overline{\boldsymbol{y}_{a..}}^\top - \overline{\boldsymbol{y}_{...}}^\top\right)^\top$ converges to a centered normal distribution with variance $\boldsymbol{V}_{\boldsymbol{\alpha}}$, defined in Theorem 31 and $\boldsymbol{V}_{\boldsymbol{\alpha}}$ is the function of $\boldsymbol{f}(0)$, the spectral density plays an important role in this paper. Using the Moore–Penrose inverse is essential since $\boldsymbol{V}_{\boldsymbol{\alpha}}$, defined in Theorem 31, is a non-singular matrix. Actually, it can be seen from the fact $\sum_{i_1=1}^{a} {}^{\boldsymbol{\alpha}}\boldsymbol{V}_{i_1 i_2} = \boldsymbol{O}_p$.

To describe the assumptions, we define, for a random variables $\{Y_t\}$, the cumulant of order ℓ of $(Y_1, \dots, Y_\ell)$ as

$$\mathrm{cum}(Y_1, \dots, Y_\ell) := \sum_{(\nu_1, \dots, \nu_p)} (-1)^{p-1}(p-1)! \left(\mathrm{E}\prod_{j \in \nu_1} Y_{\nu_1}\right) \cdots \left(\mathrm{E}\prod_{j \in \nu_p} Y_{\nu_p}\right),$$

where the summation $\sum_{(\nu_1, \dots, \nu_p)}$ extends over all partitions $(\nu_1, \dots, \nu_p)$ of $\{1, 2, \dots, \ell\}$ (see Brillinger 1981, p.19).

We make the following assumptions to construct the test for the hypothesis defined in (2).

Assumption 31 (A1) For all $\ell \in \mathbb{N}$, $(i_1, \dots, i_\ell) \in \{1, \dots, a\}^\ell$, $(j_1, \dots, j_\ell) \in \{1, \dots, b\}^\ell$, and $(d_1, \dots, d_\ell) \in \{1, \dots, p\}^\ell$ it holds that

$$\sum_{s_2, \dots, s_\ell = -\infty}^{\infty} \left\{ \left(1 + \sum_{k=2}^{\ell} |s_k|\right) \left|\kappa^{i_1 \cdots i_\ell}_{j_1 \cdots j_\ell}(s_2, \dots, s_\ell; d_1, \dots, d_\ell)\right| \right\} < \infty,$$

where $\kappa^{i_1 \cdots i_\ell}_{j_1 \cdots j_\ell}(s_2, \dots, s_\ell; d_1, \dots, d_\ell) = \mathrm{cum}\{e_{i_1 j_1 0 d_1}, e_{i_2 j_2 s_2 d_2}, \dots, e_{i_\ell j_\ell s_\ell d_\ell}\}$.

(A2) The function $W(\cdot)$ is a real, bounded, nonnegative and even such that $\int_{-\infty}^{\infty} W(t)\,\mathrm{d}t = 1$ and $\int_{-\infty}^{\infty} W^2(t)\mathrm{d}t < \infty$ with a bounded derivative.

(A3) It holds that $\mathrm{rank}(\hat{\boldsymbol{V}}_{n,\boldsymbol{\alpha}})$ converges in probability to $\mathrm{rank}(\boldsymbol{V}_{\boldsymbol{\alpha}})$ as $\min_{\substack{i=1,\dots,a\\ j=1,\dots,b}} n_{ij} \to \infty$.

Remark 32 (A1) is a natural assumption in the context of time series analysis (see Brillinger (1981, p.26)). The asymptotic normality of $\left(\overline{\boldsymbol{y}_{1\cdot\cdot}}^\top - \overline{\boldsymbol{y}_{\cdot\cdot\cdot}}^\top \ \dots \ \overline{\boldsymbol{y}_{a\cdot\cdot}}^\top - \overline{\boldsymbol{y}_{\cdot\cdot\cdot}}^\top\right)^\top$ can be shown by (A1), which can be relaxed by Theorem 2.1 of Hosoya and Taniguchi (1982). In conjunction with (A2), it can be seen that $\hat{\boldsymbol{f}}_n(\lambda)$ is a consistent estimator of $\boldsymbol{f}(\lambda)$ (see Brillinger (1981, Corollaries 5.6.1 and 5.6.2 and Theorem 5.9.1)). (A3) is a technical assumption: from Koliha (2001, Corollary 1.8), we found that $\hat{\boldsymbol{V}}_{n,\boldsymbol{\alpha}}^-$ converges in probability to $\boldsymbol{V}_{\boldsymbol{\alpha}}^-$, which is defined by replacing $\hat{\boldsymbol{f}}_n(\lambda)$ with $\boldsymbol{f}(\lambda)$ in $\hat{\boldsymbol{V}}_{n,\boldsymbol{\alpha}}^-$, as $\min_{\substack{i=1,\dots,a\\ j=1,\dots,b}} n_{ij} \to \infty$.

We can derive the asymptotic null distribution of $T_{n,\boldsymbol{\alpha}}$ by employing Rao and Mitra (1971, Theorem 9.2.3, p.173) (see Lemma A.1 in Supplementary Material).

Theorem 31 *Suppose Assumption 31. Under $H_{\boldsymbol{\alpha}}$, $T_{n,\boldsymbol{\alpha}}$ converges in distribution to chi-square distribution with $r_{\boldsymbol{\alpha}}$ degrees of freedom as $\min_{\substack{i=1,\dots,a\\ j=1,\dots,b}} n_{ij} \to \infty$, where $r_{\boldsymbol{\alpha}} := \mathrm{rank}(\boldsymbol{V}_{\boldsymbol{\alpha}})$ and $\boldsymbol{V}_{\boldsymbol{\alpha}} := ({}^{\boldsymbol{\alpha}}\boldsymbol{V}_{i_1 i_2})_{i_1,i_2=1,\dots,a}$ with*

$$ {}^{\boldsymbol{\alpha}}\boldsymbol{V}_{i_1 i_2} := \boldsymbol{\zeta}^{i_1 i_2}_{\cdot\cdot} - \boldsymbol{\zeta}^{i_1 \cdot}_{\cdot\cdot} - \boldsymbol{\zeta}^{\cdot i_2}_{\cdot\cdot} + \boldsymbol{\zeta}^{\cdot\cdot}_{\cdot\cdot}. $$

Here a subscript dot denotes taking the average with respect to the corresponding element.

Let τ is a nominal level. From Theorem 31, we can construct an asymptotically size τ test if we reject $H_{\boldsymbol{\alpha}}$ when $T_{n,\boldsymbol{\alpha}} \geq \chi^2_{\hat{r}_{n,\boldsymbol{\alpha}}}[1-\tau]$, where $\hat{r}_{n,\boldsymbol{\alpha}} := \mathrm{rank}(\hat{\boldsymbol{V}}_{n,\boldsymbol{\alpha}})$ and $\chi^2_{\hat{r}_{n,\boldsymbol{\alpha}}}[1-\tau]$ denotes the upper τ-percentiles of the chi-square distribution with $\hat{r}_{n,\boldsymbol{\alpha}}$ degrees of freedom. The next theorem ensures the test has a fundamental property with respect to power.

Theorem 32 *Suppose Assumption 31. Then, the proposed test based on $T_{n,\boldsymbol{\alpha}}$ is consistent, i.e., under $K_{\boldsymbol{\alpha}}$, the power of the test converges to one as*

$$ \min_{\substack{i=1,\dots,a\\ j=1,\dots,b}} n_{ij} \to \infty. $$

The nontrivial power of the test can be derived by considering the local alternative $K_{\boldsymbol{\alpha}}^{(n)}$ defined, for any ap-by-ap symmetric, positive definite matrix ${}^{\boldsymbol{\alpha}}\boldsymbol{H} = ({}^{\boldsymbol{\alpha}}\boldsymbol{H}_{i_1 i_2})_{i_1,i_2=1\dots,a}$ with a p-by-p matrix ${}^{\boldsymbol{\alpha}}\boldsymbol{H}_{i_1 i_2}$, as

$$ K_{\boldsymbol{\alpha}}^{(n)}\ {}^{\boldsymbol{\alpha}}\boldsymbol{\Sigma} := \frac{{}^{\boldsymbol{\alpha}}\boldsymbol{H}}{n}. $$

