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Bartlett correction of frequency domain empirical likelihood for time series with unknown innovation variance

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Abstract
The Bartlett correction is a desirable feature of the likelihood inference, which yields the confidence region for parameters with improved coverage probability. This study examines the Bartlett correction for the frequency domain empirical likelihood (FDEL), based on the Whittle likelihood of linear time series models. Nordman and Lahiri (Ann Stat 34:3019–3050, 2006) showed that the FDEL does not have an ordinary Chi-squared limit when the innovation is non-Gaussian with unknown variance, which restricts the use of the FDEL inference in time series. We show that, by profiling the innovation variance out of the Whittle likelihood function, the FDEL is Chi-squared-distributed and Bartlett correctable. In particular, the order of the coverage error of the confidence region can be reduced from $O(n^{-1})$ to $O(n^{-2})$.

Keywords Coverage error · Edgeworth expansion · Periodogram · Whittle likelihood

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1 Introduction

Empirical likelihood (EL) is a widely used nonparametric likelihood, introduced by Owen (1988, 1990). It has two main features that are analogous to the ordinary parametric likelihood. First, the EL is asymptotically Chi-squared-distributed, which is a nonparametric version of Wilks' theorem of the ordinary log-likelihood ratio test statistic. Second, the Bartlett correction can be applied to improve the approximation to the asymptotic distribution. Recent surveys and discussions of the EL and related methods can be found in Kitamura (2007), Smith (2007) and Nordman and Lahiri (2014).

The EL has been extended to accommodate a particular dependent structure. Monti (1997) used the score function of the Whittle likelihood to derive the limiting Chi-squared distribution of the EL for a Gaussian short-memory time series. Nordman and Lahiri (2006) formulated the frequency domain empirical likelihood (FDEL) for a general framework that includes Monti (1997) as a special case (see also Ogata and Taniguchi 2009; Kakizawa 2013). In particular, their method can be applied to both short- and long-memory time series with possible non-Gaussian distributions. However, Nordman and Lahiri (2006) showed that the Chi-squared approximation is valid only when the innovation is Gaussian or when the innovation variance is known. In other words, when the innovation of the time series is non-Gaussian and the innovation variance is unknown, the FDEL does not have a Chi-squared limit. Hence, the FDEL inference is not directly applicable.

The Bartlett correction has been well studied in the independent setting. For example, Chen and Cui (2006, 2007) proved that the EL with moment restrictions is Bartlett correctable even in the presence of a nuisance parameter. See Cribari-Neto and Cordeiro (1996) for a review of Bartlett-type correction methods under the independent setting. However, the Bartlett correction is relatively less explored for a dependent data. In the time series context, Chan and Liu (2010) proved that the FDEL is Bartlett correctable for Gaussian short-memory time series. Chan et al. (2014) extended Chan and Liu (2010) to Gaussian long-memory time series. More recently, Chen et al. (2016) showed that the FDEL is Bartlett correctable if the process is a non-Gaussian short-memory time series, and the innovation variance is known. However, the case of a non-Gaussian time series with unknown variance remains unresolved because Nordman and Lahiri (2006) showed that the FDEL does not have a Chi-squared limit when the innovation variance is unknown. For a detailed discussion of the Bartlett correctability of other test statistics in a time series, see Taniguchi and Kakizawa (2000).

Although the EL (or FDEL) is Bartlett correctable under the aforementioned settings, the implementation of the Bartlett correction is computationally challenging. To apply the Bartlett correction to a dataset of size n, the statistic is rescaled by an adjustment factor $(1 + b/n)$. The constant b depends on the higher-order moments of the underlying process and is, in general, difficult to estimate. Except for some simple situations, e.g., the statistical inference for the population mean in the independent and identically distributed (i.i.d.) settings (e.g., DiCiccio et al. 1991), the constant b typically has to be estimated by a bootstrap procedure (see Hall and La Scala 1990 for i.i.d. data and Monti 1997 for time series), which significantly increases the computational burden. Moreover, to the best of our knowledge, there is little theoretical evidence to support the validity of the bootstrap procedure.

In this study, we address Wilks' phenomenon and the Bartlett correctability of the FDEL for a non-Gaussian, linear short-memory time series. By profiling the unknown variance out of the FDEL, we show that the FDEL is Chi-squared-distributed. Thus, the inference for the parameters can be performed based on the profiled FDEL. Furthermore, we show that this statistic is Bartlett correctable. In particular, the coverage errors of the resulting confidence intervals can be reduced from $O(n^{-1})$ to $O(n^{-2})$. By deriving a closed form expression of the Bartlett correction factor b, an estimator of b can be constructed. We show the consistency of the estimator and, using a Monte Carlo simulation, demonstrate that the performance is comparable with the bootstrap procedure.

The rest of this paper is organized as follows. Section 2 reviews the Bartlett correction of the Whittle-type FDEL for short-memory time series. In Sect. 3, we show that the FDEL is Bartlett correctable for both Gaussian and non-Gaussian short-memory time series with unknown innovation variances. Finally, Sect. 4 presents simulation studies that demonstrate the finite sample performance of the Bartlett correction. Proofs are given in the Supplementary Material.

Throughout the paper, the following notations are adopted: $O(1)(O_p(1))$ denotes a term (a random variable) that is bounded (in probability); $o(1)(o_p(1))$ denotes a term (a random variable) converging to zero (in probability); for two real sequences a_n and b_n, "$a_n \sim b_n$" means that $a_n/b_n \stackrel{n\to\infty}{\longrightarrow} 1$; and $\mathbb{R}^k$ is the Euclidean space with dimension k. The rth cumulant of a random variable X is denoted by $\kappa_r(X)$. For $\boldsymbol{x} = (x_1, \ldots, x_d)' \in \mathbb{R}^d$, $\|\boldsymbol{x}\| = (x_1^2 + \cdots + x_d^2)^{1/2}$ denotes the l_2-norm of $\boldsymbol{x}$.

