ON MOMENTS OF ORDER STATISTICS FROM INDEPENDENT BINOMIAL POPULATIONS*

SHANTI S. GUPTA AND S. PANCHAPAKESAN

(Received Aug. 1, 1972)

1. Introduction and summary

In practice many problems arise which involve order statistics from discrete distributions, in particular, from binomial distributions while dealing with attribute type or categorical data. Order statistics from discrete distributions have earlier been studied by Siotani [4], Khatri [3] and Gupta [1]. The present paper is concerned with the order statistics from independent binomial populations.

Let X_i $(i=1, 2, \cdots, M)$ be the number of successes in N independent Bernoulli trials with p as the probability of a success in each trial. Let $X_{(1)} \leqq X_{(2)} \leqq \cdots \leqq X_{(M)}$ denote the ordered X_i. The discrete variable X_i or $X_{(i)}$ takes values $0, 1, \cdots, N$ for all $i=1, 2, \cdots, M$. Section 2 gives the expressions for the distribution function and the first two moments of $X_{(i)}$. The joint distribution of $X_{(i)}$ and $X_{(j)}$ $(i<j)$ is derived (Section 3) and an explicit form for the covariance is provided in the case of $i=1$ and $j=M$. The distribution of generalized range, $X_{(j)}-X_{(i)}$, is obtained in Section 4 with emphasis on the special cases: (i) $i=1$, $j=M$ and (ii) $j=i+1$. The present paper provides tables for the mean and variance of $X_{(1)}$ and $X_{(M)}$ for $M=1(1)10$, $N=1(1)15$ and $p=0.1(0.1)0.5$. Tables of the cumulative distribution function of $X_{(1)}$ and $X_{(M)}$ are also given for some selected values of p. Some applications and discription of these tables are given in the last section.

2. Moments of order statistics

We adopt the following notations:

$$b(\alpha) \equiv b(\alpha; p, N) = \binom{N}{\alpha} p^{\alpha}(1-p)^{N-\alpha}, \qquad \alpha=0, 1, \cdots, N,$$

* This research was supported by the Office of Naval Research Contract N00014-67-A-0226-0014 and by the Aerospace Research Laboratories Contract AF33(615)67C1244 at Purdue University. Reproduction in whole or in part is permitted for any purpose of The United States Government.

$$B(s)=\sum_{\alpha=0}^{s} b(\alpha) ,$$

$$B(p, q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} ,$$

$$I_p(a, b)=\frac{1}{B(a, b)}\int_0^p u^{a-1}(1-u)^{b-1} du , \qquad a, b>0 .$$

Let $p_i(x)$ be the probability that the ith order statistic $X_{(i)}$ is equal to x and let $P_i(x)=\mathrm{P}\{X_{(i)}\leqq x\}$ be the c.d.f. of $X_{(i)}$. Khatri [3] has obtained the following results.

$$(2.1) \qquad p_i(x)=\sum_{k=0}^{i-1}\sum_{m=0}^{M-i}\frac{M!}{(i-1-k)!\,(k+m+1)!\,(M-i-m)!} \times\{B(x-1)\}^{i-1-k}\{b(x)\}^{k+m+1}\{1-B(x)\}^{M-i-m}$$

where $B(x-1)=0$ for $x=0$. This can be rewritten as

$$(2.2) \qquad p_i(x)=i\binom{M}{i}\int_{B(x-1)}^{B(x)}\omega^{i-1}(1-\omega)^{M-i}\,d\omega = I_{B(x)}(i, M-i+1)-I_{B(x-1)}(i, M-i+1) .$$

Further

$$(2.3) \qquad P_i(x)=i\binom{M}{i}\int_0^{B(x)}\omega^{i-1}(1-\omega)^{M-i}\,d\omega=I_{B(x)}(i, M-i+1) ,$$

$$(2.4) \qquad \mathrm{E}(X_{(i)})=\sum_{x=0}^{N-1}[1-P_i(x)]$$

and

$$(2.5) \qquad \mathrm{E}(X_{(i)}^2)=2\sum_{x=0}^{N-1}x[1-P_i(x)]+\sum_{x=0}^{N-1}[1-P_i(x)] .$$

It should be pointed out that the expressions (2.1) through (2.5) hold for any discrete distribution with proper choice of the probability mass $b(\alpha)$. Also, it can be easily seen that for both discrete and continuous random variables

$$(2.6) \qquad P_i(x)=\sum_{t=i}^{M}\binom{M}{t}[B(x)]^t[1-B(x)]^{M-t}$$

where $B(x)$ is to be interpreted as the usual c.d.f. in the continuous case. Now (2.3) follows from (2.6), since

$$(2.7) \qquad \sum_{t=i}^{M}\binom{M}{t}p^t(1-p)^{M-t}=\frac{1}{B(i, M-i+1)}\int_0^p\omega^{i-1}(1-\omega)^{M-i}\,d\omega$$

where $0\leqq p<1$. (When $B(x)=1$, (2.3) is obviously true.) Now (2.2) follows at once from (2.3) by noting that

$$p_i(x)=P_i(x)-P_i(x-1)\ .$$

For the special cases $i=1$ and $i=M$, we obtain the following results from (2.3), (2.4) and (2.5).

$$p_1(x)=[1-B(x-1)]^M-[1-B(x)]^M \tag{2.8}$$

$$P_1(x)=1-[1-B(x)]^M \tag{2.9}$$

$$p_M(x)=[B(x)]^M-[B(x-1)]^M \tag{2.10}$$

$$P_M(x)=[B(x)]^M \tag{2.11}$$

$$\mathrm{E}\,(X_{(1)})=\sum_{x=0}^{N-1}[1-B(x)]^M \tag{2.12}$$

$$\mathrm{E}\,(X_{(1)}^2)=2\sum_{x=0}^{N-1}x[1-B(x)]^M+\mathrm{E}\,(X_{(1)}) \tag{2.13}$$

$$\mathrm{E}\,(X_{(M)})=\sum_{x=0}^{N-1}[1-\{B(x)\}^M] \tag{2.14}$$

$$\mathrm{E}\,(X_{(M)}^2)=2\sum_{x=0}^{N-1}x[1-\{B(x)\}^M]+\mathrm{E}\,(X_{(M)})\ . \tag{2.15}$$

3. Joint distribution of $X_{(i)}$ and $X_{(j)}$, $i<j$

Let $p_{i,j}(x, y)$ $(i<j)$ be the probability that $X_{(i)}$ is equal to x and $X_{(j)}$ is equal to y and let $P_{i,j}(x, y)=\mathrm{P}(X_{(i)}\leqq x,\ X_{(j)}\leqq y)$, If $x\geqq y$,

$$\begin{aligned} P_{i,j}(x, y) &= \mathrm{P}\{X_{(j)}\leqq y\} \\ &= j\binom{M}{j}\int_0^{B(y)} u^{i-1}(1-u)^{M-j}\,du\ . \end{aligned} \tag{3.1}$$

If $x<y$, then a combinatorial argument leads to

$$\begin{aligned} P_{i,j}(x, y) &= \sum_{\substack{s=i\\ s+t\leqq M}}^{M}\sum_{t=0}^{M-j}\frac{M!}{s!\,(M-s-t)!\,t!}\{B(x)\}^s\{B(y)-B(x)\}^{M-s-t}\{1-B(y)\}^t \\ &= \sum_{s=i}^{j}\sum_{t=0}^{M-j}\frac{M!}{s!\,(M-s-t)!\,t!}\{B(x)\}^s\{B(y)-B(x)\}^{M-s-t}\{1-B(y)\}^t \\ &\quad +(1-\delta_{jM})\sum_{s=j+1}^{M}\sum_{t=0}^{M-s}\frac{M!}{s!\,(M-s-t)!\,t!}\{B(x)\}^s\{B(y)-B(x)\}^{M-s-t}\{1-B(y)\}^t \end{aligned}$$

where δ_{jM} is the Kronecker delta.

By repeated application of the results

$$\sum_{t=a}^{n}\binom{n}{t}p^t(k-p)^{n-t}=\frac{1}{B(a,\,n-a+1)}\int_0^p u^{a-1}(k-u)^{n-a}\,du$$

and

$$\sum_{t=0}^{b}\binom{n}{t}p^t(k-p)^{n-t}=k^n-\frac{1}{B(b+1,\,n-b)}\int_0^p u^b(k-u)^{n-b-1}\,du$$

where $0\leqq p<1$ and $p<k$, we obtain

$$(3.2)\qquad P_{i,j}(x,\,y)=i\binom{M}{i}\int_0^{B(x)}u^{i-1}(1-u)^{M-i}\,du \\ -\frac{M!}{(i-1)!\,(j-i-1)!\,(M-j)!} \\ \times\int_{B(y)}^{1}dv\int_0^{B(x)}\omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\,d\omega\,.$$

Now we can write

$$(3.3)\qquad p_{i,j}(x,\,y)=\begin{cases}0\,, & \text{if } x>y\\ P_{i,j}(x,\,x)-P_{i,j}(x-1,\,x)\,, & \text{if } x=y\\ P_{i,j}(x,\,y)-P_{i,j}(x-1,\,y) & \\ \qquad -P_{i,j}(x,\,y-1)+P_{i,j}(x-1,\,y-1)\,, & \text{if } x<y\,.\end{cases}$$

Khatri [3] has obtained the joint distribution directly in the form

$$(3.4)\qquad p_{i,j}(x,\,y)=\frac{M!}{(i-1)!\,(j-i-1)!\,(M-j)!}\iint\omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\,d\omega\,dv$$

where the double integration is performed over the region given by

$$v\geqq\omega\,,$$
$$\mathrm{P}(x)\geqq\omega\geqq\mathrm{P}(x-1)\,,$$
$$\mathrm{P}(y)\geqq v\geqq\mathrm{P}(y-1)\,.$$

But Khatri has obtained the c.d.f. $P_{i,j}(x,\,y)$ $(x\leqq y)$ in the form

$$(3.5)\qquad P_{i,j}(x,\,y)=\frac{M!}{(i-1)!\,(j-i-1)!\,(M-j)!} \\ \times\int_{B(x)}^{B(y)}dv\int_0^{B(x)}\omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\,d\omega \\ +j\binom{M}{j}\int_0^{B(x)}v^{j-1}(1-v)^{M-j}\,dv$$

which is valid only for $x\leqq y$.

