Supplementary Material for "On the usage of randomized p-values in the Schweder–Spjøtvoll estimator" by Anh-Tuan Hoang and Thorsten Dickhaus

In Section 4.2 we analyze the MSE of the Schweder-Spjøtvoll estimator in case the LFC p-values are independent or positively dependent. We employed the multiple Z-tests model and applied a pairwise correlation coefficient $\rho = 0$ or $\rho > 0$ on the test statistics. We can also determine the dependency structure among the LFC p-values directly by defining their copula. In case of independent LFC p-values, their joint copula is the product copula. In case of positively dependent LFC p-values we consider the Gumbel-Hougaard copula, defined as

$$C_\nu(x_1, \ldots, x_m) = \exp\left[-\left(\sum_{j=1}^{m} -\ln(x_j)\right)^{1/\nu}\right],$$

where $\nu \geq 1$. For increasing ν the degree of dependence increases.

We employed the same model as in the left graph of Figure 2 in the paper, i.e. the multiple Z-tests model with $\pi_0 = 0.7$ and $\theta_j(\vartheta) = 2.5/\sqrt{n_j}$ if H_j is false and $\theta_j(\vartheta) = -1/\sqrt{n_j}$ if H_j is true, $m = 500$ and $n_j = 50$.

Figures 1 and 2 illustrate the effect of the copula of the p-values utilized in $\hat{\pi}_0$ on its variance and its MSE, respectively, in our context. On the left we assumed independent LFC p-values and on the right we assumed that the LFC p-values had the Gumbel-Hougaard copula as a joint copula with copula parameter $\nu = 2$. The values were calculated via Monte-Carlo Simulation with 100,000 repetitions.

In the left graph of Figure 1, the variance of $\hat{\pi}_0(1/2, c)$ is decreasing in c, cf. also Lemma 1. Furthermore, the variance on the left graph is always below $1/m = 1/500$. So, we may conclude here that taking into account $U_1, \ldots, U_m$ increases the variance of $\hat{\pi}_0$, but only to a magnitude which is in essentially all considered cases smaller than that of the bias reduction achieved by randomization. This is also in line with the findings of Dickhaus (2013); see the discussion around Table 2 in that paper.

In the right graph of Figure 1, the behavior of the variance of $\hat{\pi}_0(1/2, c)$ is different. Here, the randomization reduces the variance of $\hat{\pi}_0$, often by a considerable amount. This can be explained by the fact, that in the dependence structure among $p_1^{rand}(X, U_1, c), \ldots, p_m^{rand}(X, U_m, c)$ the Gumbel-Hougaard copula of $p_1^{LFC}(X), \ldots, p_m^{LFC}(X)$ and the product copula of $U_1, \ldots, U_m$ are "mixed", meaning that the degree of dependency among $p_1^{rand}(X, U_1, c), \ldots, p_m^{rand}(X, U_m, c)$ is smaller than that among $p_1^{LFC}(X), \ldots, p_m^{LFC}(X)$.

Furthermore, comparing the scalings of the vertical axes in the two graphs of Figure 1, we can confirm the previous findings by Neumann et al (2021) (and other authors), that (positively) dependent p-values lead to an increased variance of $\hat{\pi}_0$ when compared with the case of jointly stochastically independent p-values. These results are similar to our results in Section 4.2 and are intended to provide an additional example.

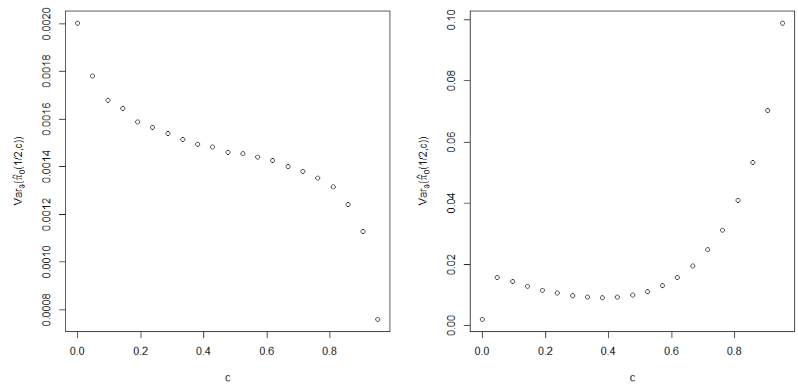

Fig. 1 The variance $\mathrm{Var}_\vartheta(\hat{\pi}_0(1/2, c))$ for $c = 0, 0.05, ..., 1$ in the multiple Z-tests model for $\pi_0 = 0.7$, and $\vartheta \in \Theta$ such that $\theta_j(\vartheta) = -1/\sqrt{50}$ if H_j is true and $\theta_j(\vartheta) = 2.5/\sqrt{50}$ if K_j is true, $j = 1, ..., m = 1,000$. The LFC-based p-values are jointly stochastically independent in the left graph and have the Gumbel-Hougaard copula with copula parameter $\nu = 2$ in the right graph.

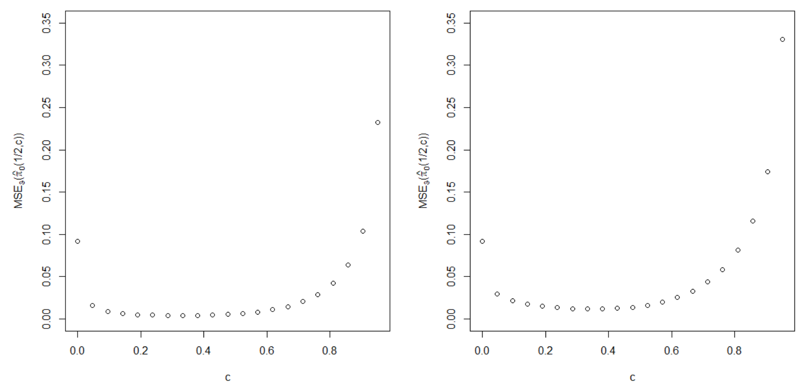

Fig. 2 The mean squared error $\mathrm{MSE}_\vartheta(\hat{\pi}_0(1/2, c))$ for $c = 0, 0.05, ..., 1$ in the multiple Z-tests model for $\pi_0 = 0.7$, and $\vartheta \in \Theta$ such that $\theta_j(\vartheta) = -1/\sqrt{50}$ if H_j is true and $\theta_j(\vartheta) = 2.5/\sqrt{50}$ if K_j is true, $j = 1, ..., m = 1,000$. The LFC-based p-values are jointly stochastically independent in the left graph and have the Gumbel-Hougaard copula with copula parameter $\nu = 2$ in the right graph.

References

Dickhaus T (2013) Randomized p-values for multiple testing of composite null hypotheses. J Stat Plann Inference 143(11):1968–1979

Neumann A, Bodnar T, Dickhaus T (2021) Estimating the proportion of true null hypotheses under dependency: a marginal bootstrap approach. J Statist Plann Inference 210:76–86, DOI 10.1016/j.jspi.2020.04.011, URL https://doi.org/10.1016/j.jspi.2020.04.011