2 Background: Bartlett correction of FDEL

Consider a stationary real-valued linear process $\{X_t; t \in \mathbb{Z}\}$ that satisfies

$$X_t = \sum_{j=0}^{\infty} a_j(\beta)\epsilon_{t-j},$$

where $\{\epsilon_t\}$ is an i.i.d. innovation process with a zero mean and unknown variance $\sigma_\epsilon^2 < \infty$, and $\beta \in \mathbb{R}^d$ is the parameter of interest. The unknown parameters are collected as $\theta \equiv (\sigma_\epsilon^2, \beta')'$, with the true value denoted by θ_0. We assume that θ_0 is an interior point of a compact parameter space $\Theta \subset (0, \infty) \times \mathbb{R}^d$. When σ_ϵ^2 is known, the parameter space reduces to $\Theta = \{\theta : \theta = \beta \in \mathbb{R}^d\}$. Let $\kappa_{\epsilon,r} = \kappa_r(\epsilon_1)$ represent the rth-order cumulant of ϵ_1. Also, denote the autocovariance function of X_t by $\gamma_\theta(k) = \mathrm{Cov}(X_t, X_{t+k})$ and the spectral density function (if it exists) by

$$f(\omega, \theta) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_\theta(k) \mathrm{e}^{-\imath k\omega}, \quad \omega \in \Pi \equiv [-\pi, \pi],$$

where $\imath = \sqrt{-1}$. We call $\{X_t\}$ a short-memory time series if the autocovariance function is absolutely summable, that is, $\sum_{k=-\infty}^{\infty} |\gamma_\theta(k)| < \infty$ (Priestley 1981). Let $J_j = J_n(\omega_j) \equiv \frac{1}{\sqrt{2\pi n}} \sum_{t=1}^{n} X_t \exp(-\imath t \omega_j)$ be the normalized discrete Fourier transform (DFT) of X_t at the Fourier frequencies $\omega_j = 2\pi j/n$ for $j = 1, \ldots, n$. The periodogram, defined by $I_n(\omega_j) = J_j J_{-j}$, is an asymptotically unbiased estimator of the spectral density function. Specifically, it was shown in Brillinger (1981) that at the true value $\theta = \theta_0$,

$$\mathrm{E}(I_n(\omega_j)) = f(\omega_j, \theta_0) + \frac{b_I(\omega_j)}{n} + o(n^{-2}),$$

where

$$b_I(\omega) = -\frac{1}{2\pi} \sum_{u=-\infty}^{\infty} |u| \gamma_{\theta_0}(u) \exp(-\imath u \omega). \tag{1}$$

Given a sample $\boldsymbol{X} = \{X_1, \ldots, X_n\}$, the Whittle likelihood in the frequency domain, which is an analog to the ordinary parametric likelihood in the time domain, is defined, up to a constant, by

$$L_W(\boldsymbol{X}, \theta) = -\sum_{j=1}^{n} \left\{ \log f(\omega_j, \theta) + \frac{I_n(\omega_j)}{f(\omega_j, \theta)} \right\}. \tag{2}$$

The Whittle estimator $\hat{\theta}_w$ of θ is defined as the solution of $\sum_{j=1}^{n} m_j(\hat{\theta}_w) = 0$, where

$$m_j(\theta) = \frac{\partial}{\partial \theta} \log f(\omega_j, \theta) \left\{ \frac{I_n(\omega_j)}{f(\omega_j, \theta)} - 1 \right\}$$

is the first derivative of the summand (2). The FDEL is then defined as

$$l_n(\theta) = -2 \log \mathcal{R}_n(\theta), \tag{3}$$

where $\mathcal{R}_n(\theta) = \max_{w_j} \left\{ \prod_{j=1}^{n} n w_j \mid w_j \geq 0,\ \sum_{j=1}^{n} w_j = 1,\ \sum_{j=1}^{n} w_j m_j(\theta) = 0 \right\}$. Monti (1997) showed that $l_n(\theta_0)$ is asymptotically Chi-squared-distributed when $\{X_t\}$ is a Gaussian short-memory time series. Nordman and Lahiri (2006) indicated that the same asymptotic result holds for non-Gaussian linear time series when σ_ϵ^2 is known, whereas $l_n(\theta_0)$ is not asymptotically Chi-squared-distributed if σ_ϵ^2 is unknown and $\{\epsilon_t\}$ is not Gaussian.

Formally, the Bartlett correction is to multiply $l_n(\theta_0)$ by a factor $1 + b/n$ (say) and conduct the statistical inference based on the Bartlett-corrected FDEL, $l_n^*(\theta_0) = l_n(\theta_0)(1 + b/n)^{-1}$. In particular, the $(1-\alpha)$-level Bartlett-corrected FDEL confidence interval with a known σ_ϵ^2 is given by

$$\mathbf{I}_{n,\alpha}^* = \left\{ \theta \mid l_n(\theta) \leq \left(1 + \frac{b}{n}\right) \chi_{d,1-\alpha}^2 \right\},$$

where $\chi^2_{d,1-\alpha}$ is the $1-\alpha$ quantile of a Chi-squared distribution with d degrees of freedom. Compared with the ordinary FDEL confidence region, $\mathbf{I}_{n,\alpha} = \{\theta \mid l_n(\theta) \leq \chi^2_{d,1-\alpha}\}$, it is important to consider whether the coverage error of $\mathbf{I}^*_{n,\alpha}$ is of a lower order. In particular, the coverage errors of $\mathbf{I}_{n,\alpha}$ and $\mathbf{I}^*_{n,\alpha}$, defined by $|P(l_n(\theta) \leq \chi^2_{d,1-\alpha}) - (1-\alpha)|$ and $|P(l_n(\theta) \leq (1+b/n)\chi^2_{d,1-\alpha}) - (1-\alpha)|$, respectively, are $O(n^{-1})$ and $o(n^{-1})$ for Gaussian short-memory time series (Chan and Liu 2010), $O(\log^6 n/n)$ and $O(\log^3 n/n)$ for Gaussian long-memory time series (Chan et al. 2014), and $O(n^{-1})$ and $O(n^{-2})$ for non-Gaussian short-memory time series (Chen et al. 2016).

