Ann Inst Stat Math (2018) 70:269–274
https://doi.org/10.1007/s10463-017-0640-y

INVITED ARTICLE: AKAIKE MEMORIAL LECTURE

Discussion

Chien-Yu Peng[1]

Received: 19 June 2017 / Published online: 9 February 2018

I would like to first congratulate Professor Wu for his path-breaking contributions to statistical sciences. His work greatly deepens our understandings about the theory of experimental design, EM algorithms and resampling. The technical ideas developed in his work (e.g., Wu 2015) are very useful for studying many statistical problems. Su and Wu (2017) proposed a conditional main effect (CME) analysis of de-aliasing method without making additional runs to save money and time. The CME analysis is based on orthogonal models (i.e., *D* optimality) and can lead to models with fewer factorial effects and smaller *p* values for the selected CME effects. The CME effects have good connections between physical/chemical mechanisms and statistical explanations. However, all illustrated examples in Su and Wu (2017) are 2^{k-p} designs with resolution IV. In the following, we apply the CME analysis to real physical experiments using a 2^{k-p} design with resolution III and a 12-run Plackett–Burman design with complex aliasing.

1 Eye focus time experiment (Montgomery 2012, Example 8.7)

A human performance analyst conducted an experiment on the eye focus time of an apparatus using a 2^{7-4}_{III} design with 8 runs. The defining relations of the design are

The Related Articles are https://doi.org/10.1007/s10463-018-0646-0; https://doi.org/10.1007/s10463-017-0639-4; https://doi.org/10.1007/s10463-017-0641-x.

✉ Chien-Yu Peng
chienyu@stat.sinica.edu.tw

1 Institute of Statistical Science, Academia Sinica, Taipei, Taiwan, ROC

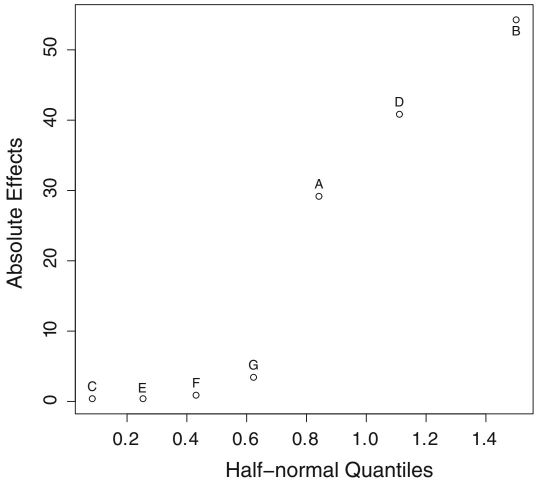

Fig. 1 Half-normal plot, eye focus time experiment

$$I = ABD = ACE = BCF = ABCG = BCDE = ACDF = CDG = ABEF$$
$$= BEG = AFG = DEF = ADEG = CEFG = BDFG = ABCDEFG,$$

and the seven factors are sharpness of vision (A), distance from target to eye (B), target shape (C), illumination level (D), target size (E), target density (F) and subject (G). The runs given in standard order, data obtaining the focus times in milliseconds and CMEs from the design are given in Table 1. The goal of this experiment was to identify the most importance factorial effects. We apply the CME analysis to this data set.

First, we use half-normal plot to identify significant factorial effects. From Fig. 1, it is clearly seen that the main effects B (distance from target to eye), D (illumination level) and A (sharpness of vision) are the most significant. There is a huge gap between A and G. Therefore, we include these three effects in the first model, denoted as model 1.1. There are four interpretations of the true effects of this experiment: (i) the main effects of A, B and D; (ii) A, B and the AB (= D) interaction; (iii) B, D and the BD (= A) interaction; or (iv) A, D and the AD (= B) interaction.