Incidentally, from (3.2) and (3.5) we get the relation

$$(3.6)\qquad \frac{M!}{(i-1)!\,(j-i-1)!\,(M-j)!}\int_{B(x)}^{1}dv\int_0^{B(x)}\omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\,d\omega \\ =i\binom{M}{i}\int_0^{B(x)}u^{i-1}(1-u)^{M-i}\,du-j\binom{M}{j}\int_0^{B(x)}u^{j-1}(1-u)^{M-j}\,du \\ =I_{B(x)}(i,\,M-i+1)-I_{B(x)}(j,\,M-j+1)\,.$$

Now

$$\mathrm{E}(X_{(i)}X_{(j)})=\sum_{x=0}^{N}\sum_{y=0}^{N} xyp_{i,j}(x, y)$$

$$=\sum_{x=0}^{N} x^2p_{i,j}(x, x)+\sum_{x=0}^{N-1}\sum_{y=x+1}^{N} xyp_{i,j}(x, y) .$$

So

$$\text{(3.7)}\qquad \mathrm{Cov}(X_{(i)}, X_{(j)})=\sum_{x=0}^{N} x^2p_{i,j}(x, x)+\sum_{x=0}^{N-1}\sum_{y=x+1}^{N} xyp_{i,j}(x, y) - \left\{\sum_{x=0}^{N-1}[1-P_i(x)]\right\}\left\{\sum_{x=0}^{N-1}[1-P_j(x)]\right\} .$$

An explicit expression for $\mathrm{Cov}(X_{(i)}, X_{(j)})$ is very complicated. However, for the special case where $i=1$ and $j=M$, we obtain by usual algebraic simplifications,

$$\text{(3.8)}\qquad \mathrm{Cov}(X_{(1)}, X_{(M)})=N\mathrm{E}(X_{(1)})-(1-\delta_{N1})\sum_{y=1}^{N-1}\sum_{x=0}^{y-1}[B(y)-B(x)]^M - \mathrm{E}(X_{(1)})\,\mathrm{E}(X_{(M)}) ,$$

where $\mathrm{E}(X_{(1)})$ and $\mathrm{E}(X_{(M)})$ are given by (2.12) and (2.14) and δ_{N1} is the Kronecker delta.

4. Distribution of $Y_{i,j}=X_{(j)}-X_{(i)}$ $(j>i)$

$Y_{i,j}$ represents a generalized range and can take values $0, 1, \cdots, N$. For $r\geqq 0$,

$$\text{(4.1)}\qquad \mathrm{P}(Y_{i,j}=r)=\sum_{k=0}^{N-r}\frac{M!}{(i-1)!\,(j-i-1)!\,(M-j)!} \times\iint_A \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\,dv\,d\omega$$

where A is the region given by

$$v\geqq\omega ,$$

$$B(k)\geqq\omega\geqq B(k-1) ,$$

$$B(k+r)\geqq v\geqq B(k+r-1) .$$

This can be rewritten as

(4.2) $$\mathrm{P}\{Y_{i,j}=r\} = \begin{cases} \displaystyle\sum_{k=0}^{N} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{\omega}^{B(k)} \omega^{i-1}(v-\omega)^{j-i-1} \\ \qquad \times (1-v)^{M-j}\,dv\,, \quad r=0\,, \\ \displaystyle\sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1}(v-\omega)^{j-i-1} \\ \qquad \times (1-v)^{M-j}\,dv\,, \quad r>0\,. \end{cases}$$

So

(4.3) $$\mathrm{E}(Y_{i,j}) = \sum_{r=1}^{N} r \sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1}(v-\omega)^{j-i-1} \times (1-v)^{M-j}\,dv\,.$$

But we also know that

(4.4) $$\begin{aligned} \mathrm{E}(Y_{i,j}) &= \mathrm{E}(X_{(j)}) - \mathrm{E}(X_{(i)}) \\ &= \sum_{x=0}^{N-1} i \binom{M}{i} \int_0^{B(x)} \omega^{i-1}(1-\omega)^{M-i}\,d\omega - \sum_{x=0}^{N-1} j \binom{M}{j} \int_0^{B(x)} \omega^{j-1} \times (1-\omega)^{M-j}\,d\omega \\ &= \sum_{x=0}^{N-1} [I_{B(x)}(i, M-i+1) - I_{B(x)}(j, M-j+1)]\,. \end{aligned}$$

The equations (4.3) and (4.4) lead to the relation

(4.5) $$\sum_{r=1}^{N} \sum_{k=0}^{N-r} r \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1}(v-\omega)^{j-i-1} \times (1-v)^{M-j}\,dv = \sum_{x=0}^{N-1} [I_{B(x)}(i, M-i+1) - I_{B(x)}(j, M-j+1)]\,.$$

An explicit expression for variance of $Y_{i,j}$ is complicated. However, for the special case $i=1$ and $j=M$, we obtain

(4.6) $$\mathrm{P}\{Y_{1,M}=r\} = \begin{cases} \displaystyle\sum_{k=0}^{N} [B(k)-B(k-1)]^M = \sum_{k=0}^{N} \{b(k)\}^M, & \text{if } r=0 \\ \displaystyle\sum_{k=0}^{N-r} [\{B(k+r)-B(k-1)\}^M - \{B(k+r)-B(k)\}^M \\ \quad - \{B(k+r-1)-B(k-1)\}^M \\ \quad + \{B(k+r-1)-B(k)\}^M]\,, & \text{if } r>0 \end{cases}$$

This has been obtained by Siotani [4] with different notations from the joint distribution of $X_{(1)}$ and $X_{(M)}$. The use of the tables of range has been made by Siotani and Ozawa [5] for testing the homogeneity of binomial populations.

Another special case of interest, is when $j=i+1$. Then we have

$$(4.7)\quad P(Y_{i,i+1}=r)$$
$$=\begin{cases} \sum_{k=0}^{N}\Big[I_{B(k)}(i, M-i+1)-I_{B(k-1)}(i, M-i+1)-\binom{M}{i}[\{B(k)\} \\ \qquad -\{B(k-1)\}^{i}][1-B(k)]^{M-i}\Big], & r=0, \\ \sum_{k=0}^{N-r}\binom{M}{i}[\{B(k)\}^{i}-\{B(k-1)\}^{i}][\{1-B(k+r-1)\}^{M-i} \\ \qquad -\{1-B(k+r)\}^{M-i}], & r>0. \end{cases}$$

In particular if $i=M-1$, we obtain

$$(4.8)\quad P(Y_{M-1,M}=r)=\begin{cases} \sum_{k=0}^{N}[\{B(k)\}^{M}-\{B(k-1)\}^{M-1} \\ \qquad \times\{MB(k)-(M-1)B(k-1)\}], & r=0 \\ \sum_{k=0}^{N-r} M[\{B(k)\}^{M-1}-\{B(k-1\}^{M-1}]b(k+r), & r>0. \end{cases}$$

5. Asymptotic results

Let $Z_i=\dfrac{X_i-Np}{\sqrt{Np(1-p)}}$. Then, for large N, $P\{X_1\leqq x\}\approx P\{Z_i\leqq z\}$, where Z_i is a normal variate with mean zero and variance unity and $z=\dfrac{x-Np}{\sqrt{Np(1-p)}}$. As the transformation from X to Z preserves order, we have

$$(5.1)\qquad P\{X_{(i)}\leqq x\}\approx P\{Z_{(i)}\leqq z\}$$
$$=i\binom{M}{i}\int_0^{\Phi(z)}u^{i-1}(1-u)^{M-i}\,du\,,$$

where

$$\Phi(z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z}e^{-\frac{t^2}{2}}dt\,.$$

Further we can write

$$(5.2)\qquad E(X_{(i)}^{r})\approx E([Z_{(i)}\sqrt{Np(1-p)}+Np]^{r})$$
$$=\sum_{\alpha=0}^{r}\binom{r}{\alpha}(Np)^{r-\frac{\alpha}{2}}(1-p)^{\frac{\alpha}{2}}E(z_{(i)}^{\alpha})\,.$$

In particular

$$\text{(5.3)} \qquad \mathrm{E}(X_{(i)}) \approx Np + \sqrt{Np(1-p)}\, \mathrm{E}(Z_{(i)})$$

and

$$\text{(5.4)} \qquad \mathrm{V}(X_{(i)}) \approx Np(1-p)\, \mathrm{V}(Z_{(i)})\ .$$

6. Applications and description of the tables

The binomial model is of interest in some statistical inference problems. For example, in a life test experiment, truncated at a fixed time, the number of failures is a binomial random variable. Hence if we assume the c.d.f. of the life distribution to be $p_i = F(t, \theta_i)$ and we are interested in testing hypotheses about p_i or, alternatively, about θ_i, then the distribution of the ordered number of failures becomes relevant especially if one is interested in a ranking or selection problems. More specifically, we give the following examples.

Sobel and Huyett [6], Gupta and Sobel [2] and Gupta [1] have discussed the problems of selection and ranking for the parameters of several binomial populations. The problems discussed deal with selecting a subset or selecting a single population using the indifference zone approach. For both formulations, the probability of a correct selection depends on the distribution of $X_{\max}$ or $X_{\max} - X$ where $X_{\max}$ is the largest of a set of independent and identical binomial random variables and X is another binomial random variable distributed independently of $X_{\max}$. For subset selection problems we are interested in evaluating the probability

$$\text{(6.1)} \qquad \mathrm{P}\{X_{\max} - X \leqq d\} = \sum_{x=0}^{N} \binom{N}{x} p^x (1-p)^{N-x} \left\{ \sum_{\alpha=0}^{x+d} \binom{N}{\alpha} p^{\alpha} (1-p)^{N-\alpha} \right\}^{M-1}$$
$$= \sum_{x=0}^{N} b(x) \{B(x+d)\}^{M-1}\ .$$

Thus, the tables similar to Table II of the present paper are useful in evaluating the above probability. The expected size of the selected subset is given by a similar expression (see (6.3) of Gupta and Sobel [2]) which can be computed with the help of such tables.

