

Supplementary Materials for “Robust High-Dimensional Regression for Data with Anomalous Responses”

by Mingyang Ren, Sanguo Zhang and Qingzhao Zhang

For Case 1 in Simulation Studies, Fig. S1 to Fig. S6 report the classification accuracy of penalized γ -logistic regression and other methods under (S1-S2). The performances of variable selection with $p = 2000$ for Case 1 and Case 2 are replaced in Table S1 and Table S2, respectively. The sensitivity analysis results of δ in adjust BIC for logistic and poisson regression are shown in Table S3, and the numerical approximation results in γ -poisson regression are in Table S4.

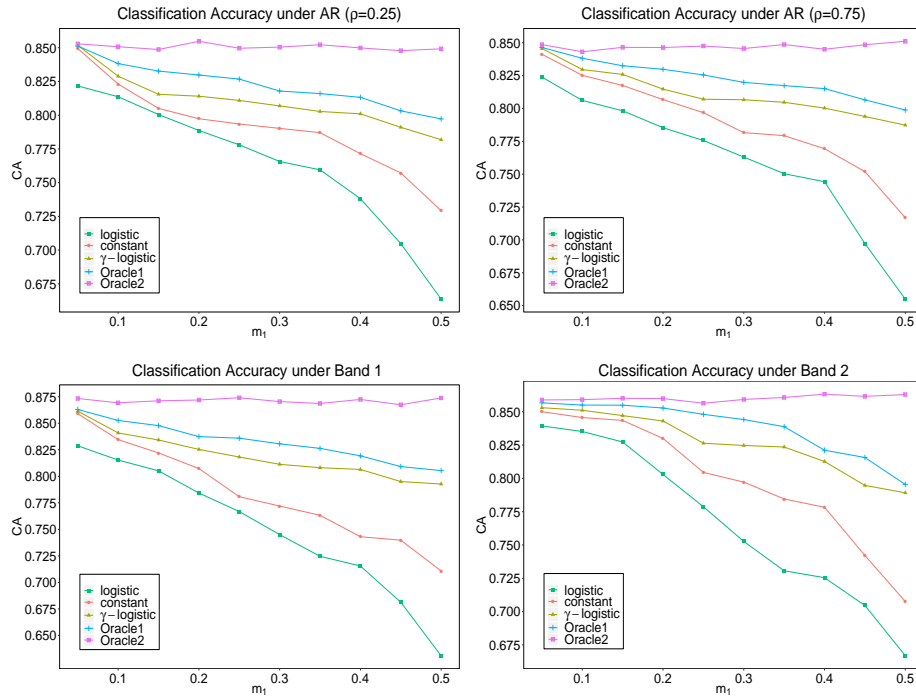


Figure S1: Simulation results of the classification accuracy under (S1) with $p = 1000$, $n = 200$.

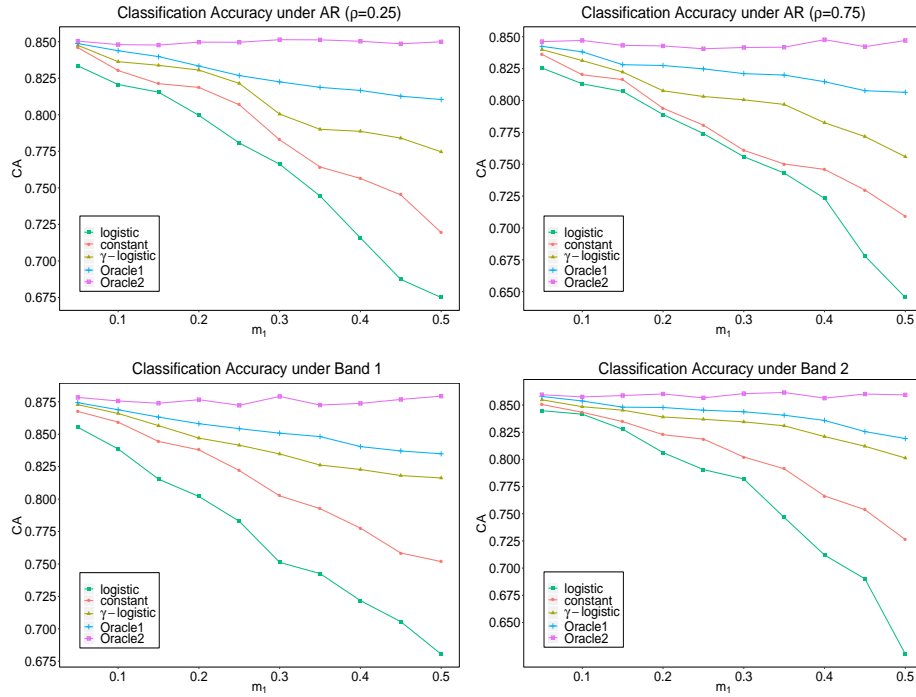


Figure S2: Simulation results of the classification accuracy under (S2) with $p = 1000$, $n = 200$.