The classic approach (e.g., DiCiccio et al. 1991 for the i.i.d. case) for establishing the Bartlett correctability of the FDEL considers the signed root statistic (SR), which is defined as $l_n(\theta_0) = W_n' W_n$, where $W_n \equiv \sqrt{n}SR \in \mathbb{R}^d$. Note that, under some regularity conditions, $\sqrt{n}SR$ is asymptotically normal and, in addition, the density function $\pi(x)$ admits the Edgeworth expansion,

$$\pi(x) = \phi(x) + \frac{r_1(x)\phi(x)}{\sqrt{n}} + \frac{r_2(x)\phi(x)}{n} + \frac{r_3(x)\phi(x)}{n^{3/2}} + O(n^{-2}),$$

where $\phi(x)$ is the probability density function (p.d.f.) of the standard normal distribution and $r_j(x)$ is a polynomial of a degree of at most $3j$ and is an odd or even function according to whether j is odd or even (e.g., Hall 1992; Taniguchi and Kakizawa 2000). If $r_2(x)$, which is a degree of at most 6, is actually a degree of 2, then $l_n(\theta_0) = W_n^{'} W_n$ is the Bartlett correctable. A sufficient condition for the terms with degrees of 4 and 6 in $r_2(x)$ to vanish is that the third- and fourth-order cumulants $\kappa_3(\sqrt{n}SR)$ and $\kappa_4(\sqrt{n}SR)$ decay to zero with rates $O(n^{-3/2})$ and $O(n^{-2})$, respectively (see Chen et al. 2016).

3 Main results

Before defining the profiled FDEL and establishing its Bartlett correctability, we impose some assumptions on the time series under consideration.

3.1 Regularity conditions (RC)

(a) The real-valued process $\{X_t\}$ has a linear representation $X_t = \sum_{j=0}^{\infty} a_j(\beta)\epsilon_{t-j}$, where $\{\epsilon_t\}$ is an i.i.d. innovation process with an unknown variance σ^2_ϵ. In addition, ϵ_t has a finite 16th-order cumulant, i.e., $\kappa_{\epsilon,16} < \infty$, and there exists a constant $\rho \in (0, 1)$ such that, for large j, $|a_j(\beta)| < \rho^{|j|}$, where $\theta = (\sigma^2_\epsilon, \beta)^{'} \in \Theta \subset \mathbb{R}^2$, and Θ is a compact set. Furthermore, $\sum_{k=-\infty}^{\infty} |k| \| \frac{\partial}{\partial\theta}\gamma_\theta(k)\| < \infty$ for all θ lies in the interior of Θ.

(b) The spectral density function for the process $\{X_t\}$, defined by

$$f(\omega, \theta) = \frac{\sigma^2_\epsilon}{2\pi} \left| \sum_{j=0}^{\infty} a_j(\beta) e^{\imath j\omega} \right|^2 \quad \text{for } \omega \in \Pi = [-\pi, \pi], \tag{4}$$

is identifiable, i.e., $\theta_1 \neq \theta_2$ implies $f(\omega, \theta_1) \neq f(\omega, \theta_2)$. Also, $f(\omega, \theta)$ is four times continuously differentiable with respect to θ in a neighborhood of θ_0, say Θ_0, and is two times continuously differentiable with respect to $\omega \in \Pi$.

(c) The innovation $(\epsilon_1, \epsilon_1^2)$ fulfills Cramér's condition, that is, there exists constants δ, $C > 0$, such that $\sup_{\|\tau\|>C} |\mathrm{E}\exp(\imath(\tau_1\epsilon_1 + \tau_2\epsilon_1^2))| \leq 1 - \delta$, where $\tau = (\tau_1, \tau_2)'$.

Remark In Condition (a), the existence of the 16th-order cumulant is stronger than that in Nordman and Lahiri (2006), as, in this study, the higher-order asymptotics are required to establish the Bartlett correctability. For simplicity and brevity in the derivation, we consider the case $\beta \in \mathbb{R}$. The general case can be established similarly with more tedious algebras. The exponential decay of the coefficient $a_j(\beta)$ is Assumption (A2) of Janas (1994). The requirement of the (uniform) summability involving the derivative of the autocovariance functions can be satisfied by stationary and invertible ARMA processes (Brockwell and Davis 1991 Chapter 3.6). Condition (b) is Assumption (A6) of Janas (1994), which is necessary to derive the higher-order cumulants of profiled score function and establish the valid Edgeworth expansion. Finally, Condition (c) is Cramér's condition for establishing the valid Edgeworth expansions for the functionals of the periodogram given in Janas (1994). Note that Conditions (a)–(c) imply Assumptions (A1)–(A5) of Janas (1994) and Janas and von Sachs (1995).