Siotani [4] has considered tests of hypotheses of the type $p_1 = p_2 = \cdots = p_M = p$ and suggested the use of the range for this case. He has constructed tables for the distribution of the range. If we are interested in testing $H: p_{(M)} \leqq p_0$, then a quick test for this is as follows:

Reject H if $x_{(1)} > x_0(\alpha, N, M)$ where α is the level of significance. To construct this test, we wish to obtain $x_0(\alpha, N, M)$ satisfying

$$\text{(6.2)} \qquad \sup_{0 < p_{(M)} \leqq p_0 < 1} \mathrm{P}[x_{(1)} > x_0(\alpha, N, M)] \leqq \alpha\ .$$

It can be shown that the probability on the left hand side of (6.2) is maximized when $p_{(M)}=p_0$. Hence the necessary constants $x_0(\alpha, N, M)$ can be obtained from the results of this paper. A similar test for the hypothesis $H: p_{(1)}\geqq p_0$, can be constructed using the rejection region $x_{(M)}<x_0'(\alpha, N, M)$.

The usual tests for outliers suggest the use of statistics $X_{(M)}-X_{(M-i)}$, $1\leqq i\leqq M-1$. The results in Section 4 deal with the distribution of statistics of this type. For the particular case $X_{(M)}-X_{(M-1)}$, a simpler form of the distribution is given by (4.8).

Description of the tables

The tables at the end give the first two moments and the c.d.f. of the largest and smallest of M independent and identical binomial random variables, each denoting the number of successes in N independent trials with p as the associated parameter. For the table of moments the range of values of common p is: $p=0.1(0.1)0.5$; the values of N are: $N=1(1)15$; the values of M are: $M=1(1)10$. These tables were computed in 1960 by Miss Ann Elmer of Bell Telephone Laboratories while one of the authors was a member of the technical staff of Bell Laboratories.

More extensive set of tables for the moments and the c.d.f. of the largest and the smallest order statistic is available and can be obtained by request from the authors.

Table 1.

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.1

N/M	1	2	3	4	5	6	7	8	9	10
1	0.1000	0.1900	0.2710	0.3439	0.4095	0.4686	0.5217	0.5695	0.6126	0.6513
	0.0900	0.1539	0.1976	0.2256	0.2418	0.2490	0.2495	0.2452	0.2373	0.2271
	0.1000	0.0100	0.0010	0.0001	0.0000					
	0.0900	0.0099	0.0010	0.0001	0.0000					
2	0.2000	0.3638	0.4983	0.6089	0.7003	0.7761	0.8392	0.8920	0.9364	0.9740
	0.1800	0.2712	0.3094	0.3169	0.3079	0.2908	0.2708	0.2509	0.2325	0.2165
	0.2000	0.0362	0.0069	0.0013	0.0002	0.0000				
	0.1800	0.0351	0.0068	0.0013	0.0002	0.0000				
3	0.3000	0.5258	0.6972	0.8289	0.9315	1.0126	1.0778	1.1314	1.1764	1.2148
	0.2700	0.3678	0.3864	0.3725	0.3485	0.3246	0.3046	0.2896	0.2795	0.2734
	0.3000	0.0742	0.0199	0.0054	0.0015	0.0004	0.0001	0.0000		
	0.2700	0.0703	0.0196	0.0054	0.0015	0.0004	0.0001	0.0000		
4	0.4000	0.6790	0.8778	1.0232	1.1328	1.2184	1.2874	1.3450	1.3945	1.4382
	0.3600	0.4524	0.4510	0.4243	0.3971	0.3766	0.3635	0.3563	0.3532	0.3525
	0.4000	0.1210	0.0408	0.0140	0.0048	0.0017	0.0006	0.0002	0.0001	0.0000
	0.3600	0.1119	0.0394	0.0138	0.0048	0.0017	0.0006	0.0002	0.0001	0.0000
5	0.5000	0.8256	1.0460	1.2022	1.3188	1.4101	1.4850	1.5487	1.6045	1.6544
	0.4500	0.5304	0.5123	0.4797	0.4544	0.4386	0.4299	0.4253	0.4225	0.4202
	0.5000	0.1744	0.0692	0.0282	0.0115	0.0047	0.0019	0.0008	0.0003	0.0001
	0.4500	0.1576	0.0655	0.0275	0.0114	0.0047	0.0019	0.0008	0.0003	0.0001
6	0.6000	0.9671	1.2058	1.3720	1.4958	1.5940	1.6754	1.7454	1.8069	1.8619
	0.5400	0.6046	0.5735	0.5386	0.5153	0.5015	0.4926	0.4857	0.4790	0.4719
	0.6000	0.2329	0.1044	0.0484	0.0226	0.0106	0.0050	0.0023	0.0011	0.0005
	0.5400	0.2058	0.0965	0.0464	0.0221	0.0105	0.0049	0.0023	0.0011	0.0005
7	0.7000	1.1047	1.3596	1.5354	1.6670	1.7720	1.8597	1.9350	2.0011	2.0597
	0.6300	0.6768	0.6353	0.5991	0.5761	0.5608	0.5487	0.5375	0.5263	0.5152
	0.7000	0.2953	0.1454	0.0746	0.0387	0.0202	0.0105	0.0055	0.0029	0.0015
	0.6300	0.2556	0.1310	0.0700	0.0374	0.0198	0.0104	0.0055	0.0029	0.0015
8	0.8000	1.2392	1.5090	1.6942	1.8334	1.9450	2.0381	2.1180	2.1875	2.2490
	0.7200	0.7477	0.6977	0.6596	0.6348	0.6162	0.6000	0.5848	0.5706	0.5574
	0.8000	0.3608	0.1913	0.1064	0.0602	0.0342	0.0194	0.0111	0.0063	0.0036
	0.7200	0.3064	0.1680	0.0976	0.0570	0.0331	0.0191	0.0110	0.0063	0.0036
9	0.9000	1.3712	1.6550	1.8494	1.9960	2.1135	2.2114	2.2949	2.3675	2.4315
	0.8100	0.8180	0.7604	0.7194	0.6912	0.6687	0.6488	0.6310	0.6153	0.6017
	0.9000	0.4288	0.2414	0.1434	0.0868	0.0530	0.0324	0.0198	0.0121	0.0074
	0.8100	0.3580	0.2066	0.1280	0.0805	0.0504	0.0314	0.0195	0.0120	0.0074
10	1.0000	1.5010	1.7982	2.0015	2.1550	2.2780	2.3801	2.4671	2.5425	2.6089
	0.9000	0.8879	0.8229	0.7782	0.7460	0.7196	0.6970	0.6777	0.6614	0.6478
	1.0000	0.4990	0.2950	0.1848	0.1185	0.0767	0.0498	0.0324	0.0211	0.0137
	0.9000	0.4100	0.2461	0.1605	0.1070	0.0715	0.0475	0.0314	0.0207	0.0136
11	1.1000	1.6292	1.9391	2.1511	2.3111	2.4390	2.5451	2.6353	2.7135	2.7825
	0.9900	0.9576	0.8852	0.8360	0.7995	0.7699	0.7452	0.7249	0.7081	0.6943
	1.1000	0.5708	0.3515	0.2302	0.1547	0.1052	0.0719	0.0492	0.0337	0.0232
	0.9900	0.4624	0.2863	0.1942	0.1359	0.0956	0.0672	0.0469	0.0327	0.0226
12	1.2000	1.7558	2.0780	2.2983	2.4644	2.5971	2.7069	2.8003	2.8813	2.9529
	1.0800	1.0270	0.9471	0.8929	0.8523	0.8199	0.7936	0.7722	0.7547	0.7399
	1.2000	0.6442	0.4105	0.2788	0.1949	0.1381	0.0985	0.0705	0.0505	0.0362
	1.0800	0.5150	0.3268	0.2287	0.1662	0.1222	0.0899	0.0659	0.0481	0.0349
13	1.3000	1.8812	2.2153	2.4435	2.6155	2.7526	2.8660	2.9626	3.0464	3.1205
	1.1700	1.0963	1.0087	0.9491	0.9047	0.8697	0.8418	0.8192	0.8004	0.7842
	1.3000	0.7188	0.4716	0.3303	0.2386	0.1751	0.1295	0.0962	0.0716	0.0533
	1.1700	0.5679	0.3676	0.2636	0.1974	0.1503	0.1149	0.0878	0.0668	0.0506
14	1.4000	2.0056	2.3511	2.5869	2.7644	2.9059	3.0229	3.1225	3.2090	3.2854
	1.2600	1.1655	1.0698	1.0049	0.9567	0.9194	0.8897	0.8655	0.8451	0.8274
	1.4000	0.7944	0.5345	0.3842	0.2853	0.2156	0.1644	0.1261	0.0969	0.0746
	1.2600	0.6211	0.4086	0.2987	0.2290	0.1794	0.1417	0.1119	0.0882	0.0693
15	1.5000	2.1289	2.4856	2.7288	2.9116	3.0572	3.1777	3.2802	3.3693	3.4480
	1.3500	1.2346	1.1306	1.0603	1.0085	0.9687	0.9371	0.9111	0.8890	0.8699
	1.5000	0.8711	0.5989	0.4402	0.3347	0.2592	0.2029	0.1599	0.1264	0.1001
	1.3500	0.6744	0.4496	0.3338	0.2608	0.2090	0.1694	0.1377	0.1119	0.0908

A missing entry in the above table denotes a zero correct to four decimal places.