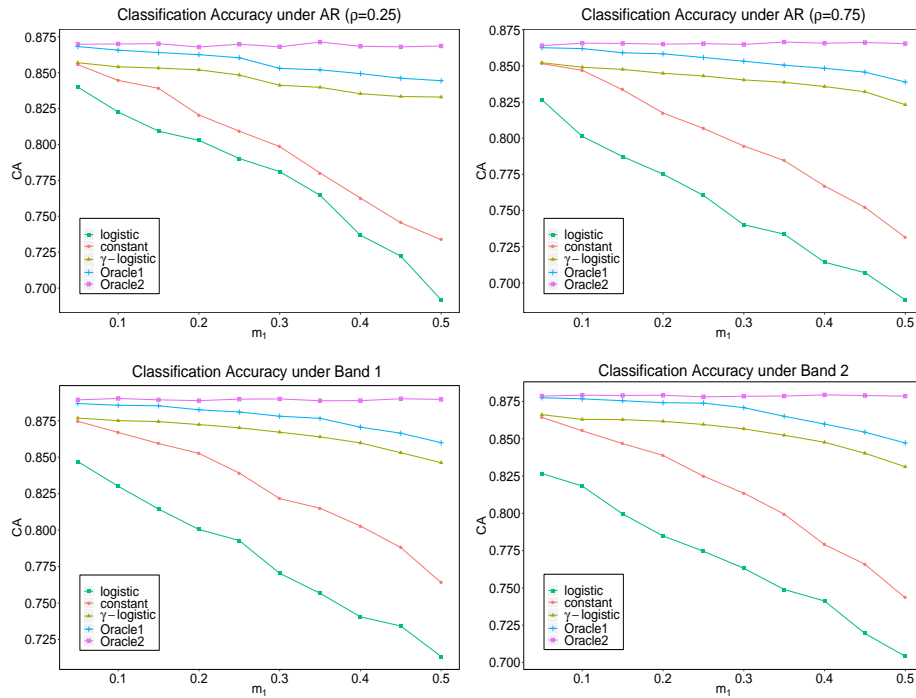


Figure S3: Simulation results of the classification accuracy under (S1) with $p = 2000$, $n = 400$.

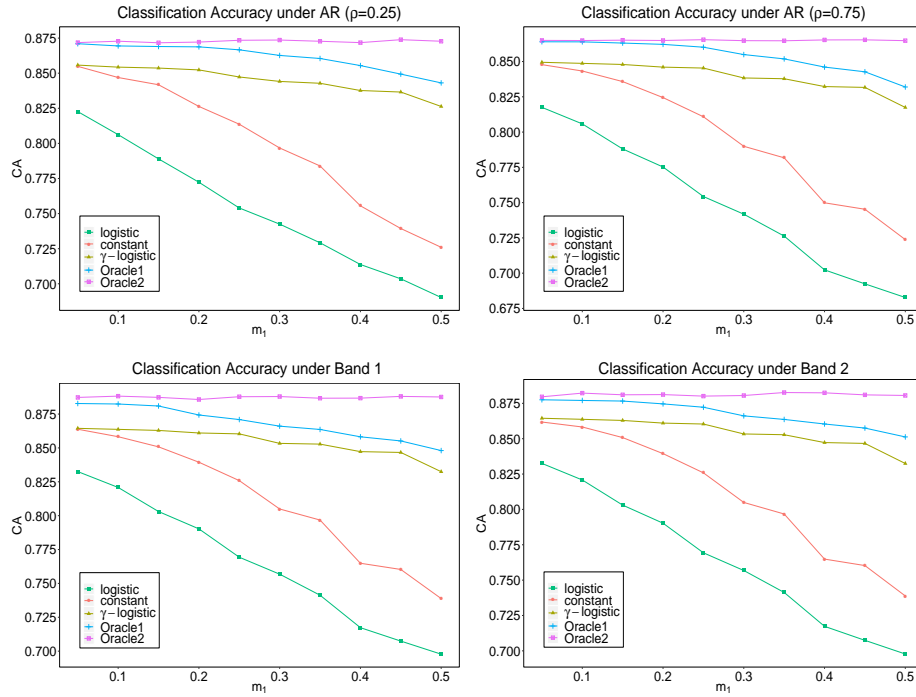


Figure S4: Simulation results of the classification accuracy under (S2) with $p = 2000$, $n = 400$.