3.2 Profiled score function

Nordman and Lahiri (2006) showed that the (not profiled) FDEL does not converge to a Chi-squared distribution when σ_ϵ^2 is unknown and $\{\epsilon_t\}$ is not Gaussian. However, as σ_ϵ^2 can be regarded as a nuisance parameter that has no effect on the dependence structure of the process, it may be profiled out of the likelihood function, as was employed by Monti (1997). To profile out σ_ϵ^2, we use the (4) and maximize the (2) with respect to σ_ϵ^2, yielding the maximizer

$$\hat{\sigma}_\epsilon^2 = \frac{1}{n}\sum_{j=1}^{n}\frac{I_n(\omega_j)}{g_j(\beta)}, \quad \text{where } g_j(\beta) = \frac{1}{2\pi}\left|\sum_{j=0}^{\infty} a_j(\beta)\mathrm{e}^{\imath j\omega}\right|^2. \tag{5}$$

Substituting σ_ϵ^2 by $\hat{\sigma}_\epsilon^2$, the profiled Whittle likelihood function becomes

$$L_{\tilde{W}}(\boldsymbol{X}, \beta) = -n\log\left(\frac{1}{n}\sum_{j=1}^{n}\frac{I_n(\omega_j)}{g_j(\beta)}\right) - \sum_{j=1}^{n}\log g_j(\beta) - n. \tag{6}$$

Differentiating the (6) with respect to β gives the profiled score function

$$\tilde{m}_j(\beta) = \frac{I_n(\omega_j)}{g_j(\beta)}\left(\frac{\partial}{\partial\beta}\log g_j(\beta) - \frac{1}{n}\sum_{j=1}^{n}\frac{\partial}{\partial\beta}\log g_j(\beta)\right). \tag{7}$$

Based on $\tilde{m}_j(\beta)$ in the (7), the profiled FDEL can be constructed as

$$\tilde{l}_n(\beta) = -4 \log \tilde{\mathcal{R}}_n(\beta), \tag{8}$$

where

$$\tilde{\mathcal{R}}_n(\beta) = \max_{w_j} \left\{ \prod_{j=1}^{n} n w_j \mid w_j \geq 0, \sum_{j=1}^{n} w_j = 1, \sum_{j=1}^{n} w_j \tilde{m}_j(\beta) = 0 \right\}.$$

Note that the scale 4 in the (8) is different from the scale 2 in the (3).

3.3 Edgeworth expansion

We need the stochastic expansion of the signed root of the FDEL to establish the asymptotic expansion for the distribution of the FDEL, which can be expressed as functionals of the first four moments and centered moments of the Whittle-type score functions $m_j(\theta)$ (e.g., Chan and Liu 2010; Chen et al. 2016). If σ_ϵ^2 is known, then the first four moments are of order $O(1)$ and the corresponding centered moments are of order $O_p(n^{-1/2})$. Based on the rates of the higher-order moments and centered moments, the asymptotic expansion for the distribution of the signed root can be obtained. In particular, the fact that $\mathrm{E}(m_j(\theta_0)) = O(1/n)$ largely simplifies the derivation of the order of the moments. On the other hand, when σ_ϵ^2 is unknown, we have

$$\mathrm{E}(\tilde{m}_j(\beta_0)) = \sigma_\epsilon^2 p_j(\beta_0) + O(n^{-1}), \tag{9}$$

where $p_j(\beta) = \frac{\partial}{\partial \beta} \log g_j(\beta) - \frac{1}{n} \sum_{j=1}^{n} \frac{\partial}{\partial \beta} \log g_j(\beta)$ and the $O(n^{-1})$ term is uniform in j. The (nonzero) constant bias in the (9) complicates the derivation. Nevertheless, by defining $\bar{\tilde{m}} = \frac{1}{n} \sum_{j=1}^{n} \tilde{m}_j(\beta_0)$, it can be shown that

$$\mathrm{E}(\bar{\tilde{m}}) = O(n^{-1}) \quad \text{and} \quad \mathrm{Var}(\bar{\tilde{m}}) = O(n^{-1}). \tag{10}$$

The orders $O(1/n)$ in the (10) indicate the possibility of establishing a valid Edgeworth expansion as a power series expansion of order $n^{-1/2}$. Let the first four moments and centered moments for $\tilde{m}_j(\beta_0)$ be, respectively,

$$\lambda_k = \mathrm{E}\left(\frac{1}{n} \sum_{j=1}^{n} \tilde{m}_j^k(\beta_0) \right) \quad \text{and} \quad \Delta_k = \frac{1}{n} \sum_{j=1}^{n} \tilde{m}_j^k(\beta_0) - \lambda_k \quad \text{for } k = 1, 2, 3, 4.$$

The following lemma evaluates the orders of λ_k and Δ_k.

Lemma 1 *Under* **RC**, *for* $k = 1, 2, 3, 4$, *we have*

$$\lambda_k = \frac{k!(\sigma_\epsilon^2)^k}{n} \sum_{j=1}^{n} p_j(\beta_0)^k + O(n^{-1}), \tag{11}$$

and

$$\Delta_k = O_p\left(\frac{1}{\sqrt{n}}\right).$$

In particular, $\lambda_1 = O(n^{-1})$ *and* $\lambda_k = O(1)$ *for* $k = 2, 3, 4$.

Similar to the definition of $\sqrt{n}SR$ in Sect. 2, we decompose, even for the profiled version, $\tilde{l}_n(\beta_0) = n\tilde{SR}^2$. Based on Lemma 1, we can establish the stochastic expansion $\sqrt{n}\tilde{SR} = \sqrt{n}(\tilde{R}_1 + \tilde{R}_2 + \tilde{R}_3 + \tilde{R}_n)$, where $\tilde{R}_j = O_p(n^{-j/2})$, $j = 1, 2, 3$, and $\tilde{R}_n = O_p(n^{-2})$. In particular, by equating the order of the terms in $n\tilde{SR}^2 = n(\tilde{R}_1 + \tilde{R}_2 + \tilde{R}_3 + \tilde{R}_n)^2$ to the order of the terms in the stochastic expansion of $\tilde{l}_n(\beta_0)$, we obtain