Table I. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p= 0.2

N/M	1	2	3	4	5	6	7	8	9	10
1	0.2000	0.3600	0.4880	0.5904	0.6723	0.7379	0.7903	0.8322	0.8658	0.8926
	0.1600	0.2304	0.2499	0.2418	0.2203	0.1934	0.1657	0.1396	0.1162	0.0958
	0.2000	0.0400	0.0080	0.0016	0.0003	0.0001	0.0000			
	0.1600	0.0384	0.0079	0.0016	0.0003	0.0001	0.0000			
2	0.4000	0.6688	0.8531	0.9829	1.0773	1.1485	1.2046	1.2505	1.2895	1.3236
	0.3200	0.3783	0.3558	0.3181	0.2860	0.2639	0.2507	0.2440	0.2417	0.2420
	0.4000	0.1312	0.0467	0.0168	0.0060	0.0022	0.0008	0.0003	0.0001	0.0000
	0.3200	0.1172	0.0447	0.0165	0.0060	0.0022	0.0008	0.0003	0.0001	0.0000
3	0.6000	0.9510	1.1703	1.3184	1.4267	1.5116	1.5818	1.6421	1.6951	1.7425
	0.4800	0.5047	0.4573	0.4177	0.3937	0.3800	0.3711	0.3637	0.3562	0.3481
	0.6000	0.2490	0.1173	0.0568	0.0277	0.0135	0.0066	0.0032	0.0016	0.0008
	0.4800	0.2089	0.1058	0.0538	0.0269	0.0133	0.0065	0.0032	0.0016	0.0008
4	0.8000	1.2180	1.4657	1.6323	1.7563	1.8551	1.9372	2.0071	2.0676	2.1206
	0.6400	0.6261	0.5642	0.5232	0.4969	0.4766	0.4584	0.4413	0.4253	0.4106
	0.8000	0.3820	0.2117	0.1226	0.0719	0.0424	0.0250	0.0148	0.0087	0.0051
	0.6400	0.3044	0.1788	0.1097	0.0671	0.0407	0.0244	0.0145	0.0086	0.0051
5	1.0000	1.4756	1.7489	1.9332	2.0711	2.1806	2.2709	2.3472	2.4128	2.4703
	0.8000	0.7465	0.6720	0.6242	0.5895	0.5612	0.5371	0.5171	0.5006	0.4873
	1.0000	0.5244	0.3222	0.2091	0.1386	0.0927	0.0622	0.0418	0.0281	0.0189
	0.8000	0.4011	0.2554	0.1749	0.1219	0.0848	0.0585	0.0401	0.0273	0.0185
6	1.2000	1.7267	2.0238	2.2242	2.3740	2.4925	2.5900	2.6723	2.7435	2.8061
	0.9600	0.8667	0.7786	0.7211	0.6786	0.6453	0.6191	0.5982	0.5814	0.5674
	1.2000	0.6733	0.4436	0.3106	0.2236	0.1630	0.1196	0.0881	0.0649	0.0479
	0.9600	0.4984	0.3326	0.2427	0.1834	0.1398	0.1065	0.0807	0.0608	0.0456
7	1.4000	1.9732	2.2924	2.5075	2.6680	2.7948	2.8992	2.9876	3.0642	3.1316
	1.1200	0.9867	0.8837	0.8162	0.7672	0.7300	0.7011	0.6776	0.6579	0.6407
	1.4000	0.8268	0.5727	0.4226	0.3219	0.2494	0.1950	0.1532	0.1207	0.0952
	1.1200	0.5961	0.4096	0.3101	0.2457	0.1987	0.1618	0.1318	0.1070	0.0865
8	1.6000	2.2162	2.5560	2.7848	2.9552	3.0899	3.2008	3.2949	3.3762	3.4478
	1.2800	1.1065	0.9876	0.9103	0.8551	0.8136	0.7808	0.7538	0.7308	0.7110
	1.6000	0.9838	0.7075	0.5423	0.4298	0.3473	0.2840	0.2338	0.1933	0.1603
	1.2800	0.6942	0.4866	0.3768	0.3069	0.2570	0.2183	0.1866	0.1596	0.1364
9	1.8000	2.4563	2.8156	3.0571	3.2369	3.3790	3.4961	3.5952	3.6809	3.7562
	1.4400	1.2259	1.0907	1.0036	0.9421	0.8956	0.8588	0.8286	0.8032	0.7817
	1.8000	1.1437	0.8468	0.6677	0.5447	0.4536	0.3828	0.3259	0.2791	0.2399
	1.4400	0.7927	0.5638	0.4431	0.3669	0.3134	0.2728	0.2402	0.2127	0.1888
10	2.0000	2.6941	3.0719	3.3254	3.5140	3.6632	3.7859	3.8898	3.9796	4.0586
	1.6000	1.3449	1.1931	1.0963	1.0281	0.9767	0.9361	0.9031	0.8755	0.8521
	2.0000	1.3059	0.9897	0.7977	0.6649	0.5661	0.4888	0.4262	0.3742	0.3301
	1.6000	0.8916	0.6413	0.5094	0.4263	0.3684	0.3252	0.2911	0.2631	0.2391
11	2.2000	2.9299	3.3252	3.5903	3.7873	3.9430	4.0711	4.1796	4.2734	4.3559
	1.7600	1.4636	1.2949	1.1882	1.1134	1.0572	1.0129	0.9770	0.9470	0.9215
	2.2000	1.4701	1.1356	0.9315	0.7897	0.6836	0.6003	0.5325	0.4760	0.4279
	1.7600	0.9909	0.7191	0.5760	0.4858	0.4230	0.3763	0.3400	0.3106	0.2860
12	2.4000	3.1640	3.5762	2.8521	4.0572	4.2192	4.3525	4.4653	4.5629	4.6487
	1.9200	1.5820	1.3962	1.2796	1.1981	1.1370	1.0891	1.0502	1.0176	0.9899
	2.4000	1.6360	1.2842	1.0685	0.9181	0.8052	0.7162	0.6437	0.5830	0.5312
	1.9200	1.0906	0.7974	0.6428	0.5455	0.4777	0.4273	0.3880	0.3565	0.3305
13	2.6000	3.3967	3.8249	4.1114	4.3242	4.4922	4.6304	4.7474	4.8486	4.9375
	2.0800	1.7001	1.4971	1.3705	1.2823	1.2164	1.1647	1.1227	1.0876	1.0578
	2.6000	1.8033	1.4350	1.2083	1.0498	0.9304	0.8361	0.7589	0.6942	0.6389
	2.0800	1.1906	0.8759	0.7100	0.6055	0.5327	0.4785	0.4362	0.4022	0.3742
14	2.8000	3.6280	4.0718	4.3683	4.5885	4.7624	4.9053	5.0263	5.1309	5.2229
	2.2400	1.8179	1.5975	1.4609	1.3660	1.2952	1.2398	1.1947	1.1572	1.1254
	2.8000	1.9720	1.5877	1.3506	1.1844	1.0588	0.9593	0.8778	0.8093	0.7506
	2.2400	1.2909	0.9548	0.7775	0.6658	0.5880	0.5300	0.4848	0.4483	0.4181
15	3.0000	3.8582	4.3169	4.6232	4.8506	5.0300	5.1775	5.3024	5.4103	5.5052
	2.4000	1.9354	1.6976	1.5510	1.4494	1.3737	1.3144	1.2664	1.2264	1.1925
	3.0000	2.1418	1.7422	1.4951	1.3214	1.1899	1.0856	0.9999	0.9277	0.8658
	2.4000	1.3915	1.0341	0.8453	0.7264	0.6435	0.5817	0.5336	0.4947	0.4626

A missing entry in the above table denotes a zero correct to four decimal places.

Table I. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.3

N/M	1	2	3	4	5	6	7	8	9	10
1	0.3000	0.5100	0.6570	0.7599	0.8319	0.8824	0.9176	0.9424	0.9596	0.9718
	0.2100	0.2499	0.2254	0.1825	0.1398	0.1038	0.0756	0.0543	0.0387	0.0274
	0.3000	0.0900	0.0270	0.0081	0.0024	0.0007	0.0002	0.0001	0.0000	0.0000
	0.2100	0.0819	0.0263	0.0080	0.0024	0.0007	0.0002	0.0001	0.0000	0.0000
2	0.6000	0.9318	1.1288	1.2566	1.3477	1.4183	1.4765	1.5264	1.5704	1.6098
	0.4200	0.4073	0.3475	0.3061	0.2833	0.2710	0.2630	0.2559	0.2483	0.2395
	0.6000	0.2682	0.1334	0.0677	0.0345	0.0176	0.0090	0.0046	0.0023	0.0012
	0.4200	0.2125	0.1170	0.0633	0.0333	0.0173	0.0089	0.0046	0.0023	0.0012
3	0.9000	1.3210	1.5566	1.7121	1.8270	1.9176	1.9917	2.0537	2.1064	2.1517
	0.6300	0.5598	0.4852	0.4401	0.4084	0.3818	0.3580	0.3371	0.3191	0.3042
	0.9000	0.4790	0.2937	0.1885	0.1229	0.0805	0.0529	0.0347	0.0228	0.0150
	0.6300	0.3458	0.2277	0.1573	0.1087	0.0742	0.0501	0.0335	0.0223	0.0148
4	1.2000	1.6942	1.9641	2.1434	2.2756	2.3790	2.4630	2.5335	2.5940	2.6471
	0.8400	0.7129	0.6200	0.5609	0.5176	0.4846	0.4594	0.4400	0.4247	0.4122
	1.2000	0.7058	0.4816	0.3482	0.2585	0.1943	0.1469	0.1114	0.0846	0.0642
	0.8400	0.4787	0.3365	0.2566	0.2020	0.1601	0.1266	0.0994	0.0776	0.0602
5	1.5000	2.0578	2.3585	2.5580	2.7049	2.8201	2.9144	2.9938	3.0623	3.1222
	1.0500	0.8661	0.7519	0.6794	0.6286	0.5911	0.5619	0.5380	0.5174	0.4993
	1.5000	0.9422	0.6852	0.5293	0.4220	0.3426	0.2810	0.2319	0.1920	0.1594
	1.0500	0.6117	0.4433	0.3511	0.2911	0.2473	0.2125	0.1830	0.1575	0.1351
6	1.8000	2.4148	2.7433	2.9611	3.1216	3.2476	3.3507	3.4375	3.5122	3.5775
	1.2600	1.0189	0.8823	0.7970	0.7378	0.6936	0.6586	0.6301	0.6063	0.5864
	1.8000	1.1852	0.8990	0.7235	0.6015	0.5102	0.4385	0.3802	0.3316	0.2903
	1.2600	0.7452	0.5497	0.4434	0.3752	0.3270	0.2906	0.2613	0.2365	0.2146
7	2.1000	2.7670	3.1211	3.3557	3.5286	3.6643	3.7754	3.8689	3.9494	4.0200
	1.4700	1.1712	1.0118	0.9133	0.8452	0.7945	0.7549	0.7230	0.6967	0.6744
	2.1000	1.4330	1.1203	0.9271	0.7918	0.6900	0.6096	0.5439	0.4889	0.4418
	1.4700	0.8791	0.6564	0.5356	0.4581	0.4035	0.3627	0.3308	0.3051	0.2835
8	2.4000	3.1153	3.4934	3.7435	3.9279	4.0727	4.1911	4.2909	4.3769	4.4523
	1.6800	1.3231	1.1404	1.0287	0.9518	0.8948	0.8504	0.8146	0.7874	0.7593
	2.4000	1.6847	1.3474	1.1379	0.9905	0.8790	0.7906	0.7181	0.6572	0.6050
	1.6800	1.0136	0.7636	0.6282	0.5415	0.4802	0.4342	0.3981	0.3691	0.3452
9	2.7000	3.4606	3.8610	4.1259	4.3210	4.4742	4.5996	4.7053	4.7963	4.8761
	1.8900	1.4745	1.2685	1.1434	1.0576	0.9942	0.9448	0.9049	0.8718	0.8438
	2.7000	1.9394	1.5792	1.3546	1.1960	1.0755	0.9797	0.9003	0.8343	0.7773
	1.8900	1.1484	0.8712	0.7211	0.6251	0.5574	0.5065	0.4664	0.4339	0.4069
10	3.0000	3.8034	4.2250	4.5036	4.7089	4.8701	5.0020	5.1132	5.2090	5.2929
	2.1000	1.6256	1.3959	1.2575	1.1628	1.0929	1.0386	0.9949	0.9585	0.9279
	3.0000	2.1966	1.8149	1.5761	1.4070	1.2782	1.1754	1.0907	1.0191	0.9574
	2.1000	1.2837	0.9791	0.8144	0.7090	0.6348	0.5790	0.5352	0.4997	0.4700
11	3.3000	4.1439	4.5857	4.8775	5.0924	5.2611	5.3992	5.5156	5.6160	5.7039
	2.3100	1.7763	1.5230	1.3710	1.2674	1.1910	1.1318	1.0840	1.0445	1.0110
	3.3000	2.4561	2.0539	1.8017	1.6226	1.4859	1.3767	1.2865	1.2101	1.1442
	2.3100	1.4192	1.0875	0.9081	0.7933	0.7124	0.6516	0.6040	0.5654	0.5334
12	3.6000	4.4826	4.9436	5.2480	5.4721	5.6480	5.7920	5.9134	6.0180	6.1098
	2.5200	1.9268	1.6496	1.4841	1.3715	1.2887	1.2244	1.1727	1.1300	1.0938
	3.6000	2.7174	2.2957	2.0307	1.8421	1.6981	1.5827	1.4873	1.4064	1.3365
	2.5200	1.5551	1.1962	1.0021	0.8778	0.7903	0.7245	0.6730	0.6312	0.5965
13	3.9000	4.8197	5.2992	5.6155	5.8484	6.0313	6.1809	6.3071	6.4158	6.5112
	2.7300	2.0769	1.7758	1.5967	1.4752	1.3859	1.3167	1.2610	1.2150	1.1761
	3.9000	2.9803	2.5399	2.2627	2.0652	1.9140	1.7928	1.6924	1.6072	1.5336
	2.7300	1.6912	1.3052	1.0963	0.9627	0.8684	0.7977	0.7422	0.6972	0.6598
14	4.2000	5.1554	5.6525	5.9804	6.2218	6.4113	6.5664	6.6971	6.8098	6.9086
	2.9400	2.2269	1.9017	1.7091	1.5785	1.4827	1.4085	1.3489	1.2996	1.2580
	4.2000	3.2446	2.7864	2.4974	2.2912	2.1333	2.0065	1.9014	1.8121	1.7348
	2.9400	1.8276	1.4144	1.1909	1.0478	0.9469	0.8711	0.8117	0.7635	0.7234
15	4.5000	5.4897	6.0040	6.3430	6.5926	6.7885	6.9488	7.0839	7.2004	7.3026
	3.1500	2.3766	2.0274	1.8211	1.6815	1.5792	1.5001	1.4365	1.3839	1.3396
	4.5000	3.5102	3.0348	2.7345	2.5200	2.3555	2.2234	2.1137	2.0205	1.9398
	3.1500	1.9642	1.5239	1.2857	1.1332	1.0256	0.9448	0.8814	0.8300	0.7873