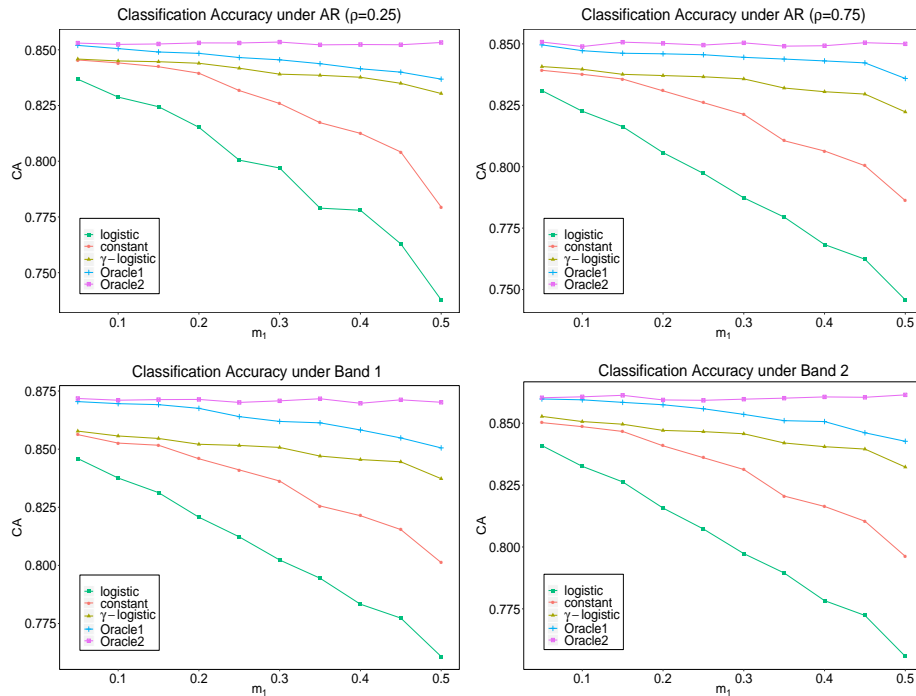


Figure S5: Simulation results of the classification accuracy under (S1) with $p = 2000$, $n = 200$.

Table S1: Mean (sd) of numbers of true/false positives (TP/FP) for variable selection under logistic regression with $p = 2000$.