Theorem 33 *Suppose Assumption 31. Then, the nontrivial power of the proposed test based on $T_{n,\boldsymbol{\alpha}}$ under $K_{\boldsymbol{\alpha}}^{(n)}$ is given by*

$$\mathbb{P}\Big(T_{n,\boldsymbol{\alpha}} \geq \chi^2_{\hat{r}_{n,\boldsymbol{\alpha}}}[1-\tau]\Big) \to \mathbb{P}\Big(\boldsymbol{Z}_{\boldsymbol{\alpha}} \boldsymbol{V}_{\boldsymbol{\alpha}}^{-} \boldsymbol{Z}_{\boldsymbol{\alpha}} \geq \chi^2_{r_{\boldsymbol{\alpha}}}[1-\tau]\Big) \quad \text{as} \min_{\substack{i=1,\ldots,a \\ j=1,\ldots,b}} n_{ij} \to \infty,$$

where $\boldsymbol{Z}_{\boldsymbol{\alpha}}$ follows an ap-dimensional centered normal distribution with variance ${}^{\boldsymbol{\alpha}}\tilde{\boldsymbol{H}} + \boldsymbol{V}_{\boldsymbol{\alpha}}$ with ${}^{\boldsymbol{\alpha}}\tilde{\boldsymbol{H}} = \left({}^{\boldsymbol{\alpha}}\tilde{\boldsymbol{H}}_{i_1 i_2}\right)_{i_1,i_2=1\ldots,a}$ defined as

$${}^{\boldsymbol{\alpha}}\tilde{\boldsymbol{H}}_{i_1 i_2} = {}^{\boldsymbol{\alpha}}\boldsymbol{H}_{i_1 i_2} - \frac{1}{a}\sum_{s=1}^{a}({}^{\boldsymbol{\alpha}}\boldsymbol{H}_{i_1 s} + {}^{\boldsymbol{\alpha}}\boldsymbol{H}_{s i_2}) + \frac{1}{a^2}\sum_{s_1,s_2=1}^{a} {}^{\boldsymbol{\alpha}}\boldsymbol{H}_{s_1 s_2}.$$

Remark 33 For fixed effects models, $\{\boldsymbol{\alpha}_i\}$ and $\{\boldsymbol{\beta}_j\}$ are defined as fixed constants such that $\sum_{i=1}^{a} \boldsymbol{\alpha}_i = \boldsymbol{0}$ and $\sum_{j=1}^{b} \boldsymbol{\beta}_j = \boldsymbol{0}$ and the hypothesis of a test for the existence of fixed effects is defined as

$$\begin{aligned} H_{\boldsymbol{\alpha},\mathrm{fix}} &: \boldsymbol{\alpha}_i = \boldsymbol{0} \text{ for all } i \in \{1,\ldots,a\} \\ &\qquad \text{vs} \quad K_{\boldsymbol{\alpha},\mathrm{fix}} : \boldsymbol{\alpha}_i \neq \boldsymbol{0} \quad \text{for some } i \in \{1,\ldots,a\}. \end{aligned}$$

The restriction is not essential: in the case of $\sum_{i=1}^{a} \boldsymbol{\alpha}_i \neq \boldsymbol{0}$, we reparameterize as $\boldsymbol{\mu}' := \boldsymbol{\mu} + \sum_{i=1}^{a} \boldsymbol{\alpha}_i$ and $\boldsymbol{\alpha}'_i := \boldsymbol{\alpha}_i - \sum_{i=1}^{a} \boldsymbol{\alpha}_i$. The test statistic $T_{n,\boldsymbol{\alpha}}$ can be applied. The asymptotic null distribution is equivalent to Theorem 31. The consistency of the test can be shown in the same manner as Theorem 32. The difference between fixed and random effects models appears in the nontrivial power under the local alternative. For fixed effects models, the local alternative is defined, for perturbations $\{{}^{\boldsymbol{\alpha}}\boldsymbol{h}_i; i = 1,\ldots,a\}$ such that $\sum_{i=1}^{a} {}^{\boldsymbol{\alpha}}\boldsymbol{h}_i = \boldsymbol{0}$, as

$$K_{\boldsymbol{\alpha},\mathrm{fix}}^{(n)} \ \boldsymbol{\alpha}_i := \frac{{}^{\boldsymbol{\alpha}}\boldsymbol{h}_i}{\sqrt{n}}.$$

Rao and Mitra (1971, Theorem 9.2.3, p.173) yields that, under $K_{\boldsymbol{\alpha},\mathrm{fix}}^{(n)}$, $T_{n,\boldsymbol{\alpha}}$ converges in distribution to the noncentral chi-square distribution with $r_{\boldsymbol{\alpha}}$ degrees of freedom and the noncentrality parameter
$\delta_{\alpha} := ({}^{\boldsymbol{\alpha}}\boldsymbol{h}_1^{\top},\ldots,{}^{\boldsymbol{\alpha}}\boldsymbol{h}_a^{\top})\boldsymbol{V}_{\boldsymbol{\alpha}}^{-}({}^{\boldsymbol{\alpha}}\boldsymbol{h}_1^{\top},\ldots,{}^{\boldsymbol{\alpha}}\boldsymbol{h}_a^{\top})^{\top}$ as $\min_{\substack{i=1,\ldots,a \\ j=1,\ldots,b}} n_{ij} \to \infty$. Therefore, the nontrivial power of the test is given by $1 - \Psi_{r_{\boldsymbol{\alpha}},\delta_{\alpha}}(\chi^2_{r_{\boldsymbol{\alpha}}}[1-\tau])$, where $\Psi_{r_{\boldsymbol{\alpha}},\delta_{\alpha}}$ is the cumulative distribution function of the noncentral chi-square distribution with $r_{\boldsymbol{\alpha}}$ degrees of freedom and the noncentrality parameter δ_{α}.

Remark 34 Since $T_{n,\boldsymbol{\alpha}}$ is independent of $\{\boldsymbol{\beta}_j\}$ as well as $T_{n,\boldsymbol{\beta}}$, which is a statistic for the test for the existence of random effects of factor B, is independent of $\{\boldsymbol{\alpha}_i\}$, these tests for fixed and random effects models can be applied to mixed effects models.

Remark 35 The Gaussian assumption on $\boldsymbol{\alpha}_i$ is not essential. Let
$(\boldsymbol{\alpha}_1^{\top},\ldots,\boldsymbol{\alpha}_a^{\top})^{\top}$ follows an *ap*-dimensional centered random vector. The null and alternative hypotheses can be described as

$$H_{\boldsymbol{\alpha},\ \text{non-Gaussian}}\ :\ \mathbb{P}(\boldsymbol{\alpha}_1 = \cdots = \boldsymbol{\alpha}_a) = 1$$

and

$$K_{\boldsymbol{\alpha},\ \text{non-Gaussian}}\ :\ \mathbb{P}(\boldsymbol{\alpha}_1 = \cdots = \boldsymbol{\alpha}_a) = 0.$$

We can show that the asymptotic size of our test is τ and the consistency of the test. Under the local alternative $K_{\boldsymbol{\alpha}}$, defined as

$$K^{(n)}_{\boldsymbol{\alpha},\ \text{non-Gaussian}}\ \boldsymbol{\alpha}_i := \frac{\mathfrak{h}_i}{\sqrt{n}},$$

where $(\mathfrak{h}_1^\top, \dots, \mathfrak{h}_a^\top)^\top$ follows an ap-dimensional centered random vector such that $\mathbb{P}(\mathfrak{h}_i = \mathbf{0}$ for all $i \in \{1, \dots, a\}) = \mathbf{0}$, it holds that, for $\overline{\mathfrak{h}}_. := \sum_{i=1}^a \mathfrak{h}_i/a$, $T_{n,\boldsymbol{\alpha}}$ converges in distribution to $((\mathfrak{h}_1^\top - \overline{\mathfrak{h}}_.^\top, \dots, \mathfrak{h}_a^\top - \overline{\mathfrak{h}}_.^\top) + \mathfrak{z}_{\boldsymbol{\alpha}}^\top)\boldsymbol{V}_{\boldsymbol{\alpha}}^-((\mathfrak{h}_1^\top - \overline{\mathfrak{h}}_.^\top, \dots, \mathfrak{h}_a^\top - \overline{\mathfrak{h}}_.^\top)^\top + \mathfrak{z}_{\boldsymbol{\alpha}})$ as $\min_{\substack{i=1,\dots,a\\ j=1,\dots,b}} n_{ij} \to \infty$, where $\mathfrak{z}_{\boldsymbol{\alpha}}$ follows an ap-dimensional centered normal distribution with variance $\boldsymbol{V}_{\boldsymbol{\alpha}}$.