Table I. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.4

N/M	1	2	3	4	5	6	7	8	9	10
1	0.4000	0.6400	0.7840	0.8704	0.9222	0.9533	0.9720	0.9832	0.9899	0.9940
	0.2400	0.2304	0.1693	0.1128	0.0717	0.0445	0.0272	0.0165	0.0100	0.0060
	0.4000	0.1600	0.0640	0.0256	0.0102	0.0041	0.0016	0.0007	0.0003	0.0001
	0.2400	0.1344	0.0599	0.0249	0.0101	0.0041	0.0016	0.0007	0.0003	0.0001
2	0.8000	1.1648	1.3606	1.4853	1.5757	1.6465	1.7041	1.7518	1.7917	1.8251
	0.4800	0.3968	0.3239	0.2834	0.2564	0.2329	0.2099	0.1871	0.1651	0.1444
	0.8000	0.4352	0.2662	0.1684	0.1075	0.0687	0.0440	0.0281	0.0180	0.0115
	0.4800	0.2970	0.2035	0.1414	0.0961	0.0640	0.0421	0.0274	0.0177	0.0114
3	1.2000	1.6573	1.8978	2.0540	2.1669	2.2534	2.3226	2.3798	2.4284	2.4708
	0.7200	0.5664	0.4719	0.4124	0.3694	0.3377	0.3146	0.2977	0.2852	0.2753
	1.2000	0.7427	0.5253	0.3932	0.3016	0.2341	0.1827	0.1430	0.1120	0.0878
	0.7200	0.4553	0.3376	0.2694	0.2215	0.1831	0.1507	0.1230	0.0996	0.0801
4	1.6000	2.1342	2.4124	2.5934	2.7248	2.8268	2.9095	2.9787	3.0378	3.0891
	0.9600	0.7361	0.6153	0.5415	0.4913	0.4545	0.4252	0.4007	0.3795	0.3611
	1.6000	1.0658	0.8097	0.6508	0.5396	0.4557	0.3894	0.3352	0.2897	0.2512
	0.9600	0.6131	0.4663	0.3831	0.3288	0.2900	0.2597	0.2343	0.2118	0.1912
5	2.0000	2.6014	2.9127	3.1156	3.2634	3.3783	3.4714	3.5491	3.6156	3.6736
	1.2000	0.9055	0.7578	0.6688	0.6080	0.5631	0.5282	0.5006	0.4781	0.4595
	2.0000	1.3986	1.1085	0.9269	0.7985	0.7013	0.6241	0.5608	0.5074	0.4615
	1.2000	0.7711	0.5946	0.4953	0.4301	0.3835	0.3486	0.3214	0.2996	0.2814
6	2.4000	3.0618	3.4031	3.6257	3.7880	3.9143	4.0169	4.1029	4.1766	4.2409
	1.4400	1.0745	0.8993	0.7949	0.7240	0.6722	0.6321	0.5999	0.5730	0.5500
	2.4000	1.7382	1.4176	1.2158	1.0723	0.9629	0.8756	0.8036	0.7429	0.6907
	1.4400	0.9295	0.7230	0.6075	0.5321	0.4780	0.4368	0.4041	0.3775	0.3556
7	2.8000	3.5172	3.8860	4.1266	4.3022	4.4390	4.5502	4.6435	4.7234	4.7932
	1.6800	1.2432	1.0402	0.9202	0.8392	0.7798	0.7340	0.6972	0.6669	0.6415
	2.8000	2.0828	1.7344	1.5141	1.3570	1.2368	1.1405	1.0608	0.9932	0.9349
	1.6800	1.0881	0.8515	0.7197	0.6339	0.5726	0.5262	0.4893	0.4591	0.4337
8	3.2000	3.9686	4.3629	4.6203	4.8082	4.9547	5.0739	5.1739	5.2597	5.3347
	1.9200	1.4115	1.1807	1.0450	0.9536	0.8869	0.8355	0.7943	0.7604	0.7317
	3.2000	2.4314	2.0572	1.8199	1.6502	1.5201	1.4156	1.3289	1.2553	1.1917
	1.9200	1.2471	0.9803	0.8321	0.7356	0.6668	0.6148	0.5737	0.5402	0.5123
9	3.6000	4.4167	4.8351	5.1081	5.3075	5.4631	5.5897	5.6960	5.7873	5.8670
	2.1600	1.5796	1.3207	1.1693	1.0676	0.9935	0.9364	0.8906	0.8530	0.8212
	3.6000	2.7832	2.3843	2.1317	1.9503	1.8108	1.6987	1.6056	1.5264	1.4578
	2.1600	1.4062	1.1093	0.9447	0.8376	0.7613	0.7035	0.6578	0.6206	0.5896
10	4.0000	4.8622	5.3032	5.5910	5.8014	5.9655	6.0991	6.2113	6.3077	6.3919
	2.4000	1.7475	1.4605	1.2932	1.1812	1.0996	1.0368	0.9866	0.9452	0.9104
	4.0000	3.1378	2.7165	2.4484	2.2560	2.1078	1.9886	1.8894	1.8049	1.7317
	2.4000	1.5656	1.2386	1.0574	2.9397	0.8558	0.7924	0.7422	0.7014	0.6672
11	4.4000	5.3054	5.7680	6.0698	6.2905	6.4627	6.6030	6.7208	6.8220	6.9105
	2.6400	1.9152	1.5999	1.4169	1.2944	1.2053	1.1369	1.0821	1.0371	0.9991
	4.4000	3.4946	3.0516	2.7693	2.5664	2.4101	2.2841	2.1792	2.0898	2.0122
	2.6400	1.7251	1.3680	1.1703	1.0420	0.9505	0.8813	0.8267	0.7822	0.7451
12	4.8000	5.7467	6.2298	6.5451	6.7755	6.9555	7.1021	7.2252	7.3311	7.4236
	2.8800	2.0828	1.7391	1.5402	1.4074	1.3108	1.2367	1.1774	1.1286	1.0876
	4.8000	3.8533	3.3897	3.0939	2.8811	2.7169	2.5845	2.4742	2.3801	2.2983
	2.8800	1.8848	1.4976	1.2834	1.1444	1.0454	0.9704	0.9113	0.8631	0.8229
13	5.2000	6.1862	6.6890	7.0172	7.2571	7.4444	7.5970	7.7253	7.8355	7.9319
	3.1200	2.2502	1.8782	1.6634	1.5201	1.4161	1.3362	1.2724	1.2199	1.1758
	5.2000	4.2138	3.7304	3.4216	3.1993	3.0277	2.8891	2.7736	2.6750	2.5893
	3.1200	2.0447	1.6273	1.3967	1.2470	1.1404	1.0597	0.9960	0.9441	0.9008
14	5.6000	6.6242	7.1460	7.4865	7.7354	7.9298	8.0883	8.2215	8.3359	8.4360
	3.3600	2.4174	2.0170	1.7863	1.6326	1.5211	1.4355	1.3671	1.3110	1.2637
	5.6000	4.5758	4.0734	3.7522	3.5207	3.3419	3.1975	3.0770	2.9741	2.8847
	3.3600	2.2046	1.7572	1.5101	1.3497	1.2355	1.1491	1.0808	1.0253	0.9789
15	6.0000	7.0608	7.6009	7.9533	8.2110	8.4122	8.5763	8.7142	8.8327	8.9364
	3.6000	2.5846	2.1557	1.9091	1.7449	1.6259	1.5346	1.4617	1.4018	1.3514
	6.0000	4.9392	4.4184	4.0852	3.8449	3.6592	3.5091	3.3839	3.2769	3.1838
	3.6000	2.3647	1.8873	1.6236	1.4526	1.3307	1.2386	1.1658	1.1065	1.0570

