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# Quantile regression based on counting process approach under semi-competing risks data

**Jin-Jian Hsieh**[1] · **Hong-Rui Wang**[1]

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**Abstract** In this paper, we investigate the quantile regression analysis for semi-competing risks data in which a non-terminal event may be dependently censored by a terminal event. The estimation of quantile regression parameters for the non-terminal event is complicated. We cannot make inference on the non-terminal event without extra assumptions. Thus, we handle this problem by assuming that the joint distribution of the terminal event and the non-terminal event follows a parametric copula model with unspecified marginal distributions. We use the stochastic property of the martingale method to estimate the quantile regression parameters under semi-competing risks data. We also prove the large sample properties of the proposed estimator, and introduce a model diagnostic approach to check model adequacy. From simulation results, it shows that the proposed estimator performs well. For illustration, we apply our proposed approach to analyze a real data.

**Keywords** Copula model · Dependent censoring · Quantile regression · Semi-competing risks data

## 1 Introduction

Quantile regression analysis has received increasing attentions in the recent literature of survival analysis, which has emerged as a significant extension of classic linear regression using the concept of conditional quantiles. Given a $(p+1) \times 1$ covariate vector $Z$ and $\gamma \in [0, 1]$, the conditional quantiles of a random variable, say $T$, are

✉ Jin-Jian Hsieh
jjhsieh@math.ccu.edu.tw

[1] Department of Mathematics, National Chung Cheng University, 168, University Rd., Min-Hsiung, Chia-Yi, Taiwan ROC

defined as $\xi_\gamma(T|Z) = \inf\{t : Pr(T \le t|Z) \ge \gamma\}$. A quantile regression model may linearly link $\xi_\gamma(T|Z)$ to $Z$ for each $0 < \gamma < 1$, that is,

$$\xi_\gamma(T|Z) = Z^T \beta_0(\gamma), \quad \gamma \in (0, 1), \tag{1}$$

where the regression parameter, $\beta_0(\gamma)$, represents the effects of covariates on the $\gamma$ quantile of $T$ and may change with $\gamma$. The model can fit the higher or lower quantile of interest, among them the median (0.5th quantile) function is a special case. According to this feature, quantile regression is very useful when data are heterogeneous on the conditional distribution or when the conditional distribution has a non-standard shape. In practice, the failure time data are often quite right skewed. An advantage of the quantile regression model is that it can provide a more complete assessment of covariate effects under conditional distribution. Quantile regression model has been widely investigated by many literatures. (Powell 1984, 1986) considered quantile regression analysis under a fixed censoring mechanism in which all the censoring times are observed. Conditional independent right censorship, given the covariates, has been assumed including Ying et al. (1995), Fitzenberger (1997), Buchinsky and Hahn (1998), Yang (1999), Portnoy (2003), Peng and Huang (2008), Yin et al. (2008), and Portnoy and Lin (2010). Under competing risks model, Peng and Fine (2009) investigated quantile regression parameter estimation, and Fan and Liu (2013) discussed the identification problem and confidence set of the quantile regression parameter. Ji et al. (2014) considered quantile regression model based on dependently censored data.

In this paper, we consider quantile regression model on the non-terminal event time for semi-competing risks data (Fine et al. 2001). The semi-competing risks data consist of a terminal event and a non-terminal event. The study of leukemia patients receiving the bone marrow transplants is an example for semi-competing risks data. The relapse time of leukemia from the bone marrow transplant is the non-terminal event and the death time is the terminal event. When the quantile covariate effect of the death time is of interest, the existing methods described in the above can be applied. But, when the quantile covariate effect of the relapse time is of main interest, the previous approaches are not appropriate due to the dependent censoring. For this problem, Hsieh et al. (2013) applied inverse probability weight (IPW) approach to construct an estimating equation of the quantile regression parameter. Li and Peng (2015) applied stochastic integral technique to construct an estimating equation for the quantile regression parameter. Here, we adopt the counting process technique to handle this problem. Because the non-terminal event is dependently censored by the terminal event, we cannot make inference on the non-terminal event without extra assumption on the dependence of the non-terminal event and the terminal event. Thus, we assume that the two events follow a parametric copula model.

The rest of the article is organized as follows. In Sect. 2, we introduce the semi-competing risks data, quantile regression model, and copula function. In Sect. 3, we propose a counting process approach to estimate quantile regression coefficients and provide a model diagnostic method. We examine the finite sample performance of our proposed approach via simulations in Sect. 4. In Sect. 5, we apply our proposed methodology to a real-data example. We conclude with some remarks in Sect. 6 and delineate the proofs of large sample properties in the Appendix.

## 2 Data and model assumptions

This section begins with an illustration of semi-competing risks data. Let $T$ and $D$ denote the time to non-terminal event and the time to terminal event, respectively. $T$ is subject to censoring by $D$ but not vice versa. Let $C$ be a censoring time. This type of data is called as semi-competing risks data. The observable data consist of $\{(X_i, Y_i, \delta_{X_i}, \delta_{Y_i}) : i = 1, \ldots, n\}$, where $X = T \wedge D \wedge C$, $Y = D \wedge C$, $\delta_X = I(T \leq D \wedge C)$, $\delta_Y = I(D \leq C)$, where $\wedge$ is the minimum operator and $I(\cdot)$ is the indicator function.

This study considers quantile regression model under semi-competing risks data. Let $\tilde{Z}$ be a $p \times 1$ discrete covariate vector and $Z = (1, \tilde{Z}^T)^T$. Consider the following linear quantile regression model on $h(T)$, where $h(\cdot)$ is a known monotonic increasing function, such that

$$\xi_\gamma(h(T)|Z) = Z^T \beta_0(\gamma), \tag{2}$$

where $0 < \gamma < 1$ and $\xi_\gamma(h(T)|Z)$ is the $(100 \times \gamma)$th quantile of $h(T)$ conditional on $Z$. Let $\epsilon_\gamma = h(T) - \beta_0^T(\gamma)Z$, where $\epsilon_\gamma$ satisfies $\Pr(\epsilon_\gamma \leq 0|Z) = \gamma$ under model (2). $\beta_0(\gamma)$ is the true quantile regression parameter of interest. Further, $C$ is assumed to be independent of $(T, D)$ conditional on $Z$.

Many papers considered the estimation of $\beta_0(\gamma)$ under conditional independent censoring given covariates, such as Ying et al. (1995), Fitzenberger (1997), Buchinsky and Hahn (1998), Yang (1999), Portnoy (2003), Peng and Huang (2008), Yin et al. (2008), and Portnoy and Lin (2010), which can be applied to quantile regression on $D$. But, these approaches are not appropriate for the quantile regression on $T$, since these approaches do not take into account the association of $(T, D)$. In this study, we specify that the association of $(T, D)$ by assuming $(T, D)$ follows a copula model as:

$$Pr(T > t, D > d|Z = z) = C_{\alpha_z}\{S_{T|Z}(t|z), S_{D|Z}(d|z)\}, \quad 0 \leq t \leq d \leq \infty, \tag{3}$$

where $S_{T|Z}(t|z)$ and $S_{D|Z}(d|z)$ are the marginal survival functions of $T$ and $D$, given $Z = z$, $C_\alpha(\cdot, \cdot)$ is a parametric copula function defined on the unit square, and $\alpha$ is an association parameter. The advantage of the copula model is that the joint survival function of $(T, D)$ can be expressed as a function of the marginal survival functions of $T$ and $D$, and the corresponding association parameter through the copula function. The Archimedean copula (AC) family is a popular subclass of the copula family, which can be expressed as:

$$C_\alpha(u, v) = \phi_\alpha^{-1}\{\phi_\alpha(u) + \phi_\alpha(v)\}, \quad 1 \geq u, v \geq 0, \tag{4}$$

where $\phi_\alpha$ is a non-increasing convex function defined on (0,1] with $\phi_\alpha(1) = 0$. This class of dependence functions includes Clayton's copula with $\phi_\alpha(s) = (s^{-\alpha} - 1)/\alpha$

and $C_\alpha(u, v) = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}$, and Frank's copula with $\phi_\alpha(s) = log\{(1 - \alpha)/(1 - \alpha^s)\}$ and $C_\alpha(u, v) = log_\alpha\{1 + (\alpha^u - 1)(\alpha^v - 1)/(\alpha - 1)\}$.