4 Two-way models with interaction and dependent errors

In the previous section, we proposed the tests for the existence of random effects for the factor A and the factor B. Next, we scrutinize interactions between factors A and B. In this section, we introduce two-way random effects models with interactions and dependent errors, and propose a test for the existence of interactions between factors A and B.

4.1 Settings

Two-way random effects models with interactions and dependent errors are defined as

$$\boldsymbol{y}_{ijt} = \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\beta}_j + \boldsymbol{\gamma}_{ij} + \boldsymbol{e}_{ijt}, \qquad i = 1, \dots, a; j = 1, \dots, b;\ t = 1, \dots, n_{ij}, \tag{5}$$

where $\boldsymbol{\gamma}_{ij} := (\gamma_{ij1}, \dots, \gamma_{ijp})^\top$ is an interaction between the i-th level of factor A and the j-th level of factor B. The difference between the models (1) and (5) is the term $\{\boldsymbol{\gamma}_{ij}\}$. We impose the same assumptions as the model (1) on $\{\boldsymbol{\alpha}_i\}$, $\{\boldsymbol{\beta}_j\}$, and $\{\boldsymbol{e}_{ijt}\}$. Besides, suppose that

$(\boldsymbol{\gamma}_{11}^\top, \boldsymbol{\gamma}_{21}^\top, \dots, \boldsymbol{\gamma}_{a1}^\top, \boldsymbol{\gamma}_{12}^\top, \dots, \boldsymbol{\gamma}_{a2}^\top, \dots, \boldsymbol{\gamma}_{1b}^\top, \dots, \boldsymbol{\gamma}_{ab}^\top)^\top$ follows an abp-dimensional centered normal distribution with variance ${}^{\gamma}\boldsymbol{\Sigma} := ({}^{\gamma}\boldsymbol{\Sigma}_{j_1 j_2})_{j_1, j_2 = 1, \dots, b}$, where ${}^{\gamma}\boldsymbol{\Sigma}_{j_1 j_2} := ({}^{\gamma}\boldsymbol{\Sigma}^{i_1 i_2}_{j_1 j_2})_{i_1, i_2 = 1, \dots, a}$ and ${}^{\gamma}\boldsymbol{\Sigma}^{i_1 i_2}_{j_1 j_2} := \mathbb{E}(\boldsymbol{\gamma}_{i_1 j_1}\boldsymbol{\gamma}_{i_2 j_2}^\top)$, and $\{\boldsymbol{\gamma}_{ij}\}$ is independent of $\{\boldsymbol{\alpha}_i\}$, $\{\boldsymbol{\beta}_j\}$, and $\{\boldsymbol{e}_{ijt}\}$.

4.2 Test for the existence of interaction effects

A test for the existence of interaction effect can be formulate as follows:

$$H_{\boldsymbol{\gamma}} \;:\; {}^{\gamma}\boldsymbol{\Sigma} = \boldsymbol{O}_{abp} \quad \text{vs} \quad K_{\boldsymbol{\gamma}} \;:\; {}^{\gamma}\boldsymbol{\Sigma} \neq \boldsymbol{O}_{abp}. \tag{6}$$

A proposed test statistic for the hypothesis (6) is defined as

$$T_{n,\boldsymbol{\gamma}} := \begin{pmatrix} \overline{\boldsymbol{y}_{11.}} - \overline{\boldsymbol{y}_{1..}} - \overline{\boldsymbol{y}_{.1.}} + \overline{\boldsymbol{y}_{...}} \\ \overline{\boldsymbol{y}_{21.}} - \overline{\boldsymbol{y}_{2..}} - \overline{\boldsymbol{y}_{.1.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{a1.}} - \overline{\boldsymbol{y}_{a..}} - \overline{\boldsymbol{y}_{.1.}} + \overline{\boldsymbol{y}_{...}} \\ \overline{\boldsymbol{y}_{12.}} - \overline{\boldsymbol{y}_{1..}} - \overline{\boldsymbol{y}_{.2.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{a2.}} - \overline{\boldsymbol{y}_{a..}} - \overline{\boldsymbol{y}_{.2.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{1b.}} - \overline{\boldsymbol{y}_{1..}} - \overline{\boldsymbol{y}_{.b.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{ab.}} - \overline{\boldsymbol{y}_{a..}} - \overline{\boldsymbol{y}_{.b.}} + \overline{\boldsymbol{y}_{...}} \end{pmatrix}^{\top} \hat{\boldsymbol{V}}_{n,\boldsymbol{\gamma}}^{-} \begin{pmatrix} \overline{\boldsymbol{y}_{11.}} - \overline{\boldsymbol{y}_{1..}} - \overline{\boldsymbol{y}_{.1.}} + \overline{\boldsymbol{y}_{...}} \\ \overline{\boldsymbol{y}_{21.}} - \overline{\boldsymbol{y}_{2..}} - \overline{\boldsymbol{y}_{.1.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{a1.}} - \overline{\boldsymbol{y}_{a..}} - \overline{\boldsymbol{y}_{.1.}} + \overline{\boldsymbol{y}_{...}} \\ \overline{\boldsymbol{y}_{12.}} - \overline{\boldsymbol{y}_{1..}} - \overline{\boldsymbol{y}_{.2.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{a2.}} - \overline{\boldsymbol{y}_{a..}} - \overline{\boldsymbol{y}_{.2.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{1b.}} - \overline{\boldsymbol{y}_{1..}} - \overline{\boldsymbol{y}_{.b.}} + \overline{\boldsymbol{y}_{...}} \\ \vdots \\ \overline{\boldsymbol{y}_{ab.}} - \overline{\boldsymbol{y}_{a..}} - \overline{\boldsymbol{y}_{.b.}} + \overline{\boldsymbol{y}_{...}} \end{pmatrix}, \tag{7}$$

where $\hat{\boldsymbol{V}}_{n,\boldsymbol{\gamma}}^{-}$ is the Moore–Penrose inverse matrix of $\hat{\boldsymbol{V}}_{n,\boldsymbol{\gamma}} := ({}^{\gamma}\hat{\boldsymbol{V}}_{j_1j_2})_{j_1,j_2=1,\ldots,b}$, ${}^{\gamma}\hat{\boldsymbol{V}}_{j_1j_2} := ({}^{\gamma}\hat{\boldsymbol{V}}_{j_1j_2}^{i_1i_2})_{i_1,i_2=1,\ldots,a}$, ${}^{\gamma}\hat{\boldsymbol{V}}_{j_1j_2}^{i_1i_2} := \sum_{v=1}^{4} \hat{\xi}_v$, $\hat{\xi}_0 := \hat{\zeta}_{j_1j_2}^{i_1i_2}$, $\hat{\xi}_1 := -\hat{\zeta}_{j_1j_2}^{.i_2} - \hat{\zeta}_{j_1j_2}^{i_1.} - \hat{\zeta}_{.j_2}^{i_1i_2} - \hat{\zeta}_{j_1.}^{i_1i_2}$, $\hat{\xi}_2 := \hat{\zeta}_{j_1j_2}^{..} + \hat{\zeta}_{.j_2}^{.i_2} + \hat{\zeta}_{j_1.}^{.i_2} + \hat{\zeta}_{.j_2}^{i_1.} + \hat{\zeta}_{j_1.}^{i_1.} + \hat{\zeta}_{..}^{i_1i_2}$, $\hat{\xi}_3 := -\hat{\zeta}_{..}^{i_1.} - \hat{\zeta}_{..}^{.i_2} - \hat{\zeta}_{j_1.}^{..} - \hat{\zeta}_{.j_2}^{..}$, $\hat{\xi}_4 := \hat{\zeta}_{..}^{..}$, a subscript dot denotes taking the average with respect to the corresponding element. As with $T_{n,\boldsymbol{\alpha}}$, $\boldsymbol{V}_{\boldsymbol{\gamma}}$ is a singular matrix since $\sum_{i_1=1}^{a}\sum_{j_1=1}^{b} {}^{\gamma}\boldsymbol{V}_{j_1j_2}^{i_1i_2} = \boldsymbol{O}_p$, and, thus, the Moor–Prose inverse is necessary.

Assumption 41 In addition to (A1) and (A2) in Assumption 31, it holds that $\mathrm{rank}(\hat{\boldsymbol{V}}_{n,\boldsymbol{\gamma}})$ converges in probability to $\mathrm{rank}(\boldsymbol{V}_{\boldsymbol{\gamma}})$ as $\min_{\substack{i=1,\ldots,a\\ j=1,\ldots,b}} n_{ij} \to \infty$.