Table I. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.5

N/M	1	2	3	4	5	6	7	8	9	10
1	0.5000	0.7500	0.8750	0.9375	0.9687	0.9844	0.9922	0.9961	0.9980	0.9990
	0.2500	0.1875	0.1094	0.0586	0.0303	0.0154	0.0078	0.0039	0.0019	0.0010
	0.5000	0.2500	0.1250	0.0625	0.0312	0.0156	0.0078	0.0039	0.0020	0.0010
	0.2500	0.1875	0.1094	0.0586	0.0303	0.0154	0.0078	0.0039	0.0019	0.0010
2	1.0000	1.3750	1.5625	1.6797	1.7617	1.8218	1.8665	1.8999	1.9249	1.9437
	0.5000	0.3594	0.2773	0.2255	0.1835	0.1469	0.1158	0.0901	0.0695	0.0531
	1.0000	0.6250	0.4375	0.3203	0.2383	0.1782	0.1335	0.1001	0.0751	0.0563
	0.5000	0.3594	0.2773	0.2255	0.1835	0.1469	0.1158	0.0901	0.0695	0.0531
3	1.5000	1.9687	2.2031	2.3511	2.4558	2.5356	2.5995	2.6525	2.6974	2.7359
	0.7500	0.5303	0.4197	0.3538	0.3107	0.2800	0.2557	0.2346	0.2149	0.1963
	1.5000	1.0312	0.7969	0.6489	0.5442	0.4644	0.4005	0.3475	0.3026	0.2641
	0.7500	0.5303	0.4197	0.3538	0.3107	0.2800	0.2557	0.2346	0.2149	0.1963
4	2.0000	2.5469	2.8203	2.9946	3.1192	3.2145	3.2906	3.3533	3.4062	3.4519
	1.0000	0.7009	0.5615	0.4796	0.4241	0.3834	0.3525	0.3287	0.3100	0.2949
	2.0000	1.4531	1.1797	1.0054	0.8808	0.7855	0.7094	0.6467	0.5938	0.5481
	1.0000	0.7009	0.5615	0.4796	0.4241	0.3834	0.3525	0.3287	0.3099	0.2949
5	2.5000	3.1152	3.4229	3.6197	3.7612	3.8701	3.9577	4.0305	4.0923	4.1457
	1.2500	0.8715	0.7023	0.6041	0.5388	0.4914	0.4547	0.4250	0.4002	0.3791
	2.5000	1.8848	1.5771	1.3803	1.2388	1.1299	1.0423	0.9695	0.9077	0.8543
	1.2500	0.8715	0.7023	0.6041	0.5388	0.4914	0.4547	0.4250	0.4002	0.3791
6	3.0000	3.6768	4.0151	4.2323	4.3888	4.5096	4.6069	4.6879	4.7569	4.8169
	1.5000	1.0420	0.8428	0.7281	0.6518	0.5966	0.5544	0.5210	0.4937	0.4707
	3.0000	2.3232	1.9849	1.7677	1.6112	1.4904	1.3931	1.3121	1.2431	1.1831
	1.5000	1.0420	0.8428	0.7281	0.6518	0.5966	0.5544	0.5210	0.4937	0.4707
7	3.5000	4.2332	4.5997	4.8354	5.0056	5.1372	5.2435	5.3321	5.4077	5.4734
	1.7500	1.2125	0.9831	0.8517	0.7647	0.7018	0.6536	0.6152	0.5837	0.5573
	3.5000	2.7668	2.4003	2.1646	1.9944	1.8628	1.7565	1.6679	1.5923	1.5266
	1.7500	1.2125	0.9831	0.8517	0.7647	0.7018	0.6536	0.6152	0.5837	0.5573
8	4.0000	4.7855	5.1783	5.4311	5.6140	5.7556	5.8701	5.9636	6.0473	6.1183
	2.0000	1.3830	1.1233	0.9752	0.8772	0.8065	0.7526	0.7097	0.6745	0.6450
	4.0000	3.2145	2.8217	2.5689	2.3860	2.2444	2.1299	2.0344	1.9527	1.8817
	2.0000	1.3830	1.1233	0.9752	0.8772	0.8065	0.7526	0.7097	0.6745	0.6450
9	4.5000	5.3346	5.7519	6.0208	6.2156	6.3664	6.4886	6.5907	6.6779	6.7539
	2.2500	1.5534	1.2634	1.0985	0.9896	0.9111	0.8512	0.8036	0.7646	0.7318
	4.5000	3.6654	3.2481	2.9792	2.7844	2.6336	2.5114	2.4093	2.3221	2.2461
	2.2500	1.5534	1.2634	1.0985	0.9896	0.9111	0.8512	0.8036	0.7646	0.7318
10	5.0000	5.8810	6.3215	6.6056	6.8114	6.9711	7.1004	7.2086	7.3011	7.3817
	2.5000	1.7239	1.4035	1.2217	1.1019	1.0156	0.9498	0.8974	0.8546	0.8186
	5.0000	4.1190	3.6785	3.3944	3.1886	3.0289	2.8996	2.7914	2.6989	2.6183
	2.5000	1.7239	1.4035	1.2217	1.1019	1.0156	0.9498	0.8974	0.8546	0.8186
11	5.5000	6.4250	6.8875	7.1860	7.4025	7.5704	7.7066	7.8205	7.9180	8.0030
	2.7500	1.8943	1.5435	1.3449	1.2141	1.1200	1.0482	0.9912	0.9444	0.9053
	5.5000	4.5750	4.1124	3.8140	3.5975	3.4296	3.2934	3.1795	3.0820	2.9970
	2.7500	1.8943	1.5435	1.3449	1.2141	1.1200	1.0482	0.9912	0.9444	0.9053
12	6.0000	6.9671	7.4506	7.7628	7.9894	8.1652	8.3079	8.4272	8.5295	8.6186
	3.0000	2.0648	1.6835	1.4681	1.3263	1.2242	1.1465	1.0848	1.0342	0.9919
	6.0000	5.0329	4.5494	4.2372	4.0106	3.8348	3.6921	3.5727	3.4705	3.3814
	3.0000	2.0648	1.6835	1.4681	1.3263	1.2242	1.1465	1.0848	1.0342	0.9919
13	6.5000	7.5074	8.0111	8.3364	8.5726	8.7560	8.9049	9.0295	9.1363	9.2294
	3.2500	2.2352	1.8235	1.5912	1.4384	1.3285	1.2448	1.1784	1.1239	1.0784
	6.5000	5.4926	4.9889	4.6636	4.4274	4.2440	4.0951	3.9705	3.8637	3.7706
	3.2500	2.2352	1.8235	1.5912	1.4384	1.3285	1.2448	1.1784	1.1239	1.0784
14	7.0000	8.0461	8.5692	8.9072	9.1526	9.3433	9.4981	9.6278	9.7389	9.8358
	3.5000	2.4056	1.9634	1.7142	1.5504	1.4328	1.3430	1.2718	1.2136	1.1648
	7.0000	5.9539	5.4308	5.0928	4.8474	4.6567	4.5019	4.3722	4.2611	4.1642
	3.5000	2.4056	1.9634	1.7142	1.5504	1.4328	1.3430	1.2718	1.2136	1.1648
15	7.5000	8.5835	9.1252	9.4754	9.7298	9.9275	10.0880	10.2225	10.3378	10.4383
	3.7500	2.5761	2.1034	1.8373	1.6625	1.5369	1.4412	1.3654	1.3032	1.2512
	7.5000	6.4165	5.8748	5.5246	5.2702	5.0725	4.9120	4.7775	4.6622	4.5617
	3.7500	2.5761	2.1034	1.8373	1.6625	1.5369	1.4412	1.3654	1.3032	1.2512

Table II.

The cumulative distribution of the largest of M order statistics from a binomial population with parameter p and N trials.