## 3 The proposed inference methods

### 3.1 The estimation of $\beta_0(\gamma)$

This section introduces the estimation of the parameter $\beta_0(\gamma)$ in model (2) under the semi-competing risks data. To specify the dependence between $T$ and $D$, we assume the Archimedean copula function on the upper wedge as

$$P(T > t, D > d|Z = z) = \phi_{\alpha_z}^{-1}\{\phi_{\alpha_z}(S_{T|Z}(t|z)) + \phi_{\alpha_z}(S_{D|Z}(d|z))\},$$
$$0 < t \le d < \infty, \tag{5}$$

where $S_{T|Z}$ and $S_{D|Z}$ are the marginal distributions of $T$ and $D$ given $Z$. Let $N_i(t) = I(X_i \le t, \delta_{x_i} = 1)$ and

$$\Lambda_T(t|z_i) = \begin{cases} -\log(\Pr(T > t|D = d_i, Z = z_i)), & \text{if } D = d_i, \\ -\log(\Pr(T > t|D > c_i, Z = z_i)), & \text{if } D > c_i. \end{cases}$$

Note that $\Lambda_T(\cdot|z_i)$ is the cumulative hazard function of $T$ conditional on $Z = z_i$ and the event "$D = d_i$" or "$D > c_i$". Let $E_i$ is the event defined from the censoring status of $D_i$, which is the event "$D_i = d_i$" or "$D_i > c_i$". Define $M_i(t) = N_i(t) - \Lambda_T(t \wedge X_i|z_i)$. Thus, under "$Z = z_i$" and the event defined by $D$ which is "$D = d_i$" or "$D > c_i$", $M_i(t)$ is the martingale process associated with the counting process $N_i(t)$ (Fleming and Harrington 1991). Then, it has $E(M_i(t)|Z = z_i, E_i) = 0$, for $t \ge 0$. Let

$$S_n^0(b(\gamma), b_0(\gamma)) = n^{-1}\sum_{i=1}^{n} Z_i\big[N_i\{h^{-1}(Z_i^T b(\gamma))\} - \Lambda_T\{h^{-1}(Z_i^T b_0(\gamma)) \wedge X_i|Z_i\}\big].$$

Thus, we have $S_n^0(\beta_0(\gamma), \beta_0(\gamma)) \xrightarrow{p} 0$ by Appendix A. Therefore, we can estimate $\beta_0(\gamma)$ by solving the following equation with respect to $b$,

$$S_n^0(b, \beta_0(\gamma)) = n^{-1}\sum_{i=1}^{n} Z_i\big[N_i\{h^{-1}(Z_i^T b)\} - \Lambda_T\{h^{-1}(Z_i^T \beta_0(\gamma)) \wedge X_i|Z_i\}\big] = 0. \tag{6}$$

However, $\Lambda_T(.|Z_i)$ is unknown. It is natural to replace $\Lambda_T(.|Z_i)$ by $\hat{\Lambda}_T(.|Z_i)$ as:

$$S_n(b, \beta_0(\gamma)) = n^{-1}\sum_{i=1}^{n} Z_i\big[N_i\{h^{-1}(Z_i^T b)\} - \hat{\Lambda}_T\{h^{-1}(Z_i^T \beta_0(\gamma)) \wedge X_i|Z_i\}\big] = 0. \tag{7}$$

For the estimation of $\Lambda_T(t|z)$, when $D = d$, we can estimate $\Lambda_T(t|z)$ by $\hat{\Lambda}_T(t|z) = -\log\{\hat{P}(T > t|D = d, Z = z)\}$, where

$$\begin{aligned}\hat{P}(T > t|D = d, Z = z) &= \frac{\hat{P}(T > t, D = d|Z = z)}{\hat{P}(D = d|Z = z)}\\ &= \phi_{\hat{\alpha}_z}^{-1'}\{\phi_{\hat{\alpha}_z}(\hat{S}_{T|Z}(t|z)) + \phi_{\hat{\alpha}_z}(\hat{S}_{D|Z}(d|z))\}\phi_{\hat{\alpha}_z}^{'}(\hat{S}_{D|Z}(d|z)),\end{aligned}$$

where $\phi_\alpha^{-1'}(t) = \frac{\partial}{\partial t}\phi_\alpha^{-1}(t)$. When $D > c$, $\Lambda_T(t|z)$ can be estimated by $\hat{\Lambda}_T(t|z) = -\log\{\hat{P}(T > t|D > c, Z = z)\}$, where

$$\begin{aligned}\hat{P}(T > t|D > c, Z = z) &= \frac{\hat{P}(T > t, D > c|Z = z)}{\hat{P}(D > C|Z = z)}\\ &= \frac{\phi_{\hat{\alpha}_z}^{-1}\{\phi_{\hat{\alpha}_z}(\hat{S}_{T|Z}(t|z)) + \phi_{\hat{\alpha}_z}(\hat{S}_{D|Z}(c|z))\}}{\hat{S}_{D|Z}(c|z)}.\end{aligned}$$

Note that $\hat{S}_{T|Z}(t|z) = \hat{P}(T > t|Z = z)$ can be obtained by the copula-graphic estimator by Lakhal et al. (2008) as:

$$\begin{aligned}&\hat{S}_{T|Z}(t|z)\\ &\quad = \phi_{\hat{\alpha}_z}^{-1}\left\{\sum_{i=1}^{n} I(X_i \le t, \delta_{x_i} = 1, Z_i = z)\{\phi_{\hat{\alpha}_z}[\hat{S}_{W|Z}(X_i|z)] - \phi_{\hat{\alpha}_z}[\hat{S}_{W|Z}(X_i^-|z)]\}\right\},\end{aligned}$$

where $W = T \wedge D$, $\hat{S}_{W|Z}(x|z) = \hat{P}(W > x|Z = z)$ can be obtained by Kaplan–Meier estimator based on $\{(x_i, \delta_{z_i}) : Z_i = z, i = 1, \ldots, n\}$, where $\delta_{z_i} = 1-(1-\delta_{x_i})(1-\delta_{y_i})$. Further, $S_{D|Z}(x|z) = P(D > x|Z = z)$ can be estimated by Kaplan–Meier estimator based on $\{(y_i, \delta_{y_i}) : Z_i = z, i = 1, \ldots, n\}$. From Lakhal et al. (2008), the estimator of $\alpha_z$ can be obtained by solving the root of the following estimating equation:

$$\begin{aligned}&\sum_{i<j, Z_i=Z_j=z} w(\tilde{X}_{ij}, \tilde{Y}_{ij}) I(\tilde{T}_{ij} \le \tilde{D}_{ij}\\ &\quad \le \tilde{C}_{ij})\left\{I((X_i - X_j)(Y_i - Y_j) > 0) - \frac{\theta_{\alpha_z}(\hat{\pi}_z(\tilde{X}_{ij}, \tilde{Y}_{ij}))}{\theta_{\alpha_z}(\hat{\pi}_z(\tilde{X}_{ij}, \tilde{Y}_{ij})) + 1}\right\} = 0,\end{aligned}$$

where $\tilde{X}_{ij} = X_i \wedge X_j$, $\tilde{Y}_{ij} = Y_i \wedge Y_j$, $\tilde{T}_{ij} = T_i \wedge T_j$, $\tilde{D}_{ij} = D_i \wedge D_j$, $\tilde{C}_{ij} = C_i \wedge C_j$, $w(.,.)$ is a weight function, $\theta_{\alpha_z}(v) = -v\phi_{\alpha_z}^{''}(v)/\phi_{\alpha_z}^{'}(v)$, and $\hat{\pi}_z(s, t) = \hat{\Pr}(T > s, D > t|Z = z) = \sum_{i=1}^{n} I(X_i > x, Y_i > y, Z_i = z)/\{n_z \hat{G}_z(y)\}$, where $n_z = \sum_{i=1}^{n} I(Z_i = z)$.

Therefore, when $h^{-1}(z_i^T \beta_0(\gamma)) \leq X_i$,

$$\hat{\Lambda}_T(h^{-1}(z_i^T \beta_0(\gamma)) \wedge X_i|z_i) = \hat{\Lambda}_T(h^{-1}(z_i^T \beta_0(\gamma))|z_i)$$
$$= \begin{cases} -\log\{\phi_{\hat{\alpha}_{z_i}}^{-1'}\{\phi_{\hat{\alpha}_{z_i}}(1-\gamma)+\phi_{\hat{\alpha}_{z_i}}(\hat{S}_{D|Z}(d_i|z_i))\}\phi'_{\hat{\alpha}_{z_i}}(\hat{S}_{D|Z}(d_i|z_i))\}, & \text{if } D = d_i, \\ -\log\{\frac{\phi_{\hat{\alpha}_{z_i}}^{-1}\{\phi_{\hat{\alpha}_{z_i}}(1-\gamma)+\phi_{\hat{\alpha}_{z_i}}(\hat{S}_{D|Z}(c_i|z_i))\}}{\hat{S}_{D|Z}(c_i|z_i)}\}, & \text{if } D > c_i. \end{cases} \tag{8}$$

When $h^{-1}(z_i^T \beta_0(\gamma)) > X_i$,

$$\hat{\Lambda}_T(h^{-1}(z_i^T \beta_0(\gamma)) \wedge X_i|z_i) = \hat{\Lambda}_T(X_i|z_i)$$
$$= \begin{cases} -\log\{\phi_{\hat{\alpha}_{z_i}}^{-1'}\{\phi_{\hat{\alpha}_{z_i}}(\hat{S}_{T|Z}(X_i|z_i))+\phi_{\hat{\alpha}_{z_i}}(\hat{S}_{D|Z}(d_i|z_i))\}\phi'_{\hat{\alpha}_{z_i}}(\hat{S}_{D|Z}(d_i|z_i))\}, & \text{if } D = d_i, \\ -\log\{\frac{\phi_{\hat{\alpha}_{z_i}}^{-1}\{\phi_{\hat{\alpha}_{z_i}}(\hat{S}_{T|Z}(X_i|z_i))+\phi_{\hat{\alpha}_{z_i}}(\hat{S}_{D|Z}(c_i|z_i))\}}{\hat{S}_{D|Z}(c_i|z_i)}\}, & \text{if } D > c_i. \end{cases} \tag{9}$$

From the above, the formulas in (8) and (9) do not involve $\beta_0(\gamma)$. But, the comparison of $h^{-1}(z_i^T \beta_0(\gamma))$ and $X_i$ is not available due to the unknown of $\beta_0(\gamma)$. To overcome this problem, we use a two-stage iterative algorithm in the following.