Next, among the significant effects in model 1.1, we use rule 1 in the CME analysis to identify a pair of two-factor interaction and its parental main effect with similar magnitudes. We have the results as follows.

$$\text{Model 1.1: } \bar{y} = 100.9125 + \underset{(7.91\text{e}-3\%)}{17.9375} B + \underset{(2.65\text{e}-2\%)}{13.1875} D + \underset{(4.43\text{e}-2\%)}{11.5625} A,$$
$$R^2 = 99.25\%,$$

$$\text{Model 1.2: } \bar{y} = 100.9125 + \underset{(1.58\text{e}-3\%)}{17.9375} B + \underset{(1.79\text{e}-3\%)}{24.75} (D|B+), \; R^2 = 99.05\%,$$

$$\text{Model 1.3: } \bar{y} = 100.9125 + \underset{(2.00\text{e}-1\%)}{13.1875} D + \underset{(2.40\text{e}-2\%)}{29.5} (B|D+), \; R^2 = 96.05\%,$$

Table 1 Design matrix, time data and CMEs from the design, eye focus time experiment

Run	A	B	C	$D = AB$	$E = AC$	$F = BC$	$G = ABC$	Time	$B\|D+$	$A\|D+$	$D\|A+$	$B\|A+$	$A\|B+$	$D\|B+$
1	−	−	−	+	+	+	−	85.5	−	−	0	0	0	0
2	+	−	−	−	−	+	+	75.1	0	0	−	−	0	0
3	−	+	−	−	+	−	+	93.2	0	0	0	0	−	−
4	+	+	−	+	−	−	−	145.4	+	+	+	+	+	+
5	−	−	+	+	−	−	+	83.7	−	−	0	0	0	0
6	+	−	+	−	+	−	−	77.6	0	0	−	−	0	0
7	−	+	+	−	−	+	−	95.0	0	0	0	0	−	−
8	+	+	+	+	+	+	+	141.8	+	+	+	+	+	+

$$\text{Model 1.4: } \bar{y} = 100.9125 + \underset{(1.32\text{e}-1\%)}{11.5625} \; A + \underset{(6.29\text{e}-3\%)}{31.125} \; (B|A+), \; R^2 = 97.47\%,$$

$$\text{Model 1.5: } \bar{y} = 100.9125 + \underset{(1.58\text{e}-3\%)}{17.9375} \; B + \underset{(1.79\text{e}-3\%)}{24.75} \; (A|B+), \; R^2 = 99.05\%,$$

$$\text{Model 1.6: } \bar{y} = 100.9125 + \underset{(2.00\text{e}-1\%)}{13.1875} \; D + \underset{(2.40\text{e}-3\%)}{29.5} \; (A|D+), \; R^2 = 96.05\%,$$

$$\text{Model 1.7: } \bar{y} = 100.9125 + \underset{(1.32\text{e}-1\%)}{11.5625} \; A + \underset{(6.29\text{e}-3\%)}{31.125} \; (D|A+), \; R^2 = 97.47\%,$$

where the p value for the factorial effect is shown in parentheses. There are two interesting results that should be noticed: (i) since the level settings of CME $B|D+$ and $A|D+$ ($D|A+$ and $B|A+$; $A|B+$ and $D|B+$) are the same, the corresponding regression coefficients and R^2 of the regression models 1.2 and 1.5 (1.3 and 1.6; 1.4 and 1.7) are the same, and (ii) because of the orthogonal effects, the regression coefficient of CME $D|B+$ in model 1.2 is the sum of the coefficients D and BD ($= A$) in model 1.1. The similar result (ii) can also be observed for all examples in Su and Wu (2017). Moreover, models 1.2 and 1.5 are better than model 1.1 because of fewer factorial effects and smaller p values for the selected CME effects. A natural question arising from the interesting results is which model (1.2 or 1.5) should be chosen and explained?

2 Cast fatigue experiment (Hunter et al. 1982)

The cast fatigue experiment used in Hunter et al. (1982) is a 12-run Plackett–Burman design with complex aliasing. The seven factors are initial structure (A), bead size (B), pressure treat (C), heat treat (D), cooling rate (E), polish (F) and final treat (G). The runs given in standard order, data obtaining the logged lifetime of the casting and a CME from the design are given in Table 2. The goal of this experiment was to identify the factorial effects that affect the casting lifetime.

It is well known for this non-regular design that any main effect has aliasing coefficient $1/3$ or $-1/3$ with any two-factor interactions that do not involve the main effect (see Wu and Hamada 2009, Chapter 9). We apply the CME analysis to analyze the cast fatigue data and compare CME analysis with the traditional regression (i.e., main effect) analysis and an analysis strategy developed by Hamada and Wu (1992).