$$\begin{aligned}
\tilde{R}_1 &= \sqrt{2}\frac{\bar{\bar{m}}}{\sqrt{\lambda_2}}, \\
\tilde{R}_2 &= \frac{\sqrt{2}}{3}\frac{\lambda_3\bar{\bar{m}}^2}{\lambda_2^{5/2}} - \frac{\sqrt{2}}{2}\frac{\bar{\bar{m}}\Delta_2}{\lambda_2^{3/2}}, \\
\tilde{R}_3 &= \frac{3\sqrt{2}}{8}\frac{\bar{\bar{m}}\Delta_2^2}{\lambda_2^{5/2}} + \frac{\sqrt{2}}{3}\frac{\bar{\bar{m}}^2\Delta_3}{\lambda_2^{5/2}} - \frac{5\sqrt{2}}{6}\frac{\lambda_3\bar{\bar{m}}^2\Delta_2}{\lambda_2^{7/2}} + \frac{4\sqrt{2}}{9}\frac{\lambda_3^2\bar{\bar{m}}^3}{\lambda_2^{9/2}} - \frac{\sqrt{2}}{4}\frac{\lambda_4\bar{\bar{m}}^3}{\lambda_2^{7/2}},
\end{aligned} \tag{12}$$

where $\tilde{R}_n = O_p(n^{-2})$ is negligible. The technical details are given in Supplementary Material. Based on the (12), we derive the higher-order cumulants of $\sqrt{n}\tilde{SR}$, which is crucial to establish the Edgeworth expansion.

Lemma 2 Under **RC**, the higher-order cumulants $\kappa_j(\sqrt{n}\tilde{SR})$, $j = 1, 2, 3, 4$, admit the asymptotic expansions

$$\kappa_1(\sqrt{n}\tilde{SR}) = \frac{\tilde{k}_{1,1}}{\sqrt{n}} + \frac{\tilde{k}_{1,2}}{n} + \frac{\tilde{k}_{1,3}}{n^{3/2}} + O(n^{-2}), \tag{13}$$

$$\begin{aligned}
\kappa_2(\sqrt{n}\tilde{SR}) &= 1 + \frac{\tilde{k}_{2,1}}{\sqrt{n}} + \frac{\tilde{k}_{2,2}}{n} + \frac{\tilde{k}_{2,3}}{n^{3/2}} + O(n^{-2}), \\
\kappa_3(\sqrt{n}\tilde{SR}) &= O(n^{-3/2}), \quad \kappa_4(\sqrt{n}\tilde{SR}) = O(n^{-2}), \\
\kappa_r(\sqrt{n}\tilde{SR}) &= O(n^{-2}) \quad \text{for } r \geq 5,
\end{aligned} \tag{14}$$

where

$$\tilde{k}_{1,1} = \frac{1}{n}\frac{\sqrt{2}}{\lambda_2^{1/2}}\sum_{j=1}^{n}\frac{p_j(\beta_0)}{g_j(\beta_0)}b_I(\omega_j) - \frac{\sqrt{2}}{6}\frac{\lambda_3}{\lambda_2^{3/2}}, \tag{15}$$

$$\tilde{k}_{2,2} = -\frac{1}{3}\frac{\lambda_3^2}{\lambda_2^3} + \frac{7}{12}\frac{\lambda_4}{\lambda_2^2} + \frac{4\sigma_\epsilon^2}{\lambda_2}\frac{1}{n}\sum_{j=1}^{n}\frac{p_j(\beta_0)^2}{g_j(\beta_0)}b_I(\omega_j) + \frac{2}{\lambda_2}\sum_{j\neq k}\frac{p_j(\beta_0)p_k(\beta_0)}{g_j(\beta_0)g_k(\beta_0)}\mathrm{cum}(I_n(\omega_j), I_n(\omega_k)), \tag{16}$$

$\tilde{k}_{1,3} \neq 0$ and $\tilde{k}_{1,2} = \tilde{k}_{2,1} = \tilde{k}_{2,3} = 0$.

For non-Gaussian time series, in $\tilde{k}_{2,2}$, $\mathrm{cum}(I_n(\omega_j), I_n(\omega_k)) = \mathrm{cum}^2(J_j, J_k) + \mathrm{cum}^2(J_j, J_{-k})$ $+\mathrm{cum}(J_j, J_{-j}, J_k, J_{-k})$. Note that the coefficients $\tilde{k}_{i,j}$, $i = 1, 2$, and $j = 1, 2, 3$, are of order $O(1)$. Also, the fourth-order cumulant $\kappa_{\epsilon,4}$, which is a characteristic of the non-Gaussian distribution, has no effect on the leading terms of $\kappa_j(\sqrt{n}\tilde{SR})$ for $j = 3$ up to order $O(n^{-3/2})$ and $j = 4$ up to order $O(n^{-2})$. This fact makes the profiled FDEL (and the Bartlett correction) valid under both Gaussian and non-Gaussian short-memory time series.

By Lemma 2, the Edgeworth expansion for $\sqrt{n}\tilde{SR}$ can be obtained using the standard argument (e.g., Hall 1992). In particular, the p.d.f. of $\sqrt{n}\tilde{SR}$ admits the asymptotic expansion

$$\tilde{\pi}(x) = \phi(x) + \frac{\tilde{r}_1(x)\phi(x)}{\sqrt{n}} + \frac{\tilde{r}_2(x)\phi(x)}{n} + \frac{\tilde{r}_3(x)\phi(x)}{n^{3/2}} + O(n^{-2}), \tag{17}$$

where

$$\tilde{r}_1(x) = \sqrt{n}\left\{\kappa_1(\sqrt{n}\tilde{SR})x + \frac{1}{6}\kappa_3(\sqrt{n}\tilde{SR})(x^3 - 3x)\right\} = \tilde{k}_{1,1}x + O(n^{-1}),$$

$$\begin{aligned}\tilde{r}_2(x) &= \frac{n}{2}\{\kappa_2(\sqrt{n}\tilde{SR}) - 1 + \kappa_1(\sqrt{n}\tilde{SR})^2\}(x^2 - 1)\\ &= \frac{1}{2}(\tilde{k}_{1,1}^2 + \tilde{k}_{2,2})(x^2 - 1) + O(n^{-1}).\end{aligned}$$

Here, $\tilde{k}_{1,1}$ and $\tilde{k}_{2,2}$ are given in Eqs. (15) and (16), respectively, and $\tilde{r}_3(x)$ is an odd polynomial with a degree of no more than 9. For the profiled FDEL method, $\tilde{r}_1(x)$ and $\tilde{r}_2(x)$ are different from those in the ordinary (non-profiled) FDEL counterparts in Chen et al. (2016). However, the Edgeworth expansion of the profiled FDEL is the same for both Gaussian and non-Gaussian short-memory time series because $r_2(x)$ is still of degree 2 whether or not the fourth-order cumulant $\kappa_{\epsilon,4}$ exists.