p=.25

	M 1	2	3	4	5	6	7	8	9	10
N=1 X=0	0.75000	0.56250	0.42187	0.31641	0.23730	0.17798	0.13348	0.10011	0.07508	0.05631
N=2 X=0	0.56250	0.31641	0.17798	0.10011	0.05631	0.03168	0.01782	0.01002	0.00564	0.00317
1	0.93750	0.87891	0.82397	0.77248	0.72420	0.67893	0.63650	0.59672	0.55942	0.52446
N=3 X=0	0.42187	0.17798	0.07508	0.03168	0.01336	0.00564	0.00238	0.00100	0.00042	0.00018
1	0.84375	0.71191	0.60068	0.50682	0.42763	0.36081	0.30444	0.25687	0.21673	0.18287
2	0.98437	0.96899	0.95385	0.93895	0.92428	0.90984	0.89562	0.88163	0.86785	0.85429
N=4 X=0	0.31641	0.10011	0.03168	0.01002	0.00317	0.00100	0.00032	0.00010	0.00003	0.00001
1	0.73828	0.54506	0.40241	0.29709	0.21934	0.16193	0.11955	0.08826	0.06516	0.04811
2	0.94922	0.90102	0.85526	0.81183	0.77060	0.73147	0.69433	0.65907	0.62560	0.59383
3	0.99609	0.99220	0.98833	0.98447	0.98062	0.97679	0.97297	0.96917	0.96539	0.96162
N=5 X=0	0.23730	0.05631	0.01336	0.00317	0.00075	0.00018	0.00004	0.00001		
1	0.63281	0.40045	0.25341	0.16036	0.10148	0.06422	0.04064	0.02572	0.01627	0.01030
2	0.89648	0.80368	0.72049	0.64591	0.57905	0.51911	0.46537	0.41720	0.37401	0.33530
3	0.98437	0.96899	0.95385	0.93895	0.92428	0.90984	0.89562	0.88163	0.86785	0.85429
4	0.99902	0.99805	0.99707	0.99610	0.99513	0.99415	0.99318	0.99221	0.99124	0.99028
N=6 X=0	0.17798	0.03168	0.00564	0.00100	0.00018	0.00003	0.00001			
1	0.53394	0.28509	0.15222	0.08127	0.04340	0.02317	0.01237	0.00661	0.00353	0.00188
2	0.83057	0.68984	0.57296	0.47588	0.39525	0.32828	0.27266	0.22646	0.18809	0.15622
3	0.96240	0.92622	0.89139	0.85788	0.82563	0.79458	0.76471	0.73596	0.70829	0.68166
4	0.99536	0.99074	0.98615	0.98157	0.97702	0.97249	0.96798	0.96349	0.95902	0.95457
5	0.99976	0.99951	0.99927	0.99902	0.99878	0.99854	0.99829	0.99805	0.99780	0.99756
N=7 X=0	0.13348	0.01782	0.00238	0.00032	0.00004	0.00001				
1	0.44495	0.19798	0.08809	0.03919	0.01744	0.00776	0.00345	0.00154	0.00068	0.00030
2	0.75641	0.57215	0.43278	0.32736	0.24762	0.18730	0.14168	0.10716	0.08106	0.06131
3	0.92944	0.86386	0.80291	0.74626	0.69361	0.64467	0.59918	0.55691	0.51761	0.48109
4	0.98712	0.97441	0.96186	0.94947	0.93725	0.92517	0.91326	0.90150	0.88989	0.87843
5	0.99866	0.99732	0.99598	0.99464	0.99330	0.99197	0.99064	0.98931	0.98798	0.98665
6	0.99994	0.99988	0.99982	0.99976	0.99969	0.99963	0.99957	0.99951	0.99945	0.99939
N=8 X=0	0.10011	0.01002	0.00100	0.00010	0.00001					
1	0.36708	0.13475	0.04946	0.01816	0.00667	0.00245	0.00090	0.00033	0.00012	0.00004
2	0.67854	0.46042	0.31242	0.21199	0.14384	0.09760	0.06623	0.04494	0.03049	0.02069
3	0.88618	0.78532	0.69594	0.61673	0.54654	0.48433	0.42921	0.38036	0.33707	0.29870
4	0.97270	0.94615	0.92032	0.89520	0.87076	0.84699	0.82387	0.80138	0.77950	0.75822
5	0.99577	0.99156	0.98737	0.98320	0.97904	0.97491	0.97079	0.96668	0.96260	0.95853
6	0.99962	0.99924	0.99886	0.99847	0.99809	0.99771	0.99733	0.99695	0.99657	0.99619
7	0.99998	0.99997	0.99995	0.99994	0.99992	0.99991	0.99989	0.99988	0.99986	0.99985
N=9 X=0	0.07508	0.00564	0.00042	0.00003						
1	0.30034	0.09020	0.02709	0.00814	0.00244	0.00073	0.00022	0.00007	0.00002	0.00001
2	0.60068	0.36081	0.21673	0.13019	0.07820	0.04697	0.02822	0.01695	0.01018	0.00612
3	0.83427	0.69601	0.58067	0.48443	0.40415	0.33717	0.28129	0.23468	0.19579	0.16334
4	0.95107	0.90454	0.86028	0.81819	0.77816	0.74009	0.70388	0.66944	0.63558	0.60553
5	0.99001	0.98011	0.97032	0.96062	0.95102	0.94151	0.93210	0.92279	0.91356	0.90443
6	0.99866	0.99732	0.99598	0.99464	0.99330	0.99197	0.99064	0.98931	0.98798	0.98665
7	0.99989	0.99979	0.99968	0.99957	0.99947	0.99936	0.99925	0.99915	0.99904	0.99893
8		0.99999	0.99999	0.99998	0.99998	0.99998	0.99997	0.99997	0.99997	0.99996
N=10 X=0	0.05631	0.00317	0.00018	0.00001						
1	0.24403	0.05955	0.01453	0.00355	0.00087	0.00021	0.00005	0.00001		
2	0.52559	0.27625	0.14519	0.07631	0.04011	0.02108	0.01108	0.00582	0.00306	0.00161
3	0.77588	0.60198	0.46706	0.36238	0.28116	0.21815	0.16926	0.13132	0.10189	0.07905
4	0.92187	0.84985	0.78345	0.72224	0.66582	0.61380	0.56585	0.52164	0.48088	0.44331
5	0.98027	0.96093	0.94198	0.92339	0.90518	0.88732	0.86982	0.85266	0.83583	0.81935
6	0.99649	0.99300	0.98952	0.98605	0.98259	0.97915	0.97572	0.97230	0.96889	0.96549
7	0.99958	0.99917	0.99875	0.99834	0.99792	0.99751	0.99709	0.99668	0.99626	0.99585
8	0.99997	0.99994	0.99991	0.99988	0.99985	0.99982	0.99979	0.99976	0.99973	0.99970
9					0.99999	0.99999	0.99999	0.99999	0.99999	0.99999

A missing entry in the above table denotes a 0 or a 1 correct to five decimal places.

Table II. (con't)

The cumulative distribution of the largest of M order statistics from a binomial population with parameter p and N trials.

p=.50

	M	1	2	3	4	5	6	7	8	9	10
N=1	X=0	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
N=2	X=0	0.25000	0.06250	0.01562	0.00391	0.00098	0.00024	0.00006	0.00002		
	1	0.75000	0.56250	0.42187	0.31641	0.23730	0.17798	0.13348	0.10011	0.07508	0.05631
N=3	X=0	0.12500	0.01562	0.00195	0.00024	0.00003					
	1	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00092
	2	0.87500	0.76562	0.66992	0.58618	0.51291	0.44880	0.39270	0.34361	0.30066	0.26308
N=4	X=0	0.06250	0.00391	0.00024	0.00002						
	1	0.31250	0.09766	0.03052	0.00954	0.00298	0.00093	0.00029	0.00009	0.00003	0.00001
	2	0.68750	0.47266	0.32495	0.22340	0.15359	0.10559	0.07260	0.04991	0.03431	0.02359
	3	0.93750	0.87891	0.82397	0.77248	0.72420	0.67893	0.63650	0.59672	0.55942	0.52446
N=5	X=0	0.03125	0.00098	0.00003							
	1	0.18750	0.03516	0.00659	0.00124	0.00023	0.00004	0.00001			
	2	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
	3	0.81250	0.66016	0.53638	0.43581	0.35409	0.28770	0.23376	0.18993	0.15432	0.12538
	4	0.96875	0.93848	0.90915	0.88074	0.85322	0.82655	0.80072	0.77570	0.75146	0.72798
N=6	X=0	0.01562	0.00024								
	1	0.10937	0.01196	0.00131	0.00014	0.00002					
	2	0.34375	0.11816	0.04062	0.01396	0.00480	0.00165	0.00057	0.00019	0.00007	0.00002
	3	0.65625	0.43066	0.28262	0.18547	0.12172	0.07988	0.05242	0.03440	0.02257	0.01481
	4	0.89062	0.79321	0.70646	0.62919	0.56037	0.49908	0.44449	0.39588	0.35258	0.31401
	5	0.98437	0.96899	0.95385	0.93895	0.92428	0.90984	0.89562	0.88163	0.86785	0.85429
N=7	X=0	0.00781	0.00006								
	1	0.06250	0.00391	0.00024	0.00002						
	2	0.22656	0.05133	0.01163	0.00263	0.00060	0.00014	0.00003	0.00001		
	3	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
	4	0.77344	0.59821	0.46267	0.35785	0.27677	0.21407	0.16557	0.12806	0.09904	0.07660
	5	0.93750	0.87891	0.82397	0.77248	0.72420	0.67893	0.63650	0.59672	0.55942	0.52446
	6	0.99219	0.98444	0.97674	0.96911	0.96154	0.95403	0.94658	0.93918	0.93184	0.92456
N=8	X=0	0.00391	0.00002								
	1	0.03516	0.00124	0.00004							
	2	0.14453	0.02089	0.00302	0.00044	0.00006	0.00001				
	3	0.36328	0.13197	0.04794	0.01742	0.00633	0.00230	0.00084	0.00030	0.00011	0.00004
	4	0.63672	0.40541	0.25813	0.16436	0.10465	0.06663	0.04243	0.02701	0.01720	0.01095
	5	0.85547	0.73183	0.62605	0.53557	0.45816	0.39194	0.33530	0.28684	0.24538	0.20991
	6	0.96484	0.93092	0.89820	0.86662	0.83615	0.80676	0.77839	0.75103	0.72462	0.69915
	7	0.99609	0.99220	0.98833	0.98447	0.98062	0.97679	0.97297	0.96917	0.96539	0.96162
N=9	X=0	0.00195									
	1	0.01953	0.00038	0.00001							
	2	0.08984	0.00807	0.00073	0.00007	0.00001					
	3	0.25391	0.06447	0.01637	0.00416	0.00106	0.00027	0.00007	0.00002		
	4	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
	5	0.74609	0.55666	0.41532	0.30987	0.23119	0.17249	0.12869	0.09602	0.07164	0.05345
	6	0.91016	0.82838	0.75396	0.68622	0.62457	0.56845	0.51738	0.47090	0.42859	0.39009
	7	0.98047	0.96132	0.94254	0.92413	0.90608	0.88839	0.87104	0.85402	0.83734	0.82099
	8	0.99805	0.99610	0.99415	0.99221	0.99027	0.98834	0.98641	0.98448	0.98256	0.98064
N=10	X=0	0.00098									
	1	0.01074	0.00012								
	2	0.05469	0.00299	0.00016	0.00001						
	3	0.17187	0.02954	0.00508	0.00087	0.00015	0.00003				
	4	0.37695	0.14209	0.05356	0.02019	0.00761	0.00287	0.00108	0.00041	0.00015	0.00006
	5	0.62305	0.38819	0.24186	0.15069	0.09389	0.05850	0.03645	0.02271	0.01415	0.00881
	6	0.82812	0.68579	0.56792	0.47031	0.38947	0.32253	0.26710	0.22119	0.18317	0.15169
	7	0.94531	0.89362	0.84475	0.79855	0.75488	0.71360	0.67457	0.63768	0.60281	0.56984
	8	0.98926	0.97863	0.96812	0.95772	0.94743	0.93725	0.92718	0.91722	0.90737	0.89762
	9	0.99902	0.99805	0.99707	0.99610	0.99513	0.99415	0.99318	0.99221	0.99124	0.99028

A missing entry in the above table denotes a 0 or a 1 correct to five decimal places.