Theorem 41 *Suppose Assumption 41. Under $H_{\boldsymbol{\gamma}}$, $T_{n,\boldsymbol{\gamma}}$ converges in distribution to chi-square distribution with $r_{\boldsymbol{\gamma}}$ degrees of freedom as $\min_{\substack{i=1,\ldots,a\\ j=1,\ldots,b}} n_{ij} \to \infty$, where $r_{\boldsymbol{\gamma}} := \mathrm{rank}(\boldsymbol{V}_{\boldsymbol{\gamma}})$ and $\boldsymbol{V}_{\boldsymbol{\gamma}}$ is defined by replacing $\hat{\boldsymbol{f}}_{j_1j_2}^{i_1i_2}(0)$ with $\boldsymbol{f}_{j_1j_2}^{i_1i_2}(0)$ in $\hat{\boldsymbol{V}}_{n,\boldsymbol{\gamma}}$.*

Therefore, we can construct an asymptotically size τ test if we reject $H_{\boldsymbol{\gamma}}$ when $T_{n,\boldsymbol{\gamma}} \geq \chi^2_{\hat{r}_{n,\boldsymbol{\gamma}}}[1-\tau]$, where $\hat{r}_{n,\boldsymbol{\gamma}} := \mathrm{rank}(\hat{\boldsymbol{V}}_{n,\boldsymbol{\gamma}})$. In the same manner as Theorem 32, the consistency of the test is shown.

Theorem 42 *Suppose Assumption 41. Then, the proposed test based on $T_{n,\boldsymbol{\gamma}}$ is consistent, i.e., under $K_{\boldsymbol{\gamma}}$, the power of the test converges to one as*

$$\min_{\substack{i=1,\ldots,a\\ j=1,\ldots,b}} n_{ij} \to \infty.$$

To evaluate the nontrivial power, we consider the following local alternative:

$$K_{\boldsymbol{\gamma}}^{(n)} := \frac{{}^{\gamma}\boldsymbol{H}}{n},$$

where abp-by-abp symmetric, positive definite matrix ${}^{\gamma}\boldsymbol{H} = \left({}^{\gamma}\boldsymbol{H}_{j_1 j_2}\right)_{j_1, j_2=1\ldots,b}$ with a ap-by-ap matrix ${}^{\gamma}\boldsymbol{H}_{j_1 j_2} := ({}^{\gamma}\boldsymbol{H}_{j_1 j_2}^{i_1 i_2})_{i_1, i_2=1,\ldots,a}$.

Theorem 43 *Suppose Assumption 41. Then, the nontrivial power of the proposed test based on $T_{n,\boldsymbol{\gamma}}$ under $K_{\boldsymbol{\gamma}}^{(n)}$ is given by*

$$\mathbb{P}\left(T_{n,\boldsymbol{\gamma}} \geq \chi^2_{\hat{r}_{n,\boldsymbol{\gamma}}}[1-\tau]\right) \to \mathbb{P}\left(\boldsymbol{Z}_{\boldsymbol{\gamma}} \boldsymbol{V}_{\boldsymbol{\gamma}}^{-} \boldsymbol{Z}_{\boldsymbol{\gamma}} \geq \chi^2_{r_{\boldsymbol{\gamma}}}[1-\tau]\right) \text{as} \min_{\substack{i=1,\ldots,a \\ j=1,\ldots,b}} n_{ij} \to \infty,$$

where $\boldsymbol{Z}_{\boldsymbol{\gamma}}$ follows an abp-dimensional centered normal distribution with variance ${}^{\gamma}\tilde{\boldsymbol{H}} + \boldsymbol{V}_{\boldsymbol{\gamma}}$, and ${}^{\gamma}\tilde{\boldsymbol{H}} = ({}^{\gamma}\tilde{\boldsymbol{H}}_{i_1 i_2})_{i_1, i_2=1\ldots,a}$ is defined by replacing $\hat{\boldsymbol{\zeta}}_{j_1 j_2}^{i_1 i_2}$ with ${}^{\gamma}\boldsymbol{H}_{j_1 j_2}^{i_1 i_2}$ in $\hat{\boldsymbol{V}}_{n,\boldsymbol{\gamma}}$.

Remark 41 When the interaction $\{\boldsymbol{\gamma}_{ij}\}$ is a non-random constant such that $\sum_{i=1}^{a} \sum_{j=1}^{b} \boldsymbol{\gamma}_{ij} = \boldsymbol{0}$, and a hypothesis for a test for the existence of interactions is defined as

$$\begin{aligned} H_{\boldsymbol{\gamma},\mathrm{fix}} &: \ \boldsymbol{\gamma}_{ij} = \boldsymbol{0} \text{ for all } (i,j) \in \{1,\ldots,a\} \times \{1,\ldots,b\} \\ \text{vs} \quad K_{\boldsymbol{\gamma},\mathrm{fix}} &: \ \boldsymbol{\gamma}_{ij} \neq \boldsymbol{0} \text{ for some } (i,j). \end{aligned}$$

The restriction $\sum_{i=1}^{a} \sum_{j=1}^{b} \boldsymbol{\gamma}_{ij} = \boldsymbol{0}$ is not essential: in the case of $\sum_{i=1}^{a} \sum_{j=1}^{b} \boldsymbol{\gamma}_{ij} \neq \boldsymbol{0}$, we reparameterize as $\boldsymbol{\mu}' := \boldsymbol{\mu} + \sum_{i=1}^{a} \sum_{j=1}^{b} \boldsymbol{\gamma}_{ij}$ and $\boldsymbol{\gamma}'_{ij} := \boldsymbol{\gamma}_{ij} - \sum_{i=1}^{a} \sum_{j=1}^{b} \boldsymbol{\gamma}_{ij}$. We can utilize the test statistic $T_{n,\boldsymbol{\gamma}}$. The asymptotic null distribution is equivalent to Theorem 41. The consistency of the test can be shown in the same manner as Theorem 42. The difference between fixed and random interactions appears in the nontrivial power under the local alternative. For fixed interactions, the local alternative is defined, for perturbations $\{{}^{\gamma}\boldsymbol{h}_{ij}; i = 1,\ldots,a, j = 1,\ldots,b\}$ such that $\sum_{i=1}^{a} \sum_{j=1}^{b} {}^{\gamma}\boldsymbol{h}_{ij} = \boldsymbol{0}$ as

$$K_{\boldsymbol{\gamma},\mathrm{fix}}^{(n)} \ \boldsymbol{\gamma}_{ij} := \frac{{}^{\gamma}\boldsymbol{h}_{ij}}{\sqrt{n}}.$$

Rao and Mitra (1971, Theorem 9.2.3, p.173) yields that, under $K_{\boldsymbol{\gamma},\mathrm{fix}}^{(n)}$, $T_{n,\boldsymbol{\gamma}}$ converges in distribution to the noncentral chi-square distribution with $r_{\boldsymbol{\gamma}}$ degrees of freedom and the noncentrality parameter

$$\delta_\gamma := \begin{pmatrix} \overline{{}^\gamma\boldsymbol{h}_{11}} - \overline{{}^\gamma\boldsymbol{h}_{1.}} - \overline{{}^\gamma\boldsymbol{h}_{.1}} \\ \overline{{}^\gamma\boldsymbol{h}_{21}} - \overline{{}^\gamma\boldsymbol{h}_{2.}} - \overline{{}^\gamma\boldsymbol{h}_{.1}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{a1}} - \overline{{}^\gamma\boldsymbol{h}_{a.}} - \overline{{}^\gamma\boldsymbol{h}_{.1}} \\ \overline{{}^\gamma\boldsymbol{h}_{12}} - \overline{{}^\gamma\boldsymbol{h}_{1.}} - \overline{{}^\gamma\boldsymbol{h}_{.2}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{a2}} - \overline{{}^\gamma\boldsymbol{h}_{a.}} - \overline{{}^\gamma\boldsymbol{h}_{.2}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{1b}} - \overline{{}^\gamma\boldsymbol{h}_{1.}} - \overline{{}^\gamma\boldsymbol{h}_{.b}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{ab}} - \overline{{}^\gamma\boldsymbol{h}_{a.}} - \overline{{}^\gamma\boldsymbol{h}_{.b}} \end{pmatrix}^\top \boldsymbol{V}_\gamma^- \begin{pmatrix} \overline{{}^\gamma\boldsymbol{h}_{11}} - \overline{{}^\gamma\boldsymbol{h}_{1.}} - \overline{{}^\gamma\boldsymbol{h}_{.1}} \\ \overline{{}^\gamma\boldsymbol{h}_{21}} - \overline{{}^\gamma\boldsymbol{h}_{2.}} - \overline{{}^\gamma\boldsymbol{h}_{.1}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{a1}} - \overline{{}^\gamma\boldsymbol{h}_{a.}} - \overline{{}^\gamma\boldsymbol{h}_{.1}} \\ \overline{{}^\gamma\boldsymbol{h}_{12}} - \overline{{}^\gamma\boldsymbol{h}_{1.}} - \overline{{}^\gamma\boldsymbol{h}_{.2}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{a2}} - \overline{{}^\gamma\boldsymbol{h}_{a.}} - \overline{{}^\gamma\boldsymbol{h}_{.2}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{1b}} - \overline{{}^\gamma\boldsymbol{h}_{1.}} - \overline{{}^\gamma\boldsymbol{h}_{.b}} \\ \vdots \\ \overline{{}^\gamma\boldsymbol{h}_{ab}} - \overline{{}^\gamma\boldsymbol{h}_{a.}} - \overline{{}^\gamma\boldsymbol{h}_{.b}} \end{pmatrix},$$