Correlation	n	Methods	S0			S1			S2			
			TP	FP	$\ \hat{\beta} - \beta_0\ _2$	TP	FP	$\ \hat{\beta} - \beta_0\ _2$	TP	FP	$\ \hat{\beta} - \beta_0\ _2$	
AR $\rho = 0.25$	200	logistic	15.32(1.16)	0.27(0.66)	2.33(0.97)	11.44(3.15)	1.95(2.08)	4.15(1.05)	13.55(2.95)	2.92(1.86)	4.31(1.02)	
		constant	15.36(1.17)	0.29(0.62)	2.26(0.84)	11.75(2.82)	1.88(1.92)	3.87(0.99)	13.96(2.14)	2.07(1.21)	3.96(0.84)	
		γ -logistic	15.25(1.22)	0.34(0.89)	2.29(0.77)	14.95(1.21)	0.74(0.98)	2.68(1.13)	15.18(0.89)	0.55(0.93)	2.54(0.95)	
		Oracle1	16.00(0.00)	0.00(0.00)	1.63(0.48)	16.00(0.00)	0.00(0.00)	1.94(0.54)	16.00(0.00)	0.00(0.00)	1.87(0.52)	
		Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	1.70(0.39)	16.00(0.00)	0.00(0.00)	1.68(0.46)	
		400	logistic	15.71(0.53)	0.19(0.53)	2.19(0.91)	12.67(2.71)	1.89(1.72)	3.91(1.01)	13.68(2.65)	1.64(1.02)	3.78(0.85)
	constant	15.73(0.58)	0.20(0.57)	2.02(0.75)	13.02(1.89)	1.68(1.41)	3.04(0.93)	14.75(1.90)	1.53(1.11)	2.89(0.64)		
	γ -logistic	15.63(0.64)	0.28(0.67)	2.62(0.46)	15.31(0.88)	0.67(0.53)	1.94(0.86)	15.45(0.87)	0.42(0.81)	2.14(0.41)		
	Oracle1	16.00(0.00)	0.00(0.00)	1.02(0.25)	16.00(0.00)	0.00(0.00)	1.21(0.35)	16.00(0.00)	0.00(0.00)	1.27(0.38)		
	Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	1.05(0.28)	16.00(0.00)	0.00(0.00)	1.07(0.31)		
	AR $\rho = 0.75$	200	logistic	14.93(1.17)	0.45(0.66)	2.56(1.19)	9.96(4.96)	3.18(2.84)	4.86(1.34)	12.64(2.81)	3.95(1.94)	4.97(0.85)
			constant	14.82(1.23)	0.53(0.70)	2.64(1.24)	10.88(3.66)	2.81(1.98)	4.07(1.15)	13.84(1.98)	2.26(1.46)	4.06(0.93)
γ -logistic			14.54(1.55)	0.61(0.81)	2.88(1.06)	14.25(1.97)	1.11(1.05)	3.41(0.93)	14.46(1.17)	0.86(1.12)	3.05(0.76)	
Oracle1			16.00(0.00)	0.00(0.00)	1.85(0.46)	16.00(0.00)	0.00(0.00)	2.20(0.48)	16.00(0.00)	0.00(0.00)	2.29(0.56)	
Oracle2			-	-	-	16.00(0.00)	0.00(0.00)	1.82(0.43)	16.00(0.00)	0.00(0.00)	1.92(0.48)	
400			logistic	15.49(0.73)	0.43(0.86)	2.38(1.26)	11.75(3.19)	2.34(1.56)	4.42(1.12)	13.31(2.12)	2.42(1.64)	3.54(1.34)
constant		15.52(0.74)	0.41(0.97)	1.95(0.98)	12.56(2.64)	1.96(1.67)	3.94(0.86)	13.92(1.76)	1.75(1.32)	3.24(1.12)		
γ -logistic		15.40(0.86)	0.51(0.72)	2.72(0.88)	15.12(1.08)	0.86(1.03)	2.76(1.25)	15.22(0.82)	0.68(0.94)	2.15(1.14)		
Oracle1		16.00(0.00)	0.00(0.00)	1.18(0.24)	16.00(0.00)	0.00(0.00)	1.37(0.43)	16.00(0.00)	0.00(0.00)	1.45(0.46)		