Because (7) may not be continuous, an exact root may not exist. Here, we apply the generalized solutions method (Fygenson and Ritov 1994) for this problem. Thus, the solution of Eq. (7) with respect to $b$ is equivalent to the minimizer of the following $L_1$ type function with respect to $b$,

$$U_n(b, \beta_0(\gamma)) = \left(\sum_{i=1}^n \delta_{x_i} \left|h(X_i) - b^T Z_i\right|\right) + \left|M - b^T \sum_{l=1}^n -Z_l \delta_{xl}\right| + \left|M - b^T \sum_{k=1}^n 2Z_k \hat{\Lambda}(h^{-1}(Z_i^T \beta_0(\gamma)) \wedge X_i|Z_i)\right|, \tag{10}$$

where $M$ is an extremely large positive value to bound $\left|b^T \sum_{l=1}^n -Z_l \delta_{xl}\right|$ and $\left|b^T \sum_{k=1}^n 2Z_k \hat{\Lambda}(h^{-1}(Z_i^T \beta_0(\gamma)) \wedge X_i|Z_i)\right|$ from above for all $b's$ in the compact parameter space of $\beta_0(\gamma)$. To overcome the comparison problem due to the unknown of $\beta_0(\gamma)$, we suggest the following two-stage algorithm for the $L_1$-type function $U_n(b, \beta_0(\gamma))$.

Step1 Select an initial value $b^{(0)}$ and set $i = 1$.

Step2 Let $\beta_0(\gamma) = b^{(i-1)}$. The $L_1$-type function is $U_n(b, b^{(i-1)})$. Compare $h^{-1}(z_i^T b^{(i-1)})$ and $X_i$ to determine $\hat{\Lambda}_T(h^{-1}(z_i^T b^{(i-1)}) \wedge X_i|z_i)$. When $h^{-1}(z_i^T b^{(i-1)}) \leq X_i$, $\hat{\Lambda}_T(h^{-1}(z_i^T b^{(i-1)}) \wedge X_i|z_i) =$ (8); when $h^{-1}(z_i^T b^{(i-1)}) > X_i$, $\hat{\Lambda}_T(h^{-1}(z_i^T b^{(i-1)}) \wedge X_i|z_i) =$ (9).

Step3 Minimize $U_n(b, b^{(i-1)})$ with respect to $b$ and obtain the minimizer $b^{(i)}$. Then, set $i = i + 1$.

Step4 Repeat step2 and step3 until $b^{(i)}, i = 1, 2, \ldots,$ converge.

By the algorithm with the $L_1$-type function $U_n(b, \beta_0(\gamma))$, we can obtain the solution of (7).

Since the variance of the proposed estimator is difficult to estimate, we use the bootstrap approach (Efron 1979; Efron and Tibishirani 1993) to estimate the variance of $\hat{\beta}(\gamma)$. Based on the bootstrap approach, we can sample data $\{(X_i^{'}, Y_i^{'}, \delta_{x_i}^{'}, \delta_{y_i}^{'}, Z_i^{'}) : (i = 1, \ldots, n)\}$ from the original data. Based on the bootstrapping sample, we can obtain $\hat{\beta}^{'}(\gamma)$. Repeating the re-sampling procedure $B$ times, we can obtain $\{\hat{\beta}_b^{'}(\gamma) : b = 1, \ldots, B\}$ and hence we can compute the variance of $\hat{\beta}(\gamma)$ by

$$V_{\hat{\beta}(\gamma)} = \frac{1}{B-1} \sum_{b=1}^{B} (\hat{\beta}_b^{'}(\gamma) - \bar{\beta}^{'}(\gamma))^2,$$

where $\bar{\beta}^{'}(\gamma) = \sum_{i=b}^{B} \hat{\beta}_b^{'}(\gamma)/B$. Then, we can construct the $(1-\alpha)$ confidence interval for $\beta(\gamma)$ as $\hat{\beta}(\gamma) \pm z_{1-\alpha/2} V_{\hat{\beta}(\gamma)}^{1/2}$, where $z_{1-\alpha/2} = \Phi^{-1}(1-\alpha/2)$, and $\Phi(\cdot)$ is the cumulative distribution function of a standard normal random variable. We can also use the bootstrap percentile method to construct the $(1-\alpha)$ confidence interval of $\beta_0(\gamma)$ as $[\hat{\beta}_{(B\times\alpha/2)}^{'}(\gamma), \hat{\beta}_{(B\times(1-\alpha/2))}^{'}(\gamma)]$, where $\hat{\beta}_{(b)}^{'}(\gamma), b = 1, \ldots, B$ are the order statistics of $\hat{\beta}_b^{'}(\gamma), b = 1, \ldots, B$.

### 3.2 Asymptotic properties of the proposed estimator

In this subsection, we provide the large sample properties for the consistency and weak convergence of the proposed estimator $\hat{\beta}(\gamma)$. Firstly, we introduce regularity conditions as follows:

C1. $Z$ is uniformly bounded.
C2. $\beta(\gamma)$ is bounded.
C3. $\mu^{'}(\beta_0(\gamma))$ is non-singular , where $\mu^{'}(\beta_0(\gamma)) = \frac{\partial \mu(b)}{\partial b}|_{b=\beta_0(\gamma)}$.
C4. $\bar{S}(b) \neq 0$ for $b \neq \beta_0(\gamma)$ and $\liminf_{\|b\|\to\infty} \|\bar{S}(b)\| > 0$, where $\beta_0(\gamma)$ is the true value.

Under the regularity conditions, we have the following theorems.

**Theorem 1** *Under assumptions of models* (2) *and* (5) *and conditions C1, C2, and C4,* $\hat{\beta}(\gamma)$ *is a consistent estimator.*

**Theorem 2** *Under assumptions of models* (2) *and* (5) *and conditions C1, C2, and C3,* $n^{1/2}\{\hat{\beta}(\gamma) - \beta_0(\gamma)\}$ *converges weakly to a joint normal with zero mean.*

The proof of Theorem 1 is presented in Appendix A and the proof of Theorem 2 is shown in Appendix B.

### 3.3 Model diagnosis

Model checking is conducted to ensure high confidence in performing model-based inference. Model checking for quantile regression models with complete data has been developed by Zheng (1998), Horowitz and Spokoiny (2002), and He and Zhu (2003). However, these approaches can be applied to uncensored data only. Here, we develop a model checking method for quantile regression model under semi-competing risks data. We use martingale residual technique to construct an approach to check model (2). We define the residuals, $e_i(\gamma) = N_i(h^{-1}(Z_i^T \hat{\beta}(\gamma))) - \hat{\Lambda}_T\big[h^{-1}(Z_i^T \hat{\beta}(\gamma)) \wedge X_i|Z_i\big]$, $i = 1, \ldots, n$, and consider the following statistic:

$$\ell_n(\gamma) = n^{-1/2} \sum_{i=1}^{n} q(Z_i) e_i(\gamma),$$

where $q(\cdot)$ is a known bounded weight function. Similar to the arguments in Lin et al. (1993) and Peng and Fine (2009), $\ell_n(\gamma)$ converges weakly to a zero-mean Gaussian process if model (2) is specified correctly. Therefore, we propose the following statistic:

$$U = n^{-1/2} \sum_{i=1}^{n} \frac{q(Z_i) e_i(\gamma)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an estimator of the standard deviation of $\ell_n(\gamma)$, which can be obtained by the bootstrap method. Thus, $U$ converges to the standard normal distribution when the considered model is correct. We can reject the model assumption (2) if $|U| > Z_{\alpha/2}$, where $Z_{\alpha/2}$ is the quantile of $N(0, 1)$ and $\alpha$ is the level of significance. If there are $K$ candidate models under consideration, we compute the absolute value of $U_k$ for each model as $|U_k|, k = 1, \ldots, K$, and choose the one with the smallest value.