3.4 Bartlett correction

Based on the Edgeworth expansion (17), the next theorem shows that the profiled FDEL is asymptotically Chi-squared-distributed; hence, the coverage error of the profiled FDEL confidence interval is $O(n^{-1})$.

Theorem 1 *Under* **RC**, *for a sufficiently large* n,

$$P(\tilde{l}_n(\beta_0) \leq \chi^2_{1,1-\alpha}) = 1 - \alpha + O(n^{-1}).$$

Recall that the key for establishing the Bartlett correctability of the profiled FDEL is to check that $\kappa_3(\sqrt{n}\tilde{SR}) = O(n^{-3/2})$ and $\kappa_4(\sqrt{n}\tilde{SR}) = O(n^{-2})$ (see Lemma 2). Therefore, the coverage error of the resulting profiled Bartlett-corrected FDEL confidence interval is reduced to $O(n^{-2})$.

Theorem 2 *Under* **RC**, *the profiled FDEL is the Bartlett correctable, that is,*

$$P(\tilde{l}_n(\beta_0) \leq (1 + \tilde{b}/n)\chi^2_{1,1-\alpha}) = 1 - \alpha + O(n^{-2}),$$

where $\tilde{b} = \tilde{k}^2_{1,1} + \tilde{k}_{2,2}$ *and* $\tilde{k}_{1,1}$, $\tilde{k}_{2,2}$ *are given in (15) and (16), respectively.*

3.5 Bartlett correction factor

In the profiled FDEL, the Bartlett correction factor $1 + \tilde{b}/n$ is, in general, difficult to compute and is usually obtained by the bootstrap method (see Monti 1997; Chen et al. 2016). The bootstrap resampling procedure is applied as if the set of periodogram ordinates constitutes an independent sample. However, the periodogram ordinates are only asymptotically independent, not exactly independent for general weakly dependent processes. It is not clear whether this approximation would affect the coverage accuracy of the bootstrapped profiled FDEL confidence intervals. To the best of our knowledge, theoretical supports for such a bootstrap procedure have not been established yet. However, if a $\sqrt{n}$-consistent estimator for $\tilde{b}$, say $\hat{\tilde{b}}$, is available, then replacing the factor $(1 + \tilde{b}/n)$ in Theorem 2 with $(1 + \hat{\tilde{b}}/n)$ will increase the coverage error to order $O_p(n^{-3/2})$. That is, let $h(\cdot)$ be the density function of the χ^2_1 distribution and $\hat{\tilde{b}} - \tilde{b} = \xi_n$, where $\xi_n = O_p(n^{-1/2})$. Based on the Edgeworth expansion (17), it follows that

$$\begin{aligned}
&P\left(\tilde{l}_n(\beta_0) \leq \chi^2_{1,1-\alpha}\left(1 + \frac{\hat{\tilde{b}}}{n}\right)\right) \\
&= P\left(n\tilde{SR}^2_0 \leq \chi^2_{1,1-\alpha}\left(1 + \frac{\tilde{b}}{n} + \frac{\xi_n}{n}\right)\right) \\
&= P\left(\chi^2_1 \leq \chi^2_{1,1-\alpha}\left(1 + \frac{\tilde{b}}{n} + \frac{\xi_n}{n}\right)\right) + \int_{-\sqrt{\chi^2_{1,1-\alpha}(1+\tilde{b}/n+\xi_n/n)}}^{\sqrt{\chi^2_{1,1-\alpha}(1+\tilde{b}/n+\xi_n/n)}} \frac{\tilde{b}}{2n}(x^2 - 1)\phi(x)\mathrm{d}x
\end{aligned}$$

$$+\int_{-\sqrt{\chi^2_{1,1-\alpha}(1+\tilde{b}/n+\xi_n/n)}}^{\sqrt{\chi^2_{1,1-\alpha}(1+\tilde{b}/n+\xi_n/n)}}\left\{\frac{\tilde{r}_1(x)}{n^{1/2}}+\frac{\tilde{r}_3(x)}{n^{3/2}}\right\}\phi(x)\mathrm{d}x+O(n^{-2})$$
$$=P(\chi^2_1\le\chi^2_{1,1-\alpha})+\frac{\tilde{b}+\xi_n}{n}\chi^2_{1,1-\alpha}h(\chi^2_{1,1-\alpha})-\frac{\tilde{b}}{2n}(1-\alpha)$$
$$+\frac{\tilde{b}}{2n}[-2\chi^2_{1,1-\alpha}h(\chi^2_{1,1-\alpha})]+\frac{\tilde{b}}{2n}(1-\alpha)+O(n^{-2})$$
$$=1-\alpha+O_p(n^{-3/2}),$$

noting that the integral $\int\{\tilde{r}_1(x)/n^{1/2}+\tilde{r}_3(x)/n^{3/2}\}\phi(x)\mathrm{d}x$ is exactly equal to zero, due to the oddness of the polynomials $\tilde{r}_1(x)$ and $\tilde{r}_3(x)$.