Oracle2		-	-	-	16.00(0.00)	0.00(0.00)	1.20(0.28)	16.00(0.00)	0.00(0.00)	1.23(0.26)		
Band 1		200	logistic	15.43(1.04)	0.23(0.64)	2.27(1.08)	11.68(3.02)	1.73(1.42)	4.08(0.85)	13.88(3.02)	2.64(1.72)	4.18(0.95)
			constant	15.48(1.15)	0.22(0.59)	2.19(1.23)	12.48(2.35)	1.65(1.72)	3.72(0.79)	14.18(1.92)	1.72(1.06)	3.84(0.78)
	γ -logistic		15.38(1.10)	0.28(0.75)	1.97(0.68)	15.22(1.18)	0.66(0.73)	2.54(0.67)	15.32(0.72)	0.48(0.84)	2.45(0.51)	
	Oracle1		16.00(0.00)	0.00(0.00)	1.50(0.36)	16.00(0.00)	0.00(0.00)	1.87(0.52)	16.00(0.00)	0.00(0.00)	1.78(0.54)	
	Oracle2		-	-	-	16.00(0.00)	0.00(0.00)	1.55(0.39)	16.00(0.00)	0.00(0.00)	1.53(0.41)	
	400		logistic	15.87(0.34)	0.11(0.31)	1.42(0.45)	12.75(2.54)	1.84(1.42)	3.42(0.86)	14.04(1.66)	1.54(1.24)	3.34(0.54)
	constant	15.84(0.37)	0.14(0.35)	1.36(0.42)	13.96(1.87)	1.16(0.96)	2.86(0.96)	14.87(1.72)	1.45(1.07)	2.78(0.42)		
	γ -logistic	15.82(0.39)	0.18(0.39)	1.40(0.38)	15.44(0.86)	0.32(0.54)	1.54(0.67)	15.48(0.92)	0.35(0.57)	1.64(0.48)		
	Oracle1	16.00(0.00)	0.00(0.00)	0.89(0.22)	16.00(0.00)	0.00(0.00)	1.18(0.36)	16.00(0.00)	0.00(0.00)	1.20(0.30)		
	Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.92(0.25)	16.00(0.00)	0.00(0.00)	1.01(0.31)		
	Band 2	200	logistic	15.36(1.21)	0.26(0.64)	2.36(1.19)	11.48(3.34)	1.90(1.93)	4.21(1.14)	13.58(3.02)	2.86(1.75)	4.28(1.06)
			constant	15.39(1.18)	0.27(0.67)	2.24(0.94)	11.94(2.67)	1.84(1.89)	3.84(1.02)	14.02(1.98)	1.98(1.06)	3.93(0.95)
γ -logistic			15.30(1.28)	0.32(0.92)	2.15(0.89)	14.97(1.08)	0.72(0.92)	2.65(1.35)	15.21(0.96)	0.62(0.97)	2.60(0.86)	
Oracle1			16.00(0.00)	0.00(0.00)	1.60(0.44)	16.00(0.00)	0.00(0.00)	1.97(0.56)	16.00(0.00)	0.00(0.00)	1.90(0.58)	
Oracle2			-	-	-	16.00(0.00)	0.00(0.00)	1.68(0.47)	16.00(0.00)	0.00(0.00)	1.65(0.40)	
400			logistic	15.74(0.44)	0.23(0.64)	2.04(0.81)	12.69(2.69)	1.91(1.56)	3.84(0.96)	13.77(2.05)	1.66(1.32)	3.71(0.76)
constant		15.76(0.43)	0.26(0.69)	1.84(0.69)	13.17(1.84)	1.60(1.36)	2.95(0.92)	14.78(1.82)	1.51(1.29)	2.86(0.57)		
γ -logistic		15.68(0.54)	0.25(0.58)	1.96(0.77)	15.33(0.91)	0.58(0.61)	1.87(0.75)	15.43(1.09)	0.45(0.91)	2.03(0.43)		
Oracle1		16.00(0.00)	0.00(0.00)	0.93(0.21)	16.00(0.00)	0.00(0.00)	1.20(0.40)	16.00(0.00)	0.00(0.00)	1.24(0.35)		
Oracle2		-	-	-	16.00(0.00)	0.00(0.00)	0.98(0.30)	16.00(0.00)	0.00(0.00)	0.94(0.29)		