## 4 Simulation studies

Here, we consider two cases to examine the finite-sample performance of the proposed method. For the first case, we consider the model,

$$\log(T) = \beta_{01}(\gamma) + \beta_{02}(\gamma) Z + \epsilon_\gamma, \tag{11}$$

where $(\beta_{01}(\gamma), \beta_{02}(\gamma)) = (-1, -1)$, and the covariate $Z$ is generated from Ber(0.5). $(\epsilon_\gamma, D)$ is generated from the Clayton copula and Frank copula with $\epsilon_\gamma$ marginally following $U(-0.5\gamma, 0.5-0.5\gamma)$ so that $\Pr(\epsilon_\gamma \leq 0) = \gamma$, and $D$ marginally following the distribution of exp(2). For the second case, we consider

$$\log(T) = b_0 + b_1(1 + Z)\epsilon, \tag{12}$$

where $Z$ is generated from Ber(0.5), $(b_0, b_1) = (-0.5, 0.5)$ and $(\epsilon, D)$ is generated from the Clayton copula and Frank copula with $\epsilon$ following the distribution of U(0,0.5) and $D$ following the distribution of exp(2). In this case, $(\beta_{01}(\gamma), \beta_{02}(\gamma)) = (b_0 + 0.5b_1\gamma, 0.5b_1\gamma)$. The censoring variable $C$ is generated from a uniform distribution on [0,12]. Three levels of Kendall's $\tau$, 0.3, 0.5, 0.7 are considered. We consider the quantile $\gamma$=0.1, 0.3, 0.5 and the sample size $n$=100 based on 400 simulations. To obtain the standard error of the proposed estimator, we use the bootstrap method with $B = 50$ (Efron and Tibishirani 1993). Under the settings, we also present Peng's (Peng and Huang 2008) estimator and Hsieh's estimator (Hsieh et al. 2013) of $\beta_0(\gamma)$. Peng and Huang (2008) proposed an estimator of $\beta_0(\gamma)$ by the counting process approach under independent right censoring data. In these settings, we present Peng's estimator with $T$ being independently censored by $D \wedge C$. Hsieh et al. (2013) suggested an estimator of $\beta_0(\gamma)$ by the IPW approach for the non-terminal event time under semi-competing risks data. The approach by Li and Peng (2015) is constructed based on quantile regression models on $T$ and $D$, and copula model on $(T, D)$. Thus, the comparison with Li and Peng (2015) is not suitable because we do not set a quantile regression model on $D$ in the simulation settings. Tables 1, 2, 3, and 4 summarize the simulation results, which present the bias of the proposed esimator (Bias), the empirical standard deviation (EmpSd), the average of estimated standard deviation (AveSd) based on the bootstrap method, the mean square error (MSE), and the coverage probability of the 95% confidence intervals (CP%). From the results, it shows that our proposed estimator has good performance. Peng's estimator produces bias when the association increases, which is caused by the method without taking account the association of $(T, D)$. In most cases, the MSE of the proposed estimator is smaller than that of Hsieh et al. (2013). We also examine the robustness for the mis-specification of copula model. We consider the first case of simulation setting with Frank copula, but the estimation procedure adopts Clayton copula for the dependence of $(T, D)$. The simulation results are presented in Table 5, which shows that the estimation of $\beta^0(\gamma)$ produces slight bias. For the model checking of copula model, we can refer to Hsieh et al. (2008).

Then, we examine the proposed model diagnostic method. We consider the accelerated failure time model for the non-terminal event time as:

$$\log(T) = \beta(\gamma)Z + \epsilon_\gamma, \tag{13}$$

where $\beta(\gamma) = -1$, $Z \sim$ 1+Ber(0.5), $\epsilon_\gamma \sim U(-0.5\gamma, 0.5 - 0.5\gamma)$ so that $\xi_\gamma(\epsilon_\gamma) = 0$, and $(\epsilon_\gamma, D)$ follow Clayton copula with $D \sim$ exp(2). We consider $\tau$=0.3, 0.5, 0.7, $\gamma$=0.1, 0.3, 0.5, and $q(z) = 1$ with sample size $n$=200 based on 200 replications. Three forms of transformation are considered: (1) $h(t) = \log(t)$; (2) $h(t) = t$; (3) $h(t) = 2(t^{1/2} - 1)$. Table 6 presents the rejection probability $\Sigma_{i=1}^{200} I(|U_i| > Z_{\alpha/2})/200$. When we choose the transformation of $h(t) = \log(t)$, the rejection probability is close to the specified level of $\alpha = 0.05$. For model selection, we choose an appropriate model with the smallest $|U|$ from the candidate models. We present the selection probability in Table 7 and it shows that the suggested model diagnosis performs well.

**Table 1** Estimations of the quantile regression parameters under model (11) with Clayton copula

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| 0.3 | 0.1 | Propose | 0.0094 | 0.0335 | 0.0320 | 0.0012 | 0.893 | −0.0034 | 0.0435 | 0.0420 | 0.0019 | 0.913 |
| | | Peng | 0.0251 | 0.0352 | 0.0354 | 0.0019 | 0.885 | −0.0156 | 0.0448 | 0.0446 | 0.0023 | 0.907 |
| | | Hsieh | 0.0057 | 0.0370 | | 0.0014 | | 0.0028 | 0.0474 | | 0.0023 | |
| | 0.3 | Propose | −0.0005 | 0.0439 | 0.0445 | 0.0019 | 0.908 | 0.0052 | 0.0594 | 0.0579 | 0.0036 | 0.925 |
| | | Peng | 0.0369 | 0.0440 | 0.0455 | 0.0033 | 0.858 | −0.0212 | 0.0578 | 0.0592 | 0.0038 | 0.912 |
| | | Hsieh | −0.0017 | 0.0435 | | 0.0019 | | 0.0065 | 0.0601 | | 0.0036 | |
| | 0.5 | Propose | −0.0035 | 0.0452 | 0.0454 | 0.0021 | 0.905 | 0.0042 | 0.0599 | 0.0601 | 0.0036 | 0.933 |
| | | Peng | 0.0357 | 0.0447 | 0.0434 | 0.0033 | 0.792 | −0.0219 | 0.0591 | 0.0587 | 0.0040 | 0.912 |
| | | Hsieh | −0.0050 | 0.0463 | | 0.0022 | | 0.0070 | 0.0603 | | 0.0037 | |
| 0.5 | 0.1 | Propose | 0.0087 | 0.0358 | 0.0355 | 0.0014 | 0.890 | −0.0027 | 0.0446 | 0.0452 | 0.0020 | 0.928 |
| | | Peng | 0.0472 | 0.0419 | 0.0417 | 0.0040 | 0.800 | −0.0315 | 0.0501 | 0.0510 | 0.0035 | 0.895 |
| | | Hsieh | 0.0083 | 0.0374 | | 0.0015 | | 0.0023 | 0.0488 | | 0.0024 | |
| | 0.3 | Propose | 0.0044 | 0.0494 | 0.0466 | 0.0025 | 0.915 | −0.0017 | 0.0610 | 0.0598 | 0.0037 | 0.915 |
| | | Peng | 0.0723 | 0.0443 | 0.0434 | 0.0072 | 0.582 | −0.0448 | 0.0587 | 0.0577 | 0.0054 | 0.855 |
| | | Hsieh | 0.0099 | 0.0509 | | 0.0027 | | −0.0042 | 0.0633 | | 0.0040 | |
| | 0.5 | Propose | 0.0001 | 0.0428 | 0.0449 | 0.0018 | 0.930 | 0.0023 | 0.0562 | 0.0595 | 0.0032 | 0.958 |
| | | Peng | 0.0638 | 0.0380 | 0.0386 | 0.0055 | 0.615 | −0.0412 | 0.0548 | 0.0540 | 0.0047 | 0.870 |
| | | Hsieh | 0.0075 | 0.0445 | | 0.0020 | | −0.0044 | 0.0585 | | 0.0034 | |
| 0.7 | 0.1 | Propose | 0.0169 | 0.0400 | 0.0404 | 0.0021 | 0.847 | −0.0101 | 0.0502 | 0.0502 | 0.0026 | 0.913 |
| | | Peng | 0.0986 | 0.0404 | 0.0439 | 0.0113 | 0.403 | −0.0683 | 0.0506 | 0.0544 | 0.0072 | 0.726 |
| | | Hsieh | 0.0272 | 0.0460 | | 0.0029 | | −0.0132 | 0.0569 | | 0.0034 | |
| | 0.3 | Propose | −0.0028 | 0.0465 | 0.0445 | 0.0022 | 0.915 | 0.0071 | 0.0597 | 0.0586 | 0.0036 | 0.925 |