As $\tilde{b}=\tilde{k}^2_{1,1}+\tilde{k}_{2,2}$, to construct $\hat{\tilde{b}}$, it is sufficient to derive the consistent estimators for $\tilde{k}_{1,1}$ and $\tilde{k}_{2,2}$. According to Eqs. (15) and (16), we estimate $\tilde{k}_{1,1}$ and $\tilde{k}_{2,2}$ by

$$\hat{\tilde{k}}_{1,1}=\frac{\sqrt{2}}{\sqrt{\hat{\lambda}_2}}\frac{1}{n}\sum_{j=1}^{n}\frac{p_j(\hat{\beta})}{g_j(\hat{\beta})}\hat{b}_I(\omega_j)-\frac{\sqrt{2}}{6}\frac{\hat{\lambda}_3}{\hat{\lambda}_2^{3/2}},\tag{18}$$

$$\hat{\tilde{k}}_{2,2}=-\frac{1}{3}\frac{\hat{\lambda}_3^2}{\hat{\lambda}_2^3}+\frac{7}{12}\frac{\hat{\lambda}_4}{\hat{\lambda}_2^2}+\frac{4\hat{\sigma}^2_\epsilon}{\hat{\lambda}_2}\frac{1}{n}\sum_{j=1}^{n}\frac{p_j(\hat{\beta})^2}{g_j(\hat{\beta})}\hat{b}_I(\omega_j)$$
$$+\frac{2}{\hat{\lambda}_2}\sum_{j\ne k}\frac{p_j(\hat{\beta})p_k(\hat{\beta})}{g_j(\hat{\beta})g_k(\hat{\beta})}\widehat{\mathrm{cum}}(I_n(\omega_j),I_n(\omega_k)),\tag{19}$$

where

$$\hat{b}_I(\omega_j)=-\frac{1}{2\pi}\sum_{u=-[n/2]+1}^{[n/2]+1}|u|\gamma_{\hat{\theta}}(u)\exp(-\imath u\omega_j)$$

is the estimator for $b_I(\omega_j)$ in the (1), $\hat{\theta}=(\hat{\sigma}^2_\epsilon,\hat{\beta})'$ is any $\sqrt{n}$-consistent estimator of $\theta_0=(\sigma^2_\epsilon,\beta_0)'$, and $\hat{\lambda}_k=\frac{k!(\hat{\sigma}^2_\epsilon)^k}{n}\sum_{j=1}^{n}p_j(\hat{\beta})^k$. By using similar arguments in the proof of Lemma 2 of Chen et al. (2016), the estimated cumulant of different periodogram ordinates can be obtained from

$$\widehat{\mathrm{cum}}(I_n(\omega_j),I_n(\omega_k))$$
$$=\left[\int_{-\pi}^{\pi}k_n(\omega_j-\lambda,\omega_k+\lambda)f(\lambda,\hat{\theta})\mathrm{d}\lambda\right]^2$$
$$+\left[\int_{-\pi}^{\pi}k_n(\omega_j+\lambda,\omega_k+\lambda)f(\lambda,\hat{\theta})\mathrm{d}\lambda\right]^2$$

$$+\frac{\hat{\kappa}_{\epsilon,4}}{4\pi^2 n^2}\sum_{r_1,r_2=-(n-1)}^{n-1}\cos(\omega_j r_1)\cos(\omega_k r_2)\sum_{s_1\in S_{r_1}}\sum_{s_2\in S_{r_2}}$$
$$\times\sum_{p=\max(-s_1,-(s_1+r_1),-s_2,-(s_2+r_2))}^{\infty}a_{p+s_1+r_1}(\hat{\beta})a_{p+s_1}(\hat{\beta})a_{p+s_2+r_2}(\hat{\beta})a_{p+s_2}(\hat{\beta}),$$

where $k_n(\lambda,\nu) = \sin(n\lambda/2)\sin(n\nu/2)/(2\pi n\sin(\lambda/2)\sin(\nu/2))$, $a_0 = 1$, $S_r = \{1,\ldots,n-r\}$ for $r \geq 0$ and $S_r = \{1-r,\ldots,n\}$ for $r \leq 0$ and $\hat{\kappa}_{\epsilon,4}$ is any $\sqrt{n}$-consistent estimator of $\kappa_{\epsilon,4}$. In practice, one can first compute the estimates of the linear time series model $\hat{\beta}$ and obtain the fitted residuals by assuming $X_t = 0$ and $\hat{\epsilon}_t = 0$ for $t < 0$, i.e., $\hat{\epsilon}_t = \sum_{j=0}^{t-1} c_j(\hat{\beta})X_{t-j}$ for $t = 1,\ldots,n$, where c_j are the coefficients of the inverse of $a(z) = \sum_{j=0}^{\infty} a_j(\hat{\beta})z^j$. Then $\hat{\kappa}_{\epsilon,4}$ can be constructed from using the ordinary method of moment. Let $\hat{\tilde{b}} = \hat{\tilde{k}}_{1,1}^2 + \hat{\tilde{k}}_{2,2}$, where $\hat{\tilde{k}}_{1,1}^2$ and $\hat{\tilde{k}}_{2,2}$ are defined in Eqs. (18) and (19), respectively. The following lemma gives an $\sqrt{n}$-consistent estimator of $\tilde{b}$:

Lemma 3 *Under* **RC***,*

$$\hat{\tilde{b}} = \tilde{b} + O_p(n^{-1/2}).$$

4 Simulation studies

In this section, Monte Carlo simulations are conducted to assess the finite sample performance of the Bartlett correction for the profiled FDEL for both Gaussian and non-Gaussian short-memory time series. For illustration, we consider the stationary AR(1) model

$$(1-\phi B)X_t = \epsilon_t, \quad \epsilon_t \overset{i.i.d.}{\sim} (0,\sigma_\epsilon^2). \tag{20}$$