Table S2: Mean (sd) of numbers of true/false positives (TP/FP) for variable selection under poisson regression with $p = 2000$.

Correlation	n	Methods	S0			S1			S2			
			TP	FP	$\ \hat{\beta} - \beta_0\ _2$	TP	FP	$\ \hat{\beta} - \beta_0\ _2$	TP	FP	$\ \hat{\beta} - \beta_0\ _2$	
AR $\rho = 0.25$	200	poisson	15.46(0.78)	0.83(0.97)	1.54(1.11)	14.76(1.64)	5.88(4.04)	2.17(1.81)	15.05(1.27)	4.98(2.92)	1.95(1.64)	
		γ -poisson	15.48(0.73)	0.79(1.08)	1.60(1.22)	15.36(0.86)	0.93(1.24)	1.76(1.54)	15.41(0.82)	0.86(1.17)	1.72(1.48)	
		Oracle1	16.00(0.00)	0.00(0.00)	0.57(0.25)	16.00(0.00)	0.00(0.00)	0.86(0.18)	16.00(0.00)	0.00(0.00)	0.70(0.12)	
		Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.55(0.27)	16.00(0.00)	0.00(0.00)	0.56(0.23)	
	400	poisson	15.52(0.68)	0.65(0.83)	1.48(1.02)	15.03(1.03)	4.32(3.12)	1.86(1.45)	15.19(1.08)	4.34(2.72)	1.75(1.35)	
		γ -poisson	15.50(0.71)	0.67(0.87)	1.55(1.23)	15.45(0.76)	0.86(1.04)	1.64(1.04)	15.46(0.78)	0.74(1.04)	1.58(1.24)	
		Oracle1	16.00(0.00)	0.00(0.00)	0.43(0.15)	16.00(0.00)	0.00(0.00)	0.60(0.15)	16.00(0.00)	0.00(0.00)	0.48(0.16)	
		Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.41(0.16)	16.00(0.00)	0.00(0.00)	0.46(0.18)	
	AR $\rho = 0.75$	200	poisson	15.18(1.42)	0.96(1.08)	2.15(1.46)	14.56(1.82)	7.68(6.64)	2.34(1.96)	14.68(1.89)	6.26(4.87)	2.28(1.38)
			γ -poisson	15.15(1.34)	0.87(0.98)	1.96(1.24)	15.10(1.42)	1.06(1.34)	2.03(1.76)	15.08(1.31)	1.01(1.19)	1.98(1.32)
			Oracle1	16.00(0.00)	0.00(0.00)	0.79(0.40)	16.00(0.00)	0.00(0.00)	0.87(0.20)	16.00(0.00)	0.00(0.00)	0.85(0.18)
			Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.77(0.42)	16.00(0.00)	0.00(0.00)	0.80(0.43)
400		poisson	15.49(0.70)	0.88(0.98)	1.52(1.04)	14.82(1.43)	5.19(3.95)	1.96(1.52)	15.09(0.39)	4.82(2.76)	1.90(1.50)	
		γ -poisson	15.48(0.74)	0.84(0.92)	1.58(0.37)	15.40(0.81)	0.95(1.12)	1.86(1.61)	15.42(0.80)	0.92(1.09)	1.67(1.38)	
		Oracle1	16.00(0.00)	0.00(0.00)	0.69(0.13)	16.00(0.00)	0.00(0.00)	0.89(0.23)	16.00(0.00)	0.00(0.00)	0.85(0.20)	
		Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.72(0.16)	16.00(0.00)	0.00(0.00)	0.68(0.14)	
Band 1		200	poisson	15.58(0.54)	0.75(0.98)	1.24(0.94)	15.03(1.41)	4.06(5.11)	1.96(1.66)	15.16(1.13)	3.76(1.50)	1.81(1.50)
			γ -poisson	15.56(0.58)	0.73(0.95)	1.32(0.99)	15.52(0.63)	0.88(1.12)	1.54(1.17)	15.53(0.69)	0.84(1.03)	1.47(1.07)
			Oracle1	16.00(0.00)	0.00(0.00)	0.40(0.16)	16.00(0.00)	0.00(0.00)	0.61(0.19)	16.00(0.00)	0.00(0.00)	0.57(0.20)
			Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.38(0.15)	16.00(0.00)	0.00(0.00)	0.41(0.19)
	400	poisson	15.80(0.40)	0.42(0.62)	1.34(0.91)	15.14(1.02)	3.74(2.69)	1.48(1.06)	15.25(0.97)	3.86(2.79)	1.44(0.97)	
		γ -poisson	15.78(0.42)	0.40(0.57)	1.39(0.97)	15.62(0.57)	0.56(0.67)	1.42(0.99)	15.71(0.46)	0.51(0.88)	1.40(0.94)	
		Oracle1	16.00(0.00)	0.00(0.00)	0.32(0.14)	16.00(0.00)	0.00(0.00)	0.48(0.20)	16.00(0.00)	0.00(0.00)	0.44(0.22)	
		Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.34(0.17)	16.00(0.00)	0.00(0.00)	0.35(0.15)	
	Band 2	200	poisson	15.50(0.52)	0.79(1.04)	1.52(1.07)	14.82(1.57)	5.65(3.82)	2.09(1.67)	15.07(1.24)	4.65(2.87)	1.92(1.67)
			γ -poisson	15.49(0.56)	0.82(1.14)	1.42(1.12)	15.41(0.69)	0.97(1.31)	1.68(1.35)	15.45(0.92)	0.92(1.28)	1.68(1.37)
			Oracle1	16.00(0.00)	0.00(0.00)	0.48(0.20)	16.00(0.00)	0.00(0.00)	0.69(0.18)	16.00(0.00)	0.00(0.00)	0.57(0.19)
			Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.50(0.22)	16.00(0.00)	0.00(0.00)	0.49(0.18)
400		poisson	15.57(0.63)	0.60(0.78)	1.43(0.97)	15.05(0.97)	4.12(3.04)	1.79(1.36)	15.21(1.10)	4.09(2.67)	1.71(1.39)	
		γ -poisson	15.54(0.68)	0.58(0.72)	1.52(1.06)	15.51(0.71)	0.77(0.89)	1.58(1.12)	15.52(0.69)	0.68(0.92)	1.51(1.16)	
		Oracle1	16.00(0.00)	0.00(0.00)	0.39(0.16)	16.00(0.00)	0.00(0.00)	0.59(0.21)	16.00(0.00)	0.00(0.00)	0.43(0.18)	
		Oracle2	-	-	-	16.00(0.00)	0.00(0.00)	0.40(0.18)	16.00(0.00)	0.00(0.00)	0.38(0.18)	

Table S3: The sensitivity analysis for δ in adjust BIC: Mean (sd) of numbers of true/false positives (TP/FP) for variable selection under logistic and poisson regression with $p = 1000$.