**Table 1** continued

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ Bias | EmpSd | Avesd | Mse | C.P. | $\beta^1(\gamma)$ Bias | EmpSd | Avesd | Mse | C.P. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Peng | 0.0950 | 0.0337 | 0.0364 | 0.0102 | 0.272 | −0.0553 | 0.0505 | 0.0512 | 0.0056 | 0.777 |
| | | Hsieh | 0.0318 | 0.0437 | | 0.0029 | | −0.0244 | 0.0578 | | 0.0039 | |
| | 0.5 | Propose | 0.0026 | 0.0408 | 0.0393 | 0.0017 | 0.942 | 0.0118 | 0.0575 | 0.0591 | 0.0035 | 0.965 |
| | | Peng | 0.0699 | 0.0328 | 0.0332 | 0.0060 | 0.436 | −0.0436 | 0.0484 | 0.0486 | 0.0042 | 0.824 |
| | | Hsieh | 0.0281 | 0.0441 | | 0.0027 | | −0.0224 | 0.0582 | | 0.0039 | |

The sample size is 100 and replications are 400

**Table 2** Estimations of the quantile regression parameters under model (12) with Clayton copula

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| 0.3 | 0.1 | Propose | 0.0044 | 0.0156 | 0.0154 | 0.0003 | 0.907 | −0.0007 | 0.0330 | 0.0348 | 0.0011 | 0.932 |
| | | Peng | 0.0103 | 0.0165 | 0.0168 | 0.0004 | 0.887 | 0.0101 | 0.0337 | 0.0377 | 0.0012 | 0.937 |
| | | Hsieh | 0.0017 | 0.0257 | | 0.0007 | | 0.0013 | 0.0422 | | 0.0017 | |
| | 0.3 | Propose | 0.0022 | 0.0214 | 0.0219 | 0.0005 | 0.925 | −0.0055 | 0.0493 | 0.0495 | 0.0025 | 0.920 |
| | | Peng | 0.0175 | 0.0210 | 0.0217 | 0.0007 | 0.882 | 0.0154 | 0.0467 | 0.0493 | 0.0024 | 0.937 |
| | | Hsieh | 0.0023 | 0.0218 | | 0.0005 | | −0.0005 | 0.0496 | | 0.0025 | |
| | 0.5 | Propose | −0.0005 | 0.0222 | 0.0220 | 0.0005 | 0.915 | −0.0065 | 0.0518 | 0.0515 | 0.0027 | 0.925 |
| | | Peng | 0.0173 | 0.0215 | 0.0210 | 0.0008 | 0.817 | 0.0185 | 0.0490 | 0.0489 | 0.0027 | 0.909 |
| | | Hsieh | 0.0002 | 0.0221 | | 0.0005 | | −0.0021 | 0.0504 | | 0.0025 | |
| 0.5 | 0.1 | Propose | 0.0043 | 0.0151 | 0.0171 | 0.0002 | 0.930 | 0.0060 | 0.0379 | 0.0389 | 0.0015 | 0.940 |
| | | Peng | 0.0198 | 0.0180 | 0.0194 | 0.0007 | 0.839 | 0.0255 | 0.0445 | 0.0431 | 0.0026 | 0.889 |
| | | Hsieh | 0.0034 | 0.0270 | | 0.0007 | | 0.0103 | 0.0502 | | 0.0026 | |
| | 0.3 | Propose | −0.0005 | 0.0219 | 0.0219 | 0.0005 | 0.905 | −0.0019 | 0.0525 | 0.0505 | 0.0028 | 0.912 |
| | | Peng | 0.0297 | 0.0198 | 0.0213 | 0.0013 | 0.706 | 0.0392 | 0.0470 | 0.0493 | 0.0037 | 0.837 |
| | | Hsieh | 0.0028 | 0.0227 | | 0.0005 | | 0.0051 | 0.0507 | | 0.0026 | |
| | 0.5 | Propose | −0.0010 | 0.0214 | 0.0216 | 0.0005 | 0.935 | −0.0052 | 0.0482 | 0.0496 | 0.0023 | 0.945 |
| | | Peng | 0.0302 | 0.0183 | 0.0193 | 0.0012 | 0.619 | 0.0346 | 0.0406 | 0.0454 | 0.0028 | 0.872 |
| | | Hsieh | 0.0045 | 0.0233 | | 0.0006 | | 0.0025 | 0.0474 | | 0.0023 | |
| 0.7 | 0.1 | Propose | 0.0031 | 0.0203 | 0.0195 | 0.0004 | 0.905 | 0.0052 | 0.0431 | 0.0424 | 0.0019 | 0.902 |
| | | Peng | 0.0405 | 0.0202 | 0.0201 | 0.0020 | 0.478 | 0.0383 | 0.0451 | 0.0468 | 0.0035 | 0.844 |
| | | Hsieh | 0.0150 | 0.0271 | | 0.0010 | | 0.0062 | 0.0589 | | 0.0035 | |
| | 0.3 | Propose | 0.0026 | 0.0227 | 0.0220 | 0.0005 | 0.922 | −0.0045 | 0.0535 | 0.0498 | 0.0029 | 0.920 |

**Table 2** continued

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| | | Peng | 0.0465 | 0.0168 | 0.0180 | 0.0024 | 0.288 | 0.0498 | 0.0412 | 0.0412 | 0.0042 | 0.766 |
| | | Hsieh | 0.0159 | 0.0209 | | 0.0007 | | 0.0211 | 0.0491 | | 0.0029 | |
| | 0.5 | Propose | 0.0029 | 0.0194 | 0.0195 | 0.0004 | 0.925 | −0.0032 | 0.0421 | 0.0436 | 0.0018 | 0.940 |
| | | Peng | 0.0348 | 0.0161 | 0.0165 | 0.0015 | 0.448 | 0.0390 | 0.0355 | 0.0370 | 0.0028 | 0.819 |
| | | Hsieh | 0.0190 | 0.0263 | | 0.0011 | | 0.0152 | 0.0468 | | 0.0024 | |

The sample size is 100 and replications are 400

**Table 3** Estimations of the quantile regression parameters under model (11) with Frank copula

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| 0.3 | 0.1 | Propose | 0.0056 | 0.0284 | 0.0287 | 0.0008 | 0.9032 | 0.0005 | 0.0362 | 0.0387 | 0.0013 | 0.941 |
| | | Peng | 0.0188 | 0.0290 | 0.0300 | 0.0012 | 0.8882 | −0.0109 | 0.0378 | 0.0394 | 0.0015 | 0.908 |
| | | Hsieh | 0.0052 | 0.0280 | | 0.0008 | | 0.0027 | 0.0402 | | 0.0016 | |
| | 0.3 | Propose | 0.0036 | 0.0396 | 0.0401 | 0.0016 | 0.9298 | −0.0020 | 0.0525 | 0.0534 | 0.0028 | 0.952 |
| | | Peng | 0.0309 | 0.0363 | 0.0376 | 0.0023 | 0.8421 | −0.0202 | 0.0500 | 0.0516 | 0.0029 | 0.924 |
| | | Hsieh | 0.0048 | 0.0383 | | 0.0015 | | −0.0021 | 0.0523 | | 0.0027 | |
| | 0.5 | Propose | 0.0024 | 0.0390 | 0.0405 | 0.0015 | 0.9399 | −0.0021 | 0.0513 | 0.0545 | 0.0026 | 0.952 |
| | | Peng | 0.0267 | 0.0355 | 0.0377 | 0.0020 | 0.8596 | −0.0196 | 0.0491 | 0.0526 | 0.0028 | 0.927 |
| | | Hsieh | 0.0004 | 0.0404 | | 0.0016 | | −0.0011 | 0.0529 | | 0.0028 | |
| 0.5 | 0.1 | Propose | 0.0118 | 0.0323 | 0.0304 | 0.0012 | 0.9048 | −0.0065 | 0.0395 | 0.0398 | 0.0016 | 0.925 |
| | | Peng | 0.0413 | 0.0316 | 0.0318 | 0.0027 | 0.7619 | −0.0275 | 0.0393 | 0.0411 | 0.0023 | 0.887 |
| | | Hsieh | 0.0113 | 0.0354 | | 0.0014 | | −0.0033 | 0.0437 | | 0.0019 | |
| | 0.3 | Propose | 0.0008 | 0.0392 | 0.0385 | 0.0015 | 0.9348 | 0.0010 | 0.0498 | 0.0525 | 0.0025 | 0.965 |
| | | Peng | 0.0432 | 0.0353 | 0.0360 | 0.0031 | 0.7494 | −0.0253 | 0.0473 | 0.0502 | 0.0029 | 0.914 |
| | | Hsieh | 0.0075 | 0.0372 | | 0.0014 | | −0.0034 | 0.0500 | | 0.0025 | |
| | 0.5 | Propose | −0.0008 | 0.0350 | 0.0381 | 0.0012 | 0.9424 | 0.0038 | 0.0466 | 0.0530 | 0.0022 | 0.960 |
| | | Peng | 0.0337 | 0.0343 | 0.0359 | 0.0023 | 0.7995 | −0.0224 | 0.0470 | 0.0514 | 0.0027 | 0.917 |
| | | Hsieh | 0.0042 | 0.0375 | | 0.0014 | | −0.0020 | 0.0499 | | 0.0025 | |
| 0.7 | 0.1 | Propose | 0.0079 | 0.0343 | 0.0311 | 0.0012 | 0.8683 | 0.0021 | 0.0471 | 0.0445 | 0.0022 | 0.924 |
| | | Peng | 0.0575 | 0.0298 | 0.0301 | 0.0042 | 0.5169 | −0.0349 | 0.0402 | 0.0405 | 0.0028 | 0.839 |
| | | Hsieh | 0.0216 | 0.0338 | | 0.0016 | | −0.0074 | 0.0484 | | 0.0024 | |
| | 0.3 | Propose | 0.0092 | 0.0368 | 0.0390 | 0.0014 | 0.9223 | 0.0124 | 0.0549 | 0.0595 | 0.0032 | 0.955 |