Table II.

The cumulative distribution of the smallest of M order statistics from a binomial population with parameter p and N trials.

p=0.25

	M	1	2	3	4	5	6	7	8	9	10
N=1	X=0	0.75000	0.93750	0.98437	0.99609	0.99902	0.99976	0.99994	0.99998		
N=2	X=0	0.56250	0.80859	0.91626	0.96336	0.98397	0.99299	0.99693	0.99866	0.99941	0.99974
	1	0.93750	0.99609	0.99976	0.99998						
N=3	X=0	0.42187	0.66577	0.80677	0.88829	0.93542	0.96266	0.97841	0.98752	0.99279	0.99583
	1	0.84375	0.97559	0.99619	0.99940	0.99991	0.99999				
	2	0.98437	0.99976								
N=4	X=0	0.31641	0.53270	0.68056	0.78163	0.85072	0.89796	0.93024	0.95231	0.96740	0.97772
	1	0.73828	0.93150	0.98207	0.99531	0.99877	0.99968	0.99992	0.99998	0.99999	
	2	0.94922	0.99742	0.99987	0.99999						
	3	0.99609	0.99998								
N=5	X=0	0.23730	0.41830	0.55634	0.66162	0.74192	0.80316	0.84987	0.88550	0.91267	0.93339
	1	0.63281	0.86517	0.95049	0.98182	0.99333	0.99755	0.99910	0.99967	0.99988	0.99996
	2	0.89648	0.98928	0.99889	0.99989	0.99999					
	3	0.98437	0,99976								
	4	0.99902									
N=6	X=0	0.17798	0.32428	0.44454	0.54340	0.62467	0.69147	0.74638	0.79152	0.82862	0.85913
	1	0.53394	0.78278	0.89876	0.95282	0.97801	0.98975	0.99522	0.99777	0.99896	0.99952
	2	0.83057	0.97129	0.99514	0.99918	0.99986	0.99998				
	3	0.96240	0.99859	0.99995							
	4	0.99536	0.99998								
	5	0.99976									
N=7	X=0	0.13348	0.24915	0.34938	0.43622	0.51148	0.57669	0.63319	0.68216	0.72458	0.76135
	1	0.44495	0.69192	0.82900	0.90508	0.94732	0.97076	0.98377	0.99099	0.99500	0.99722
	2	0.75641	0.94066	0.98555	0.99648	0.99914	0.99979	0.99995	0.99999	1.00000	
	3	0.92944	0.99502	0.99965	0.99998						
	4	0.98712	0.99983								
	5	0.99866									
	6	0.99994									
N=8	X=0	0.10011	0.19020	0.27127	0.34423	0.40988	0.46896	0.52212	0.56996	0.61302	0.65176
	1	0.36708	0.59941	0.74646	0.83953	0.89844	0.93572	0.95931	0.97425	0.98370	0.98968
	2	0.67854	0.89667	0.96678	0.98932	0.99657	0.99890	0.99965	0.99989	0.99996	0.99999
	3	0.88618	0.98705	0.99853	0.99983	0.99998					
	4	0.97270	0.99925	0.99998							
	5	0.99577	0.99998								
	6	0.99962									
	7	0.99998									
N=9	X=0	0.07508	0.14453	0.20876	0.26817	0.32312	0.37395	0.42095	0.46443	0.50464	0.54184
	1	0.30034	0.51047	0.65750	0.76036	0.83234	0.88269	0.91792	0.94257	0.95982	0.97189
	2	0.60068	0.84054	0.93632	0.97457	0.98985	0.99595	0.99838	0.99935	0.99974	0.99990
	3	0.83427	0.97253	0.99545	0.99925	0.99987	0.99998				
	4	0.95107	0.99761	0.99988	0.99999						
	5	0.99001	0.99990								
	6	0.99866									
	7	0.99989									
N=10	X=0	0.05631	0.10946	0.15961	0.20693	0.25159	0.29374	0.33351	0.37104	0.40646	0.43988
	1	0.24403	0.42850	0.56796	0.67339	0.75309	0.81334	0.85889	0.89333	0.91936	0.93904
	2	0.52559	0.77494	0.89323	0.94935	0.97597	0.98860	0.99459	0.99743	0.99878	0.99942
	3	0.77588	0.94977	0.98874	0.99748	0.99943	0.99987	0.99997	0.99999		
	4	0.92187	0.99390	0.99952	0.99996						
	5	0.98027	0.99961	0.99999							
	6	0.99649	0.99999								
	7	0.99958									
	8	0.99997									

A missing entry in this table denotes 1 correct to five decimal places.

Table II.

The cumulative distribution of the smallest of M order statistics from a binomial population with parameter p and N trials.

p=0.50

	M=1	2	3	4	5	6	7	8	9	10
N=1 X=0	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
N=2 X=0	0.25000	0.43750	0.57812	0.68359	0.76270	0.82202	0.86652	0.89989	0.92492	0.94369
1	0.75000	0.93750	0.98437	0.99609	0.99902	0.99976	0.99994	0.99998		
N=3 X=0	0.12500	0.23437	0.33008	0.41382	0.48709	0.55120	0.60730	0.65639	0.69934	0.73692
1	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
2	0.87500	0.98437	0.99805	0.99976	0.99997					
N=4 X=0	0.06250	0.12109	0.17603	0.22752	0.27580	0.32107	0.36350	0.40328	0.44058	0.47554
1	0.31250	0.52734	0.67505	0.77660	0.84641	0.89441	0.92740	0.95009	0.96569	0.97641
2	0.68750	0.90234	0.96948	0.99046	0.99702	0.99907	0.99971	0.99991	0.99997	0.99999
3	0.93750	0.99609	0.99976	0.99998						
N=5 X=0	0.03125	0.06152	0.09085	0.11926	0.14678	0.17345	0.19928	0.22430	0.24854	0.27202
1	0.18750	0.33984	0.46362	0.56419	0.64591	0.71230	0.76624	0.81007	0.84568	0.87462
2	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
3	0.81250	0.96484	0.99341	0.99876	0.99977	0.99996	0.99999			
4	0.96875	0.99902	0.99997							
N=6 X=0	0.01562	0.03101	0.04615	0.06105	0.07572	0.09016	0.10438	0.11837	0.13215	0.14571
1	0.10937	0.20679	0.29354	0.37081	0.43963	0.50092	0.55551	0.60412	0.64742	0.68599
2	0.34375	0.56934	0.71738	0.81453	0.87828	0.92012	0.94758	0.96560	0.97743	0.98519
3	0.65625	0.88184	0.95938	0.98604	0.99520	0.99835	0.99943	0.99981	0.99993	0.99998
4	0.89062	0.98804	0.99869	0.99986	0.99998					
5	0.98437	0.99976								
N=7 X=0	0.00781	0.01556	0.02325	0.03089	0.03846	0.04597	0.05342	0.06082	0.06815	0.07543
1	0.06250	0.12109	0.17603	0.22752	0.27580	0.32107	0.36350	0.40328	0.44058	0.47554
2	0.22656	0.40179	0.53733	0.64215	0.72323	0.78593	0.83443	0.97194	0.90096	0.92340
3	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
4	0.77344	0.94867	0.98837	0.99737	0.99940	0.99986	0.99997	0.99999		
5	0.93750	0.99609	0.99976	0.99998						
6	0.99219	0.99994								
N=8 X=0	0.00391	0.00780	0.01167	0.01553	0.01938	0.02321	0.02703	0.03083	0.03461	0.03838
1	0.03516	0.06908	0.10180	0.13338	0.16385	0.19324	0.22161	0.24897	0.27538	0.30085
2	0.14453	0.26817	0.37395	0.46443	0.54184	0.60806	0.66470	0.71316	0.75462	0.79009
3	0.36328	0.59459	0.74187	0.83564	0.89535	0.93337	0.95757	0.97299	0.98280	0.98905
4	0.63672	0.86803	0.95206	0.98258	0.99367	0.99770	0.99916	0.99970	0.99989	0.99996
5	0.85547	0.97911	0.99698	0.99956	0.99994	0.99999				
6	0.96484	0.99876	0.99996							
7	0.99609	0.99998								
N=9 X=0	0.00195	0.00390	0.00585	0.00779	0.00973	0.01166	0.01359	0.01552	0.01744	0.01936
1	0.01953	0.03868	0.05746	0.07587	0.09392	0.11161	0.12896	0.14598	0.16266	0.17901
2	0.08984	0.17162	0.24604	0.31378	0.37543	0.43155	0.48262	0.52910	0.57141	0.60991
3	0.25391	0.44334	0.58468	0.69013	0.76881	0.82751	0.87131	0.90398	0.92836	0.94655
4	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
5	0.74609	0.93553	0.98363	0.99584	0.99894	0.99973	0.99993	0.99998		
6	0.91016	0.99193	0.99927	0.99993	0.99999					
7	0.98047	0.99962	0.99999							
8	0.99805									
N=10 X=0	0.00098	0.00195	0.00293	0.00390	0.00487	0.00585	0.00682	0.00779	0.00875	0.00972
1	0.01074	0.02137	0.03188	0.04228	0.05257	0.06275	0.07281	0.08277	0.09263	0.10238
2	0.05469	0.10638	0.15525	0.20145	0.24512	0.28640	0.32543	0.36232	0.39719	0.43016
3	0.17187	0.31421	0.43208	0.52969	0.61053	0.67747	0.73290	0.77881	0.81683	0.84831
4	0.37695	0.61181	0.75814	0.84931	0.90611	0.94150	0.96355	0.97729	0.98585	0.99119
5	0.62305	0.85791	0.94644	0.97981	0.99239	0.99713	0.99892	0.99959	0.99985	0.99994
6	0.82812	0.97046	0.99492	0.99913	0.99985	0.99997				
7	0.94531	0.99701	0.99984	0.99999						
8	0.98926	0.99988								
9	0.99902									

A missing entry in the above table denotes a 1 correct to five decimal places.

PURDUE UNIVERSITY
SOUTHERN ILLINOIS UNIVERSITY

REFERENCES

[1] Gupta, S. S. (1966). Selection and ranking procedures and order statistics for the binomial distribution, *Proceedings of the International Symposium on classical and contagious discrete distributions held in Montreal (1963)*, Pergamon Press, New York, 219-