– Main effect analysis:

$$\text{Model 2.1: } \bar{y} = 5.73025 + \underset{(1.78\%)}{0.4576} \; F, \; R^2 = 44.51\%,$$

$$\text{Model 2.2: } \bar{y} = 5.73025 + \underset{(1.24\%)}{0.4576} \; F - \underset{(11.30\%)}{0.2581} \; D, \; R^2 = 58.67\%.$$

Table 2 Design matrix, lifetime data and a CME from the design, cast fatigue experiment

Run	Factor											Logged lifetime	$F\|G-$
	A	*B*	*C*	*D*	*E*	*F*	*G*	8	9	10	11		
1	+	+	−	+	+	+	−	−	−	+	−	6.058	+
2	+	−	+	+	+	−	−	−	+	−	+	4.733	−
3	−	+	+	+	−	−	−	+	−	+	+	4.625	−
4	+	+	+	−	−	−	+	−	+	+	−	5.899	0
5	+	+	−	−	−	+	−	+	+	−	+	7.000	+
6	+	−	−	−	+	−	+	+	−	+	+	5.752	0
7	−	−	−	+	−	+	+	−	+	+	+	5.682	0
8	−	−	+	−	+	+	−	+	+	+	−	6.607	+
9	−	+	−	+	+	−	+	+	+	−	−	5.818	0
10	+	−	+	+	−	+	+	+	−	−	−	5.917	0
11	−	+	+	−	+	+	+	−	−	−	+	5.863	0
12	−	−	−	−	−	−	−	−	−	−	−	4.809	−

– Hamada and Wu (1992) analysis:

$$\text{Model 2.3: } \bar{y} = 5.73025 + \underset{(0.02\%)}{0.4576}\ F - \underset{(0.02\%)}{0.4588}\ FG,\ R^2 = 89.25\%,$$

$$\text{Model 2.4: } \bar{y} = 5.73025 + \underset{(0.02\%)}{0.4576}\ F - \underset{(0.04\%)}{0.4193}\ FG - \underset{(14.47\%)}{0.1183}\ D,$$
$$R^2 = 91.90\%.$$

– CME analysis:

$$\text{Model 2.5: } \bar{y} = 5.73025 + \underset{(3.70\text{e}-4\%)}{0.9163}\ (F|G-),\ R^2 = 89.25\%,$$

$$\text{Model 2.6: } \bar{y} = 5.73025 + \underset{(7.12\text{e}-4\%)}{0.8791}\ (F|G-) - \underset{(13.31\%)}{0.1116}\ D,\ R^2 = 91.75\%.$$

The CME models 2.5 and 2.6 are based on models 2.3 and 2.4 proposed by Hamada and Wu (1992), respectively. Clearly, models 2.3 and 2.4 are, respectively, better than models 2.1 and 2.2 due to the significant two-factor interaction FG. By comparing the R^2 values and the p values for significant effects, the CME models 2.5 and 2.6 are better than models 2.3 and 2.4, respectively. Moreover, the CME $F|G-$ has a good engineering interpretation, i.e., at no peening, factor "polish" has a significant effect on the predicted fatigue lifetimes. The insignificant effect D in models 2.2, 2.4 and 2.6 indicates the corresponding 95% confidence interval containing zero. Note that the regression coefficient of CME $F|G-$ in model 2.5 (or 2.6) is not the sum of the coefficients F and FG in model 2.3 (or 2.4) because of the non-orthogonality of partially aliased effects. The question then arises: could we relax the assumption of the orthogonal models in the CME analysis?

The CME analysis as applied to a resolution III design and a non-regular design appears to work very well and deserves further investigation.

Acknowledgements The research was supported by the Ministry of Science and Technology (Grant No: MOST-105-2118-M-001-002) of Taiwan, Republic of China.

References

Hamada, M., Wu, C. F. J. (1992). Analysis of designed experiments with complex aliasing. *Journal of Quality Technology*, *24*, 130–137.

Hunter, G. B., Hodi, F. S., Eagar, T. W. (1982). High-cycle fatigue of weld repaired cast Ti-6AI-4V. *Metallurgical Transactions A*, *13*, 1589–1594.

Montgomery, D. C. (2012). *Design and analysis of experiments* (8th ed.). New York: Wiley.

Su, H., Wu, C. F. J. (2017). CME analysis: A new method for unraveling aliased effects in two-level fractional factorial experiments. *Journal of Quality Technology*, *49*, 1–10.

Wu, C. F. J. (2015). Post-Fisherian experimentation: From physical to virtual. *Journal of the American Statistical Association*, *110*, 612–620.

Wu, C. F. J., Hamada, M. S. (2009). *Experiments planning, analysis and optimization* (2nd ed.). New York: Wiley.