**Table 3** continued

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| | | Peng | 0.0564 | 0.0322 | 0.0321 | 0.0042 | 0.5789 | −0.0367 | 0.0455 | 0.0462 | 0.0034 | 0.872 |
| | | Hsieh | 0.0307 | 0.0337 | | 0.0021 | | −0.0240 | 0.0465 | | 0.0027 | |
| | 0.5 | Propose | 0.0202 | 0.0391 | 0.0412 | 0.0019 | 0.9073 | 0.0193 | 0.0619 | 0.0653 | 0.0042 | 0.960 |
| | | Peng | 0.0390 | 0.0311 | 0.0327 | 0.0025 | 0.7393 | −0.0240 | 0.0484 | 0.0488 | 0.0029 | 0.914 |
| | | Hsieh | 0.0252 | 0.0356 | | 0.0019 | | −0.0176 | 0.0523 | | 0.0030 | |

The sample size is 100 and replications are 400

**Table 4** Estimations of the quantile regression parameters under model (12) with Frank copula

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| 0.3 | 0.1 | Propose | 0.0037 | 0.0166 | 0.0180 | 0.0003 | 0.918 | 0.0009 | 0.0398 | 0.0401 | 0.0016 | 0.943 |
| | | Peng | 0.0157 | 0.0175 | 0.0177 | 0.0006 | 0.842 | 0.0155 | 0.0409 | 0.0405 | 0.0019 | 0.917 |
| | | Hsieh | 0.0059 | 0.0260 | | 0.0007 | | 0.0024 | 0.0418 | | 0.0017 | |
| | 0.3 | Propose | 0.0011 | 0.0249 | 0.0246 | 0.0006 | 0.938 | −0.0067 | 0.0570 | 0.0594 | 0.0033 | 0.933 |
| | | Peng | 0.0233 | 0.0199 | 0.0197 | 0.0009 | 0.737 | 0.0279 | 0.0484 | 0.0472 | 0.0031 | 0.885 |
| | | Hsieh | 0.0035 | 0.0221 | | 0.0005 | | 0.0021 | 0.0518 | | 0.0027 | |
| | 0.5 | Propose | 0.0053 | 0.0220 | 0.0244 | 0.0005 | 0.923 | −0.0115 | 0.0527 | 0.0550 | 0.0029 | 0.948 |
| | | Peng | 0.0221 | 0.0209 | 0.0190 | 0.0009 | 0.725 | 0.0172 | 0.0409 | 0.0447 | 0.0020 | 0.890 |
| | | Hsieh | 0.0070 | 0.0214 | | 0.0005 | | −0.0042 | 0.0462 | | 0.0022 | |
| 0.5 | 0.1 | Propose | 0.0045 | 0.0181 | 0.0195 | 0.0003 | 0.915 | 0.0060 | 0.0441 | 0.0437 | 0.0020 | 0.923 |
| | | Peng | 0.0317 | 0.0186 | 0.0187 | 0.0013 | 0.595 | 0.0339 | 0.0424 | 0.0442 | 0.0030 | 0.852 |
| | | Hsieh | 0.0120 | 0.0275 | | 0.0009 | | 0.0099 | 0.0554 | | 0.0032 | |
| | 0.3 | Propose | 0.0010 | 0.0242 | 0.0234 | 0.0006 | 0.900 | −0.0049 | 0.0526 | 0.0554 | 0.0028 | 0.918 |
| | | Peng | 0.0383 | 0.0176 | 0.0190 | 0.0018 | 0.445 | 0.0451 | 0.0421 | 0.0440 | 0.0038 | 0.805 |
| | | Hsieh | 0.0120 | 0.0206 | | 0.0006 | | 0.0144 | 0.0461 | | 0.0023 | |
| | 0.5 | Propose | 0.0006 | 0.0212 | 0.0220 | 0.0004 | 0.938 | −0.0043 | 0.0480 | 0.0496 | 0.0023 | 0.930 |
| | | Peng | 0.0315 | 0.0176 | 0.0173 | 0.0013 | 0.517 | 0.0316 | 0.0348 | 0.0403 | 0.0022 | 0.860 |
| | | Hsieh | 0.0123 | 0.0199 | | 0.0006 | | 0.0135 | 0.0434 | | 0.0021 | |
| 0.7 | 0.1 | Propose | 0.0050 | 0.0236 | 0.0202 | 0.0006 | 0.855 | 0.0145 | 0.0505 | 0.0402 | 0.0028 | 0.835 |
| | | Peng | 0.0516 | 0.0158 | 0.0169 | 0.0029 | 0.185 | 0.0535 | 0.0381 | 0.0397 | 0.0043 | 0.699 |
| | | Hsieh | 0.0334 | 0.0268 | | 0.0018 | | 0.0231 | 0.0634 | | 0.0046 | |
| | 0.3 | Propose | 0.0003 | 0.0242 | 0.0218 | 0.0006 | 0.900 | −0.0078 | 0.0528 | 0.0484 | 0.0028 | 0.918 |

**Table 4** continued

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| | | Peng | 0.0486 | 0.0148 | 0.0156 | 0.0026 | 0.145 | 0.0501 | 0.0327 | 0.0370 | 0.0036 | 0.707 |
| | | Hsieh | 0.0375 | 0.0183 | | 0.0017 | | 0.0359 | 0.0411 | | 0.0030 | |
| | 0.5 | Propose | 0.0042 | 0.0190 | 0.0197 | 0.0004 | 0.952 | −0.0002 | 0.0419 | 0.0459 | 0.0018 | 0.962 |
| | | Peng | 0.0346 | 0.0143 | 0.0157 | 0.0014 | 0.403 | 0.0420 | 0.0344 | 0.0357 | 0.0029 | 0.749 |
| | | Hsieh | 0.0375 | 0.0253 | | 0.0020 | | 0.0427 | 0.0548 | | 0.0048 | |

The sample size is 100 and replications are 400

**Table 5** Estimations of the quantile regression parameters for robustness settings

| $\tau$ | $\gamma$ | Method | $\beta^0(\gamma)$ | | | | | $\beta^1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Bias | EmpSd | Avesd | Mse | C.P. | Bias | EmpSd | Avesd | Mse | C.P. |
| 0.3 | 0.1 | Propose | 0.0145 | 0.0272 | 0.0284 | 0.0009 | 0.879 | −0.0074 | 0.0365 | 0.0388 | 0.0014 | 0.934 |
| | 0.3 | Propose | 0.0183 | 0.0277 | 0.0299 | 0.0011 | 0.905 | −0.0081 | 0.0369 | 0.0404 | 0.0014 | 0.930 |
| | 0.5 | Propose | 0.0330 | 0.0302 | 0.0339 | 0.0020 | 0.761 | −0.0215 | 0.0395 | 0.0441 | 0.0020 | 0.882 |
| 0.5 | 0.1 | Propose | 0.0124 | 0.0383 | 0.0416 | 0.0016 | 0.912 | −0.0051 | 0.0499 | 0.0556 | 0.0025 | 0.942 |
| | 0.3 | Propose | 0.0130 | 0.0365 | 0.0357 | 0.0015 | 0.887 | −0.0081 | 0.0470 | 0.0496 | 0.0023 | 0.949 |
| | 0.5 | Propose | 0.0127 | 0.0352 | 0.0372 | 0.0014 | 0.897 | −0.0139 | 0.0472 | 0.0493 | 0.0024 | 0.927 |
| 0.7 | 0.1 | Propose | 0.0033 | 0.0390 | 0.0402 | 0.0015 | 0.922 | −0.0033 | 0.0555 | 0.0575 | 0.0031 | 0.925 |
| | 0.3 | Propose | 0.0036 | 0.0391 | 0.0403 | 0.0015 | 0.907 | −0.0131 | 0.0503 | 0.0511 | 0.0027 | 0.942 |
| | 0.5 | Propose | −0.0076 | 0.0399 | 0.0411 | 0.0016 | 0.914 | 0.0034 | 0.0558 | 0.0533 | 0.0031 | 0.964 |