	Correlation	n	δ	S0		S1		S2	
				TP	FP	TP	FP	TP	FP
logistic	AR $\rho=0.75$	200	2	15.10(1.34)	0.07(0.26)	14.75(1.83)	0.10(0.36)	15.33(0.82)	0.19(0.42)
			4	15.17(1.32)	0.08(0.27)	14.79(1.84)	0.10(0.33)	15.35(0.81)	0.22(0.44)
			6	15.16(1.30)	0.10(0.33)	14.81(1.81)	0.11(0.35)	15.39(0.76)	0.23(0.45)
			8	15.19(1.25)	0.11(0.36)	14.85(1.88)	0.11(0.31)	15.40(0.73)	0.25(0.44)
		10	15.21(1.17)	0.17(0.47)	14.88(1.81)	0.15(0.39)	15.41(0.74)	0.27(0.47)	
		400	2	15.45(1.25)	0.10(0.33)	15.22(0.86)	0.03(0.17)	15.45(0.81)	0.20(0.40)
			4	15.49(1.17)	0.12(0.36)	15.23(0.86)	0.03(0.17)	15.45(0.78)	0.21(0.43)
			6	15.49(1.17)	0.12(0.36)	15.25(0.83)	0.05(0.22)	15.47(0.76)	0.23(0.45)
			8	15.50(1.16)	0.12(0.33)	15.25(0.84)	0.06(0.24)	15.48(0.76)	0.23(0.42)
		10	15.53(1.11)	0.16(0.33)	15.26(0.84)	0.06(0.24)	15.48(0.77)	0.26(0.46)	
	Band1	200	2	15.61(0.83)	0.02(0.14)	15.21(0.84)	0.01(0.10)	15.39(0.67)	0.08(0.27)
			4	15.65(0.82)	0.03(0.17)	15.22(0.85)	0.01(0.10)	15.40(0.65)	0.10(0.30)
			6	15.66(0.82)	0.05(0.22)	15.25(0.82)	0.02(0.14)	15.41(0.65)	0.11(0.31)
			8	15.68(0.82)	0.05(0.22)	15.28(0.79)	0.01(0.10)	15.43(0.65)	0.11(0.31)
		10	15.71(0.81)	0.06(0.24)	15.30(0.80)	0.04(0.20)	15.46(0.66)	0.15(0.39)	
		400	2	15.88(0.36)	0.03(0.17)	15.49(0.66)	0.06(0.24)	15.52(0.93)	0.04(0.20)
			4	15.90(0.30)	0.04(0.20)	15.51(0.64)	0.07(0.26)	15.52(0.90)	0.05(0.22)
			6	15.91(0.29)	0.04(0.20)	15.51(0.66)	0.07(0.26)	15.53(0.97)	0.05(0.22)
			8	15.93(0.26)	0.04(0.20)	15.52(0.64)	0.07(0.26)	15.53(0.99)	0.06(0.24)
		10	15.93(0.26)	0.04(0.20)	15.55(0.69)	0.11(0.35)	15.54(1.00)	0.06(0.24)	
poisson	AR $\rho=0.75$	200	2	15.87(0.37)	0.56(1.16)	15.83(0.40)	0.88(1.07)	15.89(0.35)	0.69(0.87)
			4	15.88(0.36)	0.58(1.18)	15.85(0.39)	0.88(1.09)	15.92(0.31)	0.70(0.87)
			6	15.90(0.30)	0.61(1.07)	15.86(0.38)	0.90(1.13)	15.94(0.28)	0.72(0.93)
			8	15.91(0.29)	0.61(1.13)	15.89(0.31)	0.91(1.18)	15.94(0.24)	0.73(0.94)
		10	15.91(0.29)	0.65(1.08)	15.90(0.30)	0.91(1.10)	15.96(0.20)	0.76(0.95)	
		400	2	15.89(0.35)	0.02(0.14)	15.97(0.17)	0.15(0.39)	15.96(0.20)	0.08(0.27)
			4	15.90(0.33)	0.02(0.14)	15.97(0.17)	0.17(0.40)	15.99(0.10)	0.10(0.30)
			6	15.90(0.30)	0.03(0.17)	15.99(0.10)	0.18(0.41)	16.00(0.00)	0.11(0.31)
			8	15.92(0.27)	0.03(0.17)	15.99(0.10)	0.18(0.42)	16.00(0.00)	0.11(0.31)
		10	15.93(0.26)	0.04(0.20)	15.99(0.10)	0.20(0.45)	16.00(0.00)	0.11(0.31)	
	Band1	200	2	15.92(0.27)	0.51(0.94)	15.87(0.34)	0.60(0.94)	15.90(0.33)	0.55(0.78)
			4	15.93(0.26)	0.52(0.97)	15.88(0.37)	0.59(0.91)	15.90(0.30)	0.54(0.76)
			6	15.96(0.20)	0.53(1.03)	15.90(0.30)	0.62(0.98)	15.91(0.29)	0.56(0.78)
			8	15.97(0.17)	0.53(1.05)	15.90(0.30)	0.62(0.98)	15.93(0.26)	0.57(0.82)
		10	15.97(0.17)	0.57(0.96)	15.91(0.29)	0.62(0.99)	15.95(0.22)	0.60(0.84)	
		400	2	15.99(0.10)	0.01(0.10)	16.00(0.00)	0.04(0.20)	15.59(0.10)	0.03(0.17)
			4	16.00(0.00)	0.02(0.14)	16.00(0.00)	0.04(0.20)	15.59(0.10)	0.05(0.22)
			6	16.00(0.00)	0.02(0.14)	16.00(0.00)	0.04(0.20)	16.00(0.00)	0.05(0.22)
			8	16.00(0.00)	0.02(0.14)	16.00(0.00)	0.04(0.20)	16.00(0.00)	0.05(0.22)
		10	16.00(0.00)	0.02(0.14)	16.00(0.00)	0.05(0.22)	16.00(0.00)	0.05(0.22)	