where $\overline{{}^\gamma\boldsymbol{h}_{.j}} := \sum_{i=1}^a {}^\gamma\boldsymbol{h}_{ij}/a$ and $\overline{{}^\gamma\boldsymbol{h}_{i.}} := \sum_{j=1}^b {}^\gamma\boldsymbol{h}_{ij}/b$ as $\min_{\substack{i=1,\dots,a \\ j=1,\dots,b}} n_{ij} \to \infty$. Therefore, the nontrivial power of the test is given by $1 - \Psi_{r_\gamma,\delta_\gamma}(\chi^2_{r_\gamma}[1-\tau])$, where $\Psi_{r_\gamma,\delta_\gamma}$ is the cumulative distribution function of the noncentral chi-square distribution with r_γ degrees of freedom and the noncentrality parameter δ_γ.

Remark 42 Since $T_{n,\gamma}$ does not depend on $\{\boldsymbol{\beta}_j\}$ and $\{\boldsymbol{\alpha}_i\}$, our results valid for fixed and mixed effects models with interactions.

Remark 43 The Gaussian assumption of interactions can be relaxed in the same manner as Remark 35.

5 Numerical study

In this section, we study the finite-sample performance of proposed tests to the existence of random effects and interaction effects in the two-way models with dependent errors. In specific, we consider two scenarios: test the random effects to the two-way models without interaction, and test interaction in the two-way models with interaction. We also investigate the tests' empirical size and power performance under different type of dependent errors.

Firstly, we generate data from model (1) and consider that $(a, b, p) = (3, 2, 1)$, sample size $n \in \{250, 500, 1000, 2000\}$, the number of iterations $R = 1000$, a significance level $\tau := 0.05$, and random effects $(\alpha_1, \alpha_2, \alpha_3)^\top$ follow a normal distribution with mean 0 and variance ${}^{\alpha}\boldsymbol{\Sigma}$ of the form ${}^{\alpha}\boldsymbol{\Sigma} = \sigma_{\boldsymbol{\alpha}}^2 I_3$ for $\sigma_{\boldsymbol{\alpha}} \in \{0, 0.1, 0.2, 0.3\}$. The case $\sigma_{\boldsymbol{\alpha}} = 0$ corresponds to the null. We set the innovation time series (dependent error) $\boldsymbol{e}_t := (e_{11t}, e_{21t}, e_{31t}, e_{12t}, , e_{22t}, e_{32t})^\top$ follows vector AR(1) models $\boldsymbol{e}_t := \Phi \boldsymbol{e}_{t-1} + \boldsymbol{\epsilon}_t$ or MA(1) models $\boldsymbol{e}_t := \boldsymbol{\epsilon}_t + \Phi \boldsymbol{\epsilon}_{t-1}$, where $\boldsymbol{\epsilon}_t := (\epsilon_{11t}, \epsilon_{21t}, \epsilon_{31t}, \epsilon_{12t}, \epsilon_{22t}, \epsilon_{32t})^\top$ are i.i.d. white noise. Here we consider two distributions for noise $\boldsymbol{\epsilon}_t$: centered normal distribution with unit variance, and the centered t-distribution with 5 degrees of freedom and unit variance. For both cases, the coefficient matrix Φ is defined as

$$\Phi := \begin{pmatrix} 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0 & c_2 & 0 & 0 & 0.5 & 0 \\ c_1 & 0 & 0.2 & 0 & 0 & 0.7 \end{pmatrix}$$

for $c_1, c_2 \in \{-0.4, -0.1, 0, 0.1, 0.4\}$. Since $T_{n,\alpha}$ does not depend on $\{\boldsymbol{\beta}_j\}$, so we need not consider $\{\boldsymbol{\beta}_j\}$. We use the Tukey–Hanning window function and a bandwidth $M_n := 3n^{1/5}$ which minimizes the mean squared error (see Hannan (1970, p.285–286). The element c_1 in Φ provides the inter-group correlation between the coordinate (1,1)-cell and the (3,2)-cell, and c_2 gives within-group correlation with respect to the factor A between the (2,1)-cell and the (2,2)-cell. The empirical size and power are computed by $\sum_{i=1}^{R} I\{T_{n,\alpha}^{(i)} \geq \chi^2_{\hat{r}_{n,\alpha}}[1-\tau]\}/R$, where $T_{n,\alpha}^{(i)}$ denotes the value of our statistic in the i-th iteration and I is an indicator function.

Figure 1 displays the empirical size and power for the test of existence of random effects defined in (2). We can see from the top panel that, the test for model with AR(1)-type innovation approaches to nominal level 0.05 when $n = 2000$, while test for the model with MA(1) error reaches the nominal level faster when n is as large as 1000 and the performance is more stable than AR(1). Besides, regarding to the distributions of $\boldsymbol{e}_t$, our test shows that there is little effect to the test performance on the distributions of disturbances under the null. On the other hand, under the alternative, our test shows that there is better power when $\boldsymbol{e}_t$ follows the normal distribution than that when $\boldsymbol{e}_t$ follows the t-distribution. As for $\sigma_{\boldsymbol{\alpha}}$, it can be seen from the plots in second to bottom panel that the power of our test becomes high when $\sigma_{\boldsymbol{\alpha}}$ becomes larger. The results for other parameters are provided in Supplementary material for space limitation.

Next, we generate data from model (5) and take interactions term $(\gamma_{11}, \gamma_{21}, \gamma_{31}, \gamma_{12}, \gamma_{22}, \gamma_{32}, \gamma_{13}, \gamma_{23}, \gamma_{33})^\top$ to follow a normal distribution with mean 0 and variance ${}^{\gamma}\boldsymbol{\Sigma}$ of the form ${}^{\gamma}\boldsymbol{\Sigma} = \sigma_\gamma^2 I_6$ for $\sigma_\gamma \in \{0, 0.1, 0.2, 0.3\}$. Other settings are the same as above. The case $\sigma_{\boldsymbol{\gamma}} = 0$ corresponds to the null. The empirical size and power are computed by $\sum_{i=1}^{R} I\{T_{n,\boldsymbol{\gamma}}^{(i)} \geq \chi^2_{\hat{r}_{n,\boldsymbol{\gamma}}}[1-\tau]\}/R$, where $T_{n,\boldsymbol{\gamma}}^{(i)}$ denotes the value of our statistic in the i-th iteration.

Figure 2 shows the empirical size and power of the tests for ${}^{\gamma}\boldsymbol{\Sigma}$. It can be seen that the results are similar to the cases above. It is interesting to observe that the size control of the test for interactions is better than that of the test for the existence of random effects, although the test statistic (7) is more complex than (4). The results for other parameters are deferred to Supplementary material for space limitation.