The sample size is 100 and replications are 400

**Table 6** The power of $U$

| Kendall's $\tau$ | Quantile $\gamma$ | $h(t) = log(t)$ | $h(t) = I(t)$ | $h(t) = 2(t^{1/2} - 1)$ |
|---|---|---|---|---|
| 0.3 | 0.1 | 0.04 | 0.80 | 0.82 |
| | 0.3 | 0.05 | 0.98 | 0.37 |
| | 0.5 | 0.05 | 0.58 | 0.27 |
| 0.5 | 0.1 | 0.06 | 0.82 | 0.53 |
| | 0.3 | 0.04 | 0.94 | 0.25 |
| | 0.5 | 0.06 | 0.23 | 0.27 |
| 0.7 | 0.1 | 0.05 | 0.75 | 0.18 |
| | 0.3 | 0.05 | 0.58 | 0.11 |
| | 0.5 | 0.05 | 0.64 | 0.13 |

The sample size is 200 and replications are 200

**Table 7** Selection probability based on $U$

| Kendall's $\tau$ | Quantile $\gamma$ | $h(t) = log(t)$ | $h(t) = I(t)$ | $h(t) = 2(t^{1/2} - 1)$ |
|---|---|---|---|---|
| 0.3 | 0.1 | 0.9 | 0.06 | 0.04 |
| | 0.3 | 0.785 | 0 | 0.215 |
| | 0.5 | 0.78 | 0.1 | 0.12 |
| 0.5 | 0.1 | 0.8 | 0.12 | 0.08 |
| | 0.3 | 0.69 | 0.01 | 0.3 |
| | 0.5 | 0.605 | 0.27 | 0.125 |
| 0.7 | 0.1 | 0.63 | 0.07 | 0.3 |
| | 0.3 | 0.625 | 0.17 | 0.205 |
| | 0.5 | 0.65 | 0.08 | 0.27 |

The sample size is 200 and replications are 200

## 5 Data analysis

In this section, we apply our proposed methodology to a real data, the Bone Marrow Transplant data, provided by Klein and Moeschberger (2003). There were 45 leukemia patients receiving bone marrow transplants with acute myelocytic leukemia high-risk remission (AML high-risk). Let $T$ be the relapse time of leukemia from bone marrow transplant, $D$ be the death time from bone marrow transplant, and $C$ be the censoring time. Thus, the observed variables are $X = T \wedge D \wedge C, Y = D \wedge C, \delta_x = I(T \leq D \wedge C)$, and $\delta_y = I(D \leq C)$, which is a semi-competing risks data. In this analysis, we would like to investigate how the time to return of platelets to normal levels, $T_p$, affects the relapse time. The covariate is coded as $Z = 0$ if $T_p \geq 19$; $Z = 1$ if $T_p < 19$, where 19 is the median of $T_p$. Thus, we consider the quantile regression model as:

$$\xi_\gamma(log(T)|Z) = \beta_0(\gamma) + \beta_1(\gamma)Z. \tag{14}$$

**Table 8** The estimations of $\beta_0(\gamma)$, $\beta_1(\gamma)$ for data analysis

| Quantile $\gamma$ | $\beta_0(\gamma)$ | | | | $\beta_1(\gamma)$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\beta}_0(\gamma)$ | Sd | 95%CI | | $\hat{\beta}_1(\gamma)$ | Sd | 95%CI | | p-value |
| 0.1 | 4.101 | 0.195 | 3.719 | 4.483 | −0.560 | 0.562 | −1.661 | 0.541 | 0.874 |
| 0.2 | 4.345 | 0.220 | 3.915 | 4.776 | 0.401 | 0.629 | −0.831 | 1.633 | 0.878 |
| 0.3 | 4.544 | 0.407 | 3.746 | 5.342 | 0.961 | 0.770 | −0.548 | 2.471 | 0.715 |
| 0.4 | 4.717 | 0.646 | 3.449 | 5.983 | 1.732 | 0.918 | −0.068 | 3.533 | 0.774 |

By the suggested methodology with Clayton copula, the results are summarized in Table 8 based on $B$=1000 bootstrap replications. From the p values in Table 8 by Sect. 3.3 with $q(z) = 1/(z+0.2)^2$, it shows that the considered model (14) is adaptive for this data set. To check the copula model, we use the model checking approach by Hsieh et al. (2008). With Clayton copula for $(T, D)$, the p values are 0.47 for $Z = 0$ and 0.82 for $Z = 1$. Thus, the Clayton copula is adaptive for the data. Further, under the one to one relationship between $\alpha$ and $\tau$, $\hat{\tau} = 0.62$ for $Z = 0$ and $\hat{\tau} = 0.83$ for $Z = 1$.

In Table 8, we present the quantile regression parameter estimator, the standard deviation, and the 95% confidence interval for quantiles 0.1, 0.2, 0.3, and 0.4. For quantiles larger than 0.5, the quantile regression parameters cannot be identified due to censoring. From the results in Table 8, it shows that the covariate effect on the quantile of the relapse is increasing as quantile increases. The 10% quantile of the relapse time for "$T_p < 19$" is 0.571 times the patients with "$T_p \geq 19$". The 20% quantile of the relapse time for "$T_p < 19$" is 1.493 times the patients with "$T_p \geq 19$". The 30% quantile of the relapse time for "$T_p < 19$" is 2.615 times the patients with "$T_p \geq 19$". The 40% quantile of the relapse time for "$T_p < 19$" is 5.653 times the patients with "$T_p \geq 19$".

## 6 Concluding remarks

Quantile regression has received increasing attention in survival analysis. In this paper, we consider the quantile regression model to analyze the nonterminal event time under semi-competing risks data. In semi-competing risks data, the failure time $T$ is dependently censored by $D$, which makes the inference on $T$ complicated. Here, we assume the Archimedean copula model to specify the dependence between $T$ and $D$. Under the Archimedean copula assumption, this study utilizes the counting process approach to construct an estimating equation for the quantile regression parameter, and provide the consistency and weak convergence properties of the proposed estimator. This study uses the bootstrap method to obtain the variance estimation of $\hat{\beta}(\gamma)$ and provides a model diagnostic approach to check the adequacy of the fitted model. From the simulation studies, it shows that the proposed methods perform well. Furthermore, we apply our proposed methodology to analyze a data set of Bone Marrow Transplant. The proposed method provides an estimator of the quantile regression parameter for a interested quantile. When multiple quantiles are interested, we need to perform the estimation procedure many times to obtain the estimators for the multiple quantiles. The restriction of the suggested approach is that it can be applied to discrete covariates only. For continuous covariates, we can group it as categorical variables or deal with it with smoothing technique, which is treated as a future work. The approach by Hsieh et al. (2013) and the proposed method do not need to set a quantile regression model on the terminal event time, but the covariates of the two approaches are restricted to discrete cases. The approach by Li and Peng (2015) needs to assume a quantile regression model on the terminal event time but the covariates of the approach could be continuous.

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## Appendix A: The Proof of Theorem 1

Define $N(t) = I(X \le t, \delta_x = 1)$,

$$\Lambda(t|z) = \begin{cases} -\log(P(T > t|D = d, Z = z)), & \text{if } D = d, \\ -\log(P(T > t|D > c, Z = z)), & \text{if } D > c, \end{cases}$$

$S_n(b) = \frac{1}{n}\sum_{i=1}^n Z_i\big[N_i(h^{-1}(Z_i^{'}b)) - \hat{\Lambda}(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)\big]$, $S_n^0(b) = \frac{1}{n}\sum_{i=1}^n Z_i\big[N_i(h^{-1}(Z_i^{'}b)) - \Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)\big]$, and $\bar{S}(b)$= the limit of $S_n^0(b)$. Let $E_i$ be the event defined from the censoring status of $D_i$, which is the event "$D_i = d_i$" or "$D_i > c_i$". Firstly, we prove that $\bar{S}(\beta_0(\gamma)) = 0$. Let $M_i(t) = N_i(t) - \Lambda(t \wedge X_i|z_i)$, where $\Lambda(t|z_i)$ is the cumulative hazard function of $T$ condition on $Z = z_i$ and the event "$D = d_i^{''}$ or "$D > c_i^{''}$. From Fleming and Harrington (1991), $M_i$ is the martingale process associated with the counting process $N_i$ under "$Z = z_i$" and the event defined by $D$ which has the form of "$D = d_i$" or "$D > c_i$". Thus, we have $E(M_i(t)|Z = z_i, E_i) = 0$, for $t \ge 0$. Then

$$E\big[N_i(h^{-1}(Z_i^{'}\beta_0(\gamma))) - \Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)|Z_i, E_i\big] = 0.$$