Table S4: Mean of $(\tau_{\text{approx1}}, \tau_{\text{approx2}})$ under poisson regression with $p = 1000$.

Correlation	n	$\gamma = 0.01$	$\gamma = 0.1$	$\gamma = 0.3$	$\gamma = 0.5$	$\gamma = 1.0$	$\gamma = 1.5$	$\gamma = 5$	$\gamma = 10$
AR $\rho = 0.25$	200	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$
	400	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$
AR $\rho = 0.75$	200	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$
	400	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$
Band 1	200	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$
	400	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$
Band 2	200	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$
	400	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$	$\langle 10^{-8}, \langle 10^{-8} \rangle$

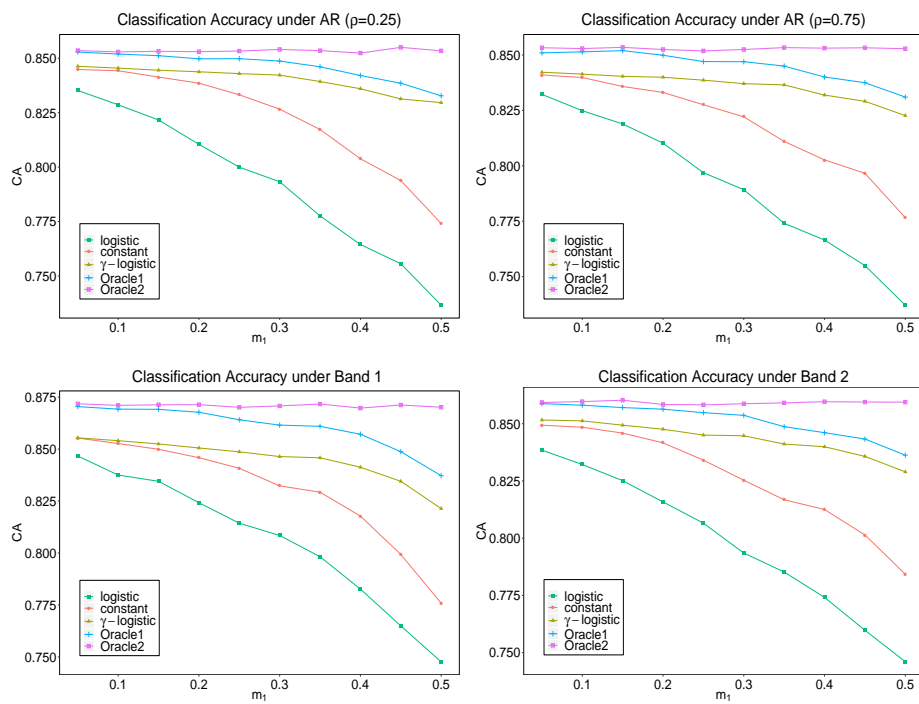


Figure S6: Simulation results of the classification accuracy under (S2) with $p = 2000$, $n = 200$.