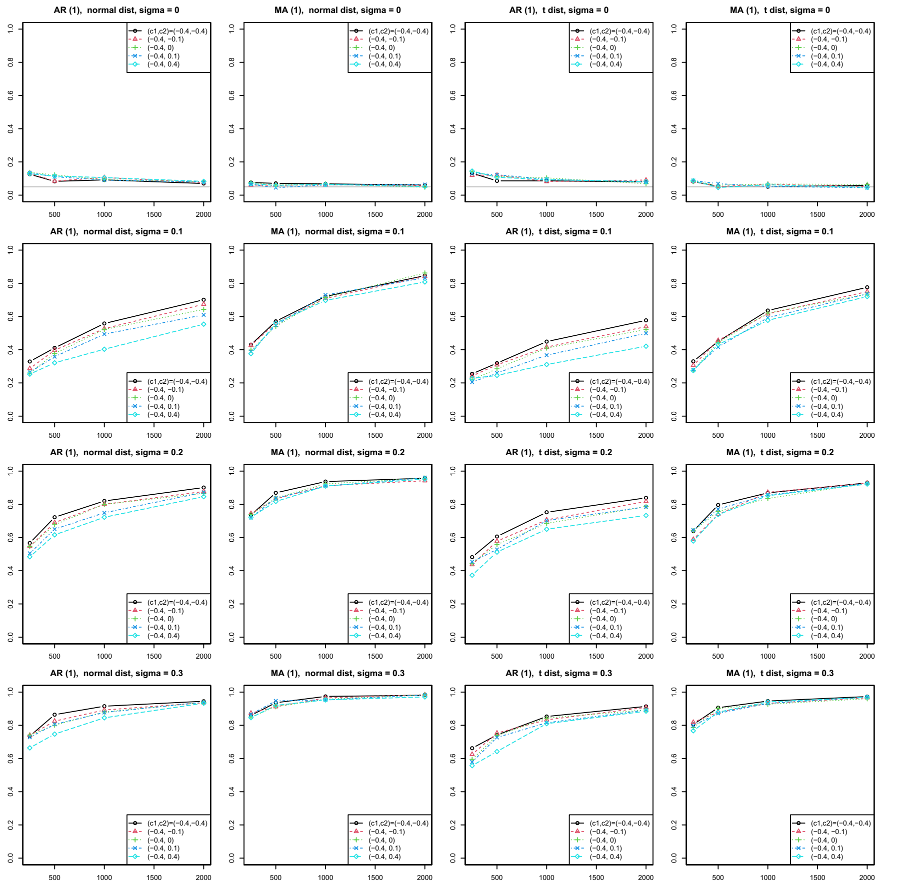

Fig. 1 Empirical size (the top panel) and power (the second to bottom panel) for the hypothesis defined in (2). The vertical and horizontal axes of each plot correspond to the average rejection probabilities over 1000 iterations and different sample size, respectively. Each panel corresponds to results for $\sigma_\alpha = 0, 0.1, 0.2, 0.3$ from top to bottom, respectively. The first panel refers to the null ($\sigma_\alpha = 0$), and the other panels correspond to the alternative ($\sigma_\alpha > 0$). The first and third columns correspond to results for AR(1) models whose disturbances follow the normal distribution and the t-distribution, respectively. The second and four columns correspond to results for MA(1) models whose disturbances follow the normal distribution and the t-distribution, respectively

6 Real data analysis

In this section, we apply the proposed statistics to the daily log-return process of 24 stock prices from January 3, 2017 to December 31, 2019, which belong to six countries (Australia, Canada, China, France, Germany, USA) and four sectors (Information Technology, Industrial, Financial, Communication Service), respectively. Each stock price data covers 733 observations. The data are collected from the website

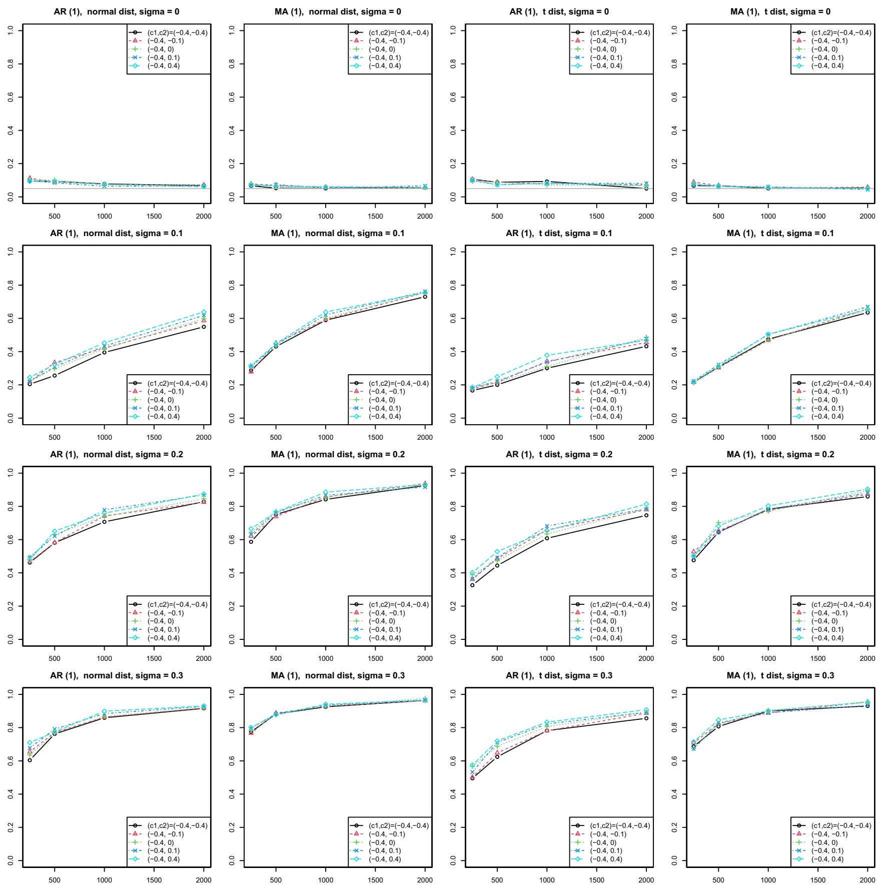

Fig. 2 Empirical size (the top panel) and power (the second to bottom panel) for the hypothesis defined in (6). The vertical and horizontal axes of each plot correspond to the average rejection probabilities over 1000 iterations and different sample size, respectively. Each panel corresponds to results for $\sigma_\gamma = 0, 0.1, 0.2, 0.3$ from top to bottom, respectively. The first panel corresponds to the null ($\sigma_\gamma = 0$), and the other panels refer to the alternative ($\sigma_\gamma > 0$). The first and third columns correspond to results for AR(1) models whose disturbances follow the normal distribution and the t-distribution, respectively. The second and four columns correspond to results for MA(1) models whose disturbances follow the normal distribution and the t-distribution, respectively

https://www.investing.com. The corresponding 24 company names for stock prices are reported in Table 1.

Figure 3 displays the time series plots of daily stock prices where each plot shows the raw data of price in four sectors for each country. It is noted that the original daily stock prices data are collected with different currencies, for example, the stock prices of American companies are shown in U.S. dollar, while that of a French company is shown in Euro. We apply our tests to the daily log-return series where the currency has no effect.

Table 1 company names

Sector \ Country	Australia	Canada
Information Technology	Computershare	CGI Inc
Industrial	Brambles	CNR
Financial	National Australia Bank	Bank of Montreal
Communication Service	CNU	BCE Inc
Sector \ Country	China	France
Information Technology	Iflytek Co Ltd	Capgemini
Industrial	CK Hutchison Holdings	Bouygues
Financial	Industrial and Commercial Bank of China	BNP Paribas
Communication Service	Tencent Holdings	Vivendi
Sector \ Country	Germany	USA
Information Technology	SAP SE	Apple Inc
Industrial	Deutsche Post	3M Company
Financial	Deutsche Bank	JPMorgan Chase & Co
Communication Service	Deutsche Telekom	Facebook, Inc

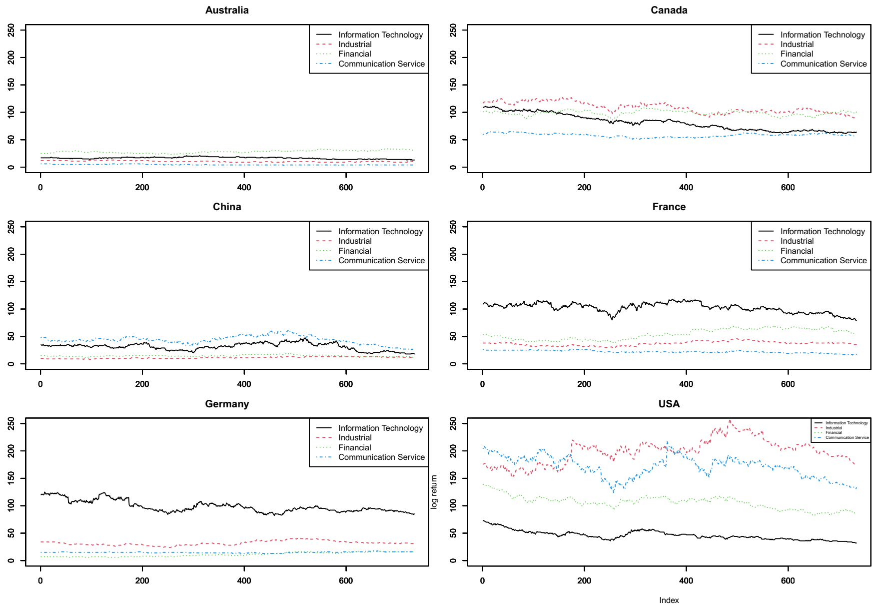

Fig. 3 Time series plots of daily stock prices from January 3, 2017 to December 31, 2019 by country