The parameter $\sigma_\epsilon^2 = \mathrm{Var}(\epsilon_1^2)$ is assumed to be unknown in the application of the profiled FDEL. All of the simulations are carried out in R, assuming that the innovation process $\{\epsilon_t\}$ follows $N(0,1)$, t_{18}, Exp(1), Uniform(0, 2), and χ_2^2. The true value for the AR parameter ϕ is chosen from $\{0.3, 0.7, 0.9\}$, which represents the weak dependence and the strong dependence, respectively. Let ϕ_0, $\hat{\phi}_{\alpha/2}$, and $\hat{\phi}_{1-\alpha/2}$ be the true value of the AR parameter and the lower and upper endpoints of the confidence intervals, respectively. In Table 1, we compute the empirical coverage errors under the 95% nominal coverage level for the sample sizes $n = 50$ and 100. In each case, 1000 replications are drawn. To calculate the critical values of the Bartlett-corrected profiled FDEL, the Bartlett correction factor $1+\tilde{b}/n$ is estimated using the analytical expression prescribed in Lemma 3 and the bootstrap method. The detailed bootstrap procedure can be found in Monti (1997) and Chen et al. (2016). In the bootstrap method, the consistent estimator for generating the bootstrapped samples is the maximum FDEL estimator and the resampling replications is set to be 500. Note that, using the bootstrap

Table 1 Empirical coverage errors of confidence intervals for AR(1) models, replications = 1000 with nominal coverage level 0.95

AR(1) model						
n		$N(0, 1)$	t_{18}	Exp(1)	Uniform(0, 2)	χ_2^2
$\phi = 0.3$						
50	FDEL	0.031	0.035	0.038	0.046	0.038
	Bart. FDEL	0.018	0.016	0.018	0.019	**0.01**
	Bart. FDEL*	0.01	0.01	**0.007**	**0.013**	0.012
	Max.lik	**0.007**	**0.008**	0.014	0.014	0.041
	Whit.lik	0.045	0.044	0.043	0.05	0.045
100	FDEL	0.015	0.024	0.025	0.031	0.027
	Bart. FDEL	**0.005**	0.013	**0.008**	0.018	**0.008**
	Bart. FDEL*	0.008	0.012	0.011	**0.001**	0.013
	Max.lik	0.01	**0.009**	0.013	0.008	0.031
	Whit.lik	0.038	0.042	0.042	0.045	0.04
$\phi = 0.7$						
50	FDEL	0.08	0.071	0.063	0.078	0.059
	Bart. FDEL	0.021	0.027	0.03	0.01	0.029
	Bart. FDEL*	**0.009**	**0.001**	**0.002**	**0.009**	**0.002**
	Max.lik	0.01	0.015	0.009	0.011	0.009
	Whit.lik	0.043	0.043	0.044	0.04	0.046
100	FDEL	0.03	0.048	0.031	0.046	0.049
	Bart. FDEL	0.011	0.026	**0.002**	**0.007**	0.021
	Bart. FDEL*	0.01	**0.017**	0.01	0.016	**0.007**
	Max.lik	**0.004**	0.018	0.006	0.014	0.015
	Whit.lik	0.036	0.032	0.047	0.047	0.042
$\phi = 0.9$						
50	FDEL	0.121	0.118	0.122	0.112	0.128
	Bart. FDEL	0.036	0.036	0.039	0.043	0.053
	Bart. FDEL*	0.031	**0.017**	**0.006**	**0.015**	**0.023**
	Max.lik	**0.017**	**0.017**	0.031	0.044	0.034
	Whit.lik	0.037	0.035	0.032	0.048	0.03
100	FDEL	0.105	0.099	0.092	0.097	0.101
	Bart. FDEL	0.037	0.03	0.028	0.028	0.029
	Bart. FDEL*	0.012	**0.015**	**0.003**	**0.009**	**0.01**
	Max.lik	**0.009**	0.018	0.032	0.012	0.019
	Whit.lik	0.049	0.036	0.043	0.038	0.039

The best performed method in each case is highlighted in bold font approximation, the coverage error of the Bartlett-corrected profiled FDEL confidence interval is generally of order $O_p(n^{-3/2})$, as shown in Sect. 3.4. For the AR(1) model (20), $\beta = \phi$, $g_j(\beta) = \frac{1}{2\pi}\frac{1}{|1-\phi\exp(-\imath\omega_j)|^2}$, and $\gamma_\theta(u) = \sigma_\epsilon^2\frac{\phi^{|u|}}{1-\phi^2}$ for $u \in \mathbb{Z}$. The $\sqrt{n}$-consistent estimator for ϕ_0 adopted here is the maximum FDEL estimator $\hat{\phi}_{MELE}$.

Table 1 reports the empirical coverage errors of the confidence intervals of the profiled FDEL and Bartlett-corrected profiled FDEL based on the bootstrap and the analytical procedures, which are denoted as FDEL, Bart. FDEL and Bart. FDEL*, respectively. All of the white noise distributions have been mean corrected; for example, Exp(1) means that the white noise is generated from Exp(1) − 1 distribution. For comparison, we also compute the empirical coverage errors of the confidence intervals based on the Gaussian maximum likelihood and the Whittle likelihood denoted as Max.lik and Whit.lik, respectively. As shown in Table 1, except for the Gaussian processes, the bootstrap Bartlett-corrected FDEL method based on the analytical and bootstrap procedures performs better than the Gaussian maximum likelihood method, the Whittle likelihood method and the profiled FDEL method without the Bartlett correction in most cases. As the sample size increases, the empirical coverage accuracies for both the profiled FDEL confidence intervals and the bootstrap Bartlett-corrected confidence intervals approach the nominal coverage level. Even for the heavy-tail distribution t_{18} with excess kurtosis $3/7$, the Bartlett correction successfully improves the coverage accuracies. It can be seen that for small values of AR coefficient, the analytical method, and the bootstrap method for the Bartlett correction give similar performances. For large values of AR coefficient, the analytical method outperforms the bootstrap counterpart.

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