Therefore, we have

$$\bar{S}(\beta_0(\gamma)) = 0 \tag{A.1}$$

Next, we show the proof of the consistency. Define $\mathcal{F}_1 = \{Z_i I(X_i \le h^{-1}(Z_i^{'}b(\gamma)))\delta_{x_i} : b(\gamma) \in R^{P+1}, Z_i : \text{bounded}\}$. $\mathcal{F}_1$ is Glivenko–Cantelli (Sect. 2.4, van der Vaart and Wellner 1996), because the class of indicator functions of polytopes in $R^{P+1}$ is Glivenko–Cantelli and $Z_i$ is bounded. Thus

$$\sup_b ||S_n^0(b) - \bar{S}(b)|| \xrightarrow{p} 0. \tag{A.2}$$

By straightforward calculation, we have

$$S_n(b) = S_n^0(b) + (S_n(b) - S_n^0(b)).$$

Note that

$$\begin{aligned}
&\sup_b \|S_n(b) - S_n^0(b)\| \\
&\quad = \left\| \frac{1}{n} \sum_{i=1}^{n} Z_i (\hat{\Lambda}(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i) - \Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)) \right\| \\
&\quad \le \frac{1}{n} \sum_{i=1}^{n} \|Z_i\| \|\hat{\Lambda}(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i) - \Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)\|. \qquad \text{(A.3)}
\end{aligned}$$

Let $M_n = \max_i \|\hat{\Lambda}(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i) - \Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)\|$. From Hsieh et al. (2013),

$$\max_i \|\hat{\Lambda}(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i) - \Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)\| \xrightarrow{p} 0.$$

Thus, $M_n \xrightarrow{p} 0$. Because $Z$ is bounded, $\frac{1}{n}\sum_{i=1}^{n} \|Z_i\|$ is bounded. Thus, we have that

$$(A.3) \le M_n \frac{1}{n} \sum_{i=1}^{n} \|Z_i\| \xrightarrow{p} 0. \qquad \text{(A.4)}$$

Therefore,

$$\begin{aligned}
&\sup_b \|S_n(b) - \bar{S}(b)\| \\
&\quad = \sup_b \|S_n^0(b) - \bar{S}(b) + S_n(b) - S_n^0(b)\| \\
&\quad \le \sup_b \|S_n^0(b) - \bar{S}(b)\| + \sup_b \|S_n(b) - S_n^0(b)\| \xrightarrow{p} 0. \\
&\text{(from } (A.2) \text{ and } (A.4)) \qquad \text{(A.5)}
\end{aligned}$$

From (A.1), we have $\bar{S}(\beta_0(\gamma)) = 0$ and $\bar{S}(b) \ne 0$ for $b \ne \beta_0(\gamma)$ by assumption C4. Consider a compact set $D_d = \{b : \|b - \beta_0(\gamma)\| \le d\}$, where $d$ is a positive constant. The continuity of $\bar{S}(b)$ implies that $\inf_{\{b:\|b-\beta_0(\gamma)\| \ge d\}} \|\bar{S}(b)\| > 0$. The uniformly convergence of $S_n(b)$ to $\bar{S}(b)$ implies that there will be no solution for $S_n(b) = 0$ outside the compact set $D_d$ when $n$ is large. Since this is true for every $d > 0$, $\hat{\beta}(\gamma)$ is consistent.

## Appendix B: The Proof of Theorem 2

Let

$$
\begin{aligned}
S_n(b,\beta_0(\gamma)) &= \frac{1}{n}\sum_{i=1}^{n} Z_i\big[N_i(h^{-1}(Z_i'b)) - \hat{\Lambda}(h^{-1}(Z_i'\beta_0(\gamma)) \wedge X_i|Z_i)\big],\\
S_n^0(b,\beta_0(\gamma)) &= \frac{1}{n}\sum_{i=1}^{n} Z_i\big[N_i(h^{-1}(Z_i'b)) - \Lambda(h^{-1}(Z_i'\beta_0(\gamma)) \wedge X_i|Z_i)\big],\\
\bar{S}(b,\beta_0(\gamma)) &= \text{ the limit of } S_n^0(b,\beta_0(\gamma)),\\
\mu(b) &= E[ZN(h^{-1}(Z'b))].
\end{aligned}
$$

By the two-stage estimation procedure and Lemma B.1 of Peng and Huang (2008), we have

$$
\begin{aligned}
&\sqrt{n}S_n(\hat{\beta}(\gamma),\beta_0(\gamma)) - \sqrt{n}S_n(\beta_0(\gamma),\beta_0(\gamma))\\
&\quad = \frac{1}{\sqrt{n}}\sum_{i=1}^{n} Z_i[N_i(h^{-1}(Z_i'\hat{\beta}(\gamma))) - N_i(h^{-1}(Z_i'\beta_0(\gamma)))]\\
&\quad = \sqrt{n}[\mu(\hat{\beta}(\gamma)) - \mu(\beta_0(\gamma))] + o_p(1).
\end{aligned}
$$

By $S_n(\hat{\beta}(\gamma),\beta_0(\gamma)) = 0$ and Taylor expression,

$$
\begin{aligned}
-\sqrt{n}S_n(\beta_0(\gamma),\beta_0(\gamma)) &= \sqrt{n}[\mu(\hat{\beta}(\gamma)) - \mu(\beta_0(\gamma))] + o_p(1)\\
&\approx \mu'(\beta_0(\gamma))\sqrt{n}(\hat{\beta}(\gamma) - \beta_0(\gamma)).
\end{aligned}
$$

Thus

$$
\sqrt{n}(\hat{\beta}(\gamma) - \beta_0(\gamma)) \approx -[\mu'(\beta_0(\gamma))]^{-1}\sqrt{n}S_n(\beta_0(\gamma),\beta_0(\gamma)), \tag{A.6}
$$

where $\mu'(\beta_0(\gamma)) = \frac{\partial\mu(b)}{\partial b}|_{b=\beta_0(\gamma)}$. Note that

$$
\begin{aligned}
&\sqrt{n}S_n(\beta_0(\gamma),\beta_0(\gamma))\\
&\quad = \sqrt{n}\frac{1}{n}\sum_{i=1}^{n} Z_i\big[N_i(h^{-1}(Z_i'\beta_0(\gamma))) - \hat{\Lambda}(h^{-1}(Z_i'\beta_0(\gamma)) \wedge X_i|Z_i)\big]\\
&\quad = \sqrt{n}\frac{1}{n}\sum_{i=1}^{n} Z_i\big[\{N_i(h^{-1}(Z_i'\beta_0(\gamma))) - \Lambda(h^{-1}(Z_i'\beta_0(\gamma)) \wedge X_i|Z_i)\}\\
&\qquad + \{\Lambda(h^{-1}(Z_i'\beta_0(\gamma)) \wedge X_i|Z_i) - \hat{\Lambda}(h^{-1}(Z_i'\beta_0(\gamma)) \wedge X_i|Z_i)\}\\
&\quad = \sqrt{n}\frac{1}{n}\sum_{i=1}^{n} Z_i\{N_i(h^{-1}(Z_i'\beta_0(\gamma))) - \Lambda(h^{-1}(Z_i'\beta_0(\gamma)) \wedge X_i|Z_i)\}
\end{aligned}
$$

$$+ \sqrt{n}\frac{1}{n}\sum_{i=1}^{n} Z_i\{\Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i) - \hat{\Lambda}(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)\}$$
$$= A + B.$$

Thus, from (A.6), we have

$$\sqrt{n}(\hat{\beta}(\gamma) - \beta_0(\gamma)) = -[\mu^{'}(\beta_0(\gamma))]^{-1}[A + B]$$
$$\Rightarrow \sqrt{n}(\hat{\beta}(\gamma) - \beta_0(\gamma)) = (-[\mu^{'}(\beta_0(\gamma))]^{-1})(A + B)$$

Define $\mathcal{F}_2 = \{N_i(h^{-1}(Z_i^{'}\beta_0(\gamma))) - \Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i) : \gamma \in (0, 1)\}$. Note that $\{N_i(h^{-1}(Z_i^{'}\beta_0(\gamma))) : \gamma \in (0, 1)\}$ is a VC-class (van der Vaart and Wellner 1996) and $\Lambda(h^{-1}(Z_i^{'}\beta_0(\gamma)) \wedge X_i|Z_i)$ is Lipschitz in $\gamma$. By the permanence properties of the Donsker class, $\mathcal{F}_2$ is a Donsker class. Thus, $A$ converges weakly to a tight Gaussian process with zero mean. Furthermore, $A$ is the form of sum of iid terms. By section 3.2 in Fleming and Harrington (1991), the appendix of Lakhal et al. (2008) and delta method, $B$, can be represented as sum of the iid terms and each term has zero mean. Thus, $\sqrt{n}(\hat{\beta}(\gamma) - \beta_0(\gamma))$ converges to joint normal with zero mean.

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