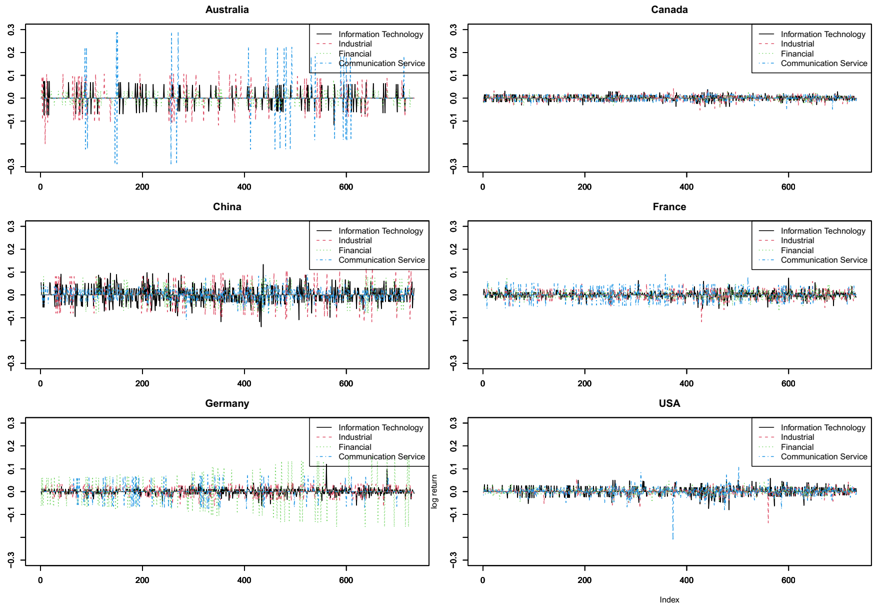

Fig. 4 Time series plots of the daily log-returns from January 3, 2017 to December 31 by country

We consider the log-returns of 4 sectors in each country, or the log-returns of 6 countries that belong to same sector, as one group. Figure 4 displays the time series plots of the log-return by country. It shows that there are sharp fluctuations in the series for Australia, China, and Germany, while fluctuations for Canada France, and USA are blunt. Figure 5 shows the heatmap of correlation matrix for 24 daily log-returns process. It shows that there exists within-group correlations in America and France. Meanwhile, there exhibits inter-group correlations between China and USA. There are small but negative correlations between log-returns in France and USA.

Our aim is to test the existence of random effects and interactions with respect to different countries or sectors. We denote by s_{ijt} the stock prices, where i, j, and t correspond to sector, country, and time, respectively. We consider the daily log-return of stock prices defined as $y_{ijt} := \log(s_{ijt}/s_{ij(t-1)})$.

We start from the simpler model without interaction and focus on the tests for the existence of random effects of country and sectors. Assume that y_{ijt} follows the two-way model (1) with $p = 1$,

$$y_{ijt} = \mu + \alpha_i + \beta_j + e_{ijt}, \qquad i = 1, \dots, 4; j = 1, \dots, 6; t = 1, \dots, 733.$$

The random effects are measured $\{\alpha_i\}$ and $\{\beta_j\}$ for sectors and countries, which correspond to the hypotheses (2) and (3), respectively. We computed that $T_{n,\alpha} = 7.43$ and $T_{n,\beta} = 4.47$. The corresponding p-values are 0.0592 and 0.484, respectively. Hence, we cannot reject the hypothesis (3) for $\{\beta_j\}$, indicating that there is no

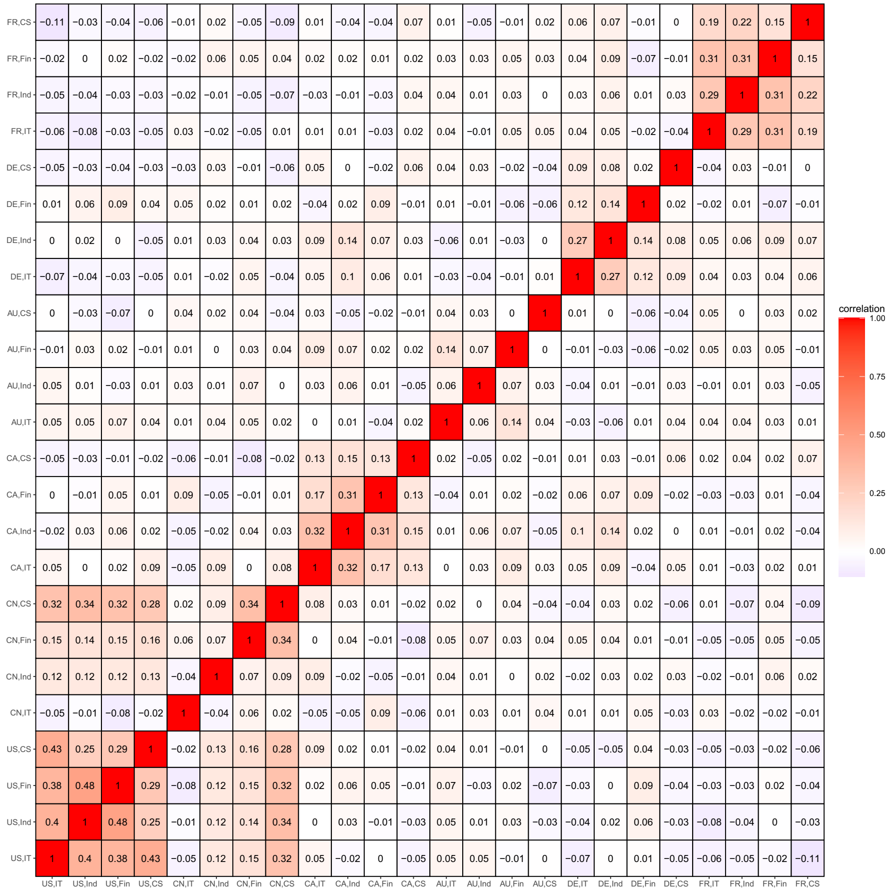

Fig. 5 Heatmap of correlation matrix of log-returns. Here, AU, CN, DE, CA, FR, and US are abbreviation for Australia, China, Germany, Canada, France, and USA, respectively. IT, Ind, Fin, and CS, are abbreviated for Information Technology, Industrial, Financial, and Communication Service, respectively. For example, "US, IT" denotes the American company for Information Technology, i.e., Apple Inc. Each square in the heatmap represents the correlation of the log-returns between the companies corresponding to the vertical and horizontal axes

significant random effects of country. On the other hand, it is reasonable to think there exists random effects $\{\alpha_i\}$ of sectors.

Next we test the existence of interaction between sectors and countries of the log-return series of the 24 companies. Assume that y_{ijt} follows the two-way model (5) with $p = 1$,

$$y_{ijt} = \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ijt}, \qquad i = 1, \dots, 4; j = 1, \dots, 6; t = 1, \dots, 733,$$

where the quantities α_i and β_j refer to random effects of sector and country, respectively, and γ_{ij} represents interactions between sectors and countries. The testing of interaction effect corresponds to the hypothesis (6). We apply the proposed test to log-return processes and obtain $T_{n,\gamma} = 16.5$ with p-value 0.352. Therefore, we cannot reject the null hypotheses (6) and, thus, there is no significant evidence for the existence of interactions between countries and sectors of these 24 companies.

In summary, there is no strong evidence to support that there exist substantial differences in the 24 daily log-returns across countries, namely country specific factor does not affect corporate profits in each country significantly. In addition, there is also no convincing evidence for the existence of interactions between countries and sectors. However, we find that there does exist random effect differences in the daily log-return series across sectors, which refers that each sector may have its unique set of factors that influence corporate earnings.

7 Discussion

7.1 Sum of squares

In the context of analysis of variance (ANOVA), the sum of squares are commonly used, provided the independence of the cells. For example, we shall consider (5) with $p = 1$. Suppose that, for any i and j, α_i, β_j, and γ_{ij} follow centered normal distributions with variances σ_α^2, σ_β^2, and σ_γ^2, respectively, Suppose that $\{e_{ijt}\}$ be a stationary Gaussian process with a spectral density $f_e(\lambda)$, any two of $\{\alpha_i\}$, $\{\beta_j\}$, $\{\gamma_{ij}\}$, and $\{e_{ijt}\}$ are independent, $\mathbb{E}(\alpha_{i_1}\alpha_{i_2}) = 0$ for any $i_1, i_2 (\neq i_1)$, $\mathbb{E}(\beta_{j_1}\beta_{j_2}) = 0$ for any $j_1, j_2 (\neq j_1)$, $\mathbb{E}(\gamma_{i_1 j_1}\gamma_{i_2 j_2}) = 0$ for any $(i_1, j_1), (i_2, j_2) (\neq (i_1, j_1))$, $\{e_{i_1 j_1 t}\}$ and $\{e_{i_2 j_2 t}\}$ are of independent for any $(i_1, j_1), (i_2, j_2) (\neq (i_1, j_1))$, and each group has a time series of equal length.

Under this setup, our hypotheses reduce to $H_\alpha : \sigma_\alpha^2 = 0$ vs $K_\alpha : \sigma_\alpha^2 \neq 0$ and $H_\gamma : \sigma_\gamma^2 = 0$ vs $K_\gamma : \sigma_\gamma^2 \neq 0$. The sum of squares statistics are defined as $S_\alpha := nb \sum_{i=1}^{a} (\bar{y}_{i..} - \bar{y}_{...})^2$ and $S_\gamma := n \sum_{i=1}^{a} \sum_{j=1}^{b} (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2$. After lengthy calculations, we can see that $S_\alpha = \boldsymbol{Y}'\boldsymbol{P}_\alpha \boldsymbol{Y}$ and $S_\gamma = \boldsymbol{Y}'\boldsymbol{P}_\gamma \boldsymbol{Y}$, where $\boldsymbol{Y} := (\, Y_{111} \; Y_{211} \cdots Y_{a11} \; Y_{121} \; Y_{221} \cdots Y_{a21} \cdots Y_{1b1} \cdots Y_{ab1} \; Y_{112} \cdots Y_{abn}\,)$, $\boldsymbol{P}_\alpha$ and $\boldsymbol{P}_\gamma$ projection matrices defined as $\boldsymbol{P}_\alpha := \mathbf{1}_{nb}\mathbf{1}'_{nb} \otimes (\boldsymbol{I}_a - \frac{1}{a}\mathbf{1}_a\mathbf{1}'_a)/(nb)$ and $\boldsymbol{P}_\gamma := \mathbf{1}_n\mathbf{1}'_n \otimes \boldsymbol{B}'_b\boldsymbol{B}_b \otimes \boldsymbol{B}'_a\boldsymbol{B}_a/n$, respectively, and B_n is an $(n-1)$-by-n matrix defined as

$$B_n := \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \dots & \dots & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \frac{1}{\sqrt{(n-1)(n-2)}} & \frac{1}{\sqrt{(n-1)(n-2)}} & \frac{1}{\sqrt{(n-1)(n-2)}} & \dots & -\sqrt{\frac{n-2}{n-1}} & 0 \\ \frac{1}{\sqrt{n(n-1)}} & \frac{1}{\sqrt{n(n-1)}} & \frac{1}{\sqrt{n(n-1)}} & \dots & \frac{1}{\sqrt{n(n-1)}} & -\sqrt{\frac{n-1}{n}} \end{pmatrix}.$$

The derivation of the projection matrices can be seen in Clarke (2008, Section 5). From Theorems S2 and S3 (Searle et al.1992, p.467), we can see that

$$S_\alpha \sim \left(nb\sigma_\alpha^2 + n\sigma_\gamma^2 + \Sigma_{|h|<n}\left(1 - \frac{|h|}{n}\right)\gamma_e(h)\right)\chi^2_{a-1},$$
$$S_\gamma \sim \left(n\sigma_\gamma^2 + \Sigma_{|h|<n}\left(1 - \frac{|h|}{n}\right)\gamma_e(h)\right)\chi^2_{(a-1)(b-1)},$$

and S_α and S_γ are independent, where $\gamma_e(h) := \mathbb{E}(e_{ijt+h}e_{ijt})$. Therefore, test statistics can be defined as $\mathcal{T}_\alpha := (b-1)\mathrm{S}_\alpha/\mathrm{S}_\gamma$, $\mathcal{T}_\gamma := S_\gamma/(2\pi\hat{f}_e(0))$, where $\hat{f}_e(0)$ is the kernel estimator of $f_e(0)$.

Under H_α, we have T_α follows $F^{a-1}_{(a-1)(b-1)}$. Then, we have a non-asymptotic size τ test if we reject H_α whenever $\mathcal{T}_\alpha \geq F^{a-1}_{(a-1)(b-1)}(\tau)$. Under K_α, the non-asymptotic power of the test can be derived as

$$\begin{aligned}
&\mathbb{P}\left(T_\alpha \geq F^{a-1}_{(a-1)(b-1)}(\tau)\right)\\
&= \mathbb{P}\left(\left(1 + \frac{b\sigma_\alpha^2}{\sigma_\gamma^2 + \frac{1}{n}\Sigma_{|h|<n}\left(1 - \frac{|h|}{n}\right)\gamma_e(h)}\right)F^{a-1}_{(a-1)(b-1)} \geq F^{a-1}_{(a-1)(b-1)}(\tau)\right).
\end{aligned}$$

Note that the larger σ_α^2 is, the closer the power of T_α is to one. If we consider two-way random effects models without interactions, the term σ_γ^2 is vanished, and, thus, we can construct a chi-squared test.

Under H_γ, it holds that $T_\gamma \Rightarrow \chi^2_{(a-1)(b-1)}$ as $n \to \infty$. Thus, we have an asymptotic size τ test if we reject H_γ whenever $\mathcal{T}_\gamma \geq \chi^2_{(a-1)(b-1)}(\tau)$. Moreover, we can easily show the consistency of the test under the alternative and the nontrivial power under the local alternative.

As mentioned above, the sum of squares statistics can be used under the independence of the cells. Since tests based the sum of squares statistics are not asymptotically distribution-free for correlated cells, we need to apply, e.g., bootstrap methods to calculate critical values.

7.2 Parametric spectral density matrix

When we assume a parametric spectral density matrix for disturbances, the unknown parameters can be estimated by the minimum discrepancy principle. Let $\{\boldsymbol{f}_{\boldsymbol{\theta}}(\lambda);\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^q\}$ be a parametric family of spectral density matrices and $\boldsymbol{\theta}_0$ is a true parameter. A minimum distance estimator is defined as

$$\hat{\boldsymbol{\theta}}_n := \arg\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \int_{-\pi}^{\pi} \mathfrak{K}\{\boldsymbol{\theta},\hat{\boldsymbol{f}}_n(\lambda), \lambda\}\mathrm{d}\lambda,$$

where $\mathfrak{K}$ is an function. If we choose

$\mathfrak{K}\{\boldsymbol{\theta},\hat{\boldsymbol{f}}_n(\lambda), \lambda\} := -\log\det(\hat{\boldsymbol{f}}_n(\lambda)\boldsymbol{f}_{\boldsymbol{\theta}}^{-1}(\lambda)) + \mathrm{tr}(\hat{\boldsymbol{f}}_n(\lambda)\boldsymbol{f}_{\boldsymbol{\theta}}^{-1}(\lambda)) - q$, this estimator reduces to the Whittle likelihood estimator. Under appropriate conditions, Taniguchi and Kakizawa (2000, Theorem 6.2.3) with Robinson (1991, Theorem 2.1) yields that $\hat{\theta}_n$ converges in probability to θ_0, and, therefore, we can show that $f_{\hat{\theta}_n}(\lambda)$ converges to $f_{\theta_0}(\lambda)$. Consequently, our theory also can be applied in the case of disturbances with a parametric spectral density.

7.3 Optimality

The optimality of hypothesis testing problems for asymptotic theory is often stated with the locally asymptotic normality (LAN). For the test for the existence of fixed effects, Hallin et al. (2022) proposed the locally asymptotically maximin test based on LAN by using the center-outward ranks for i.i.d. sequences. Thus, for the fixed effects model, the LAN approach is one direction to construct the optimal test. On the other hand, for the test for the existence of random effects, Goto et al. (2022b) showed that likelihood ratio processes for one-way model do not have the LAN property for i.i.d. settings. Due to this, the construction of optimal tests for the random effects model is more challenging. Derivation of the optimality of our tests is highly desirable, but it is beyond the scope of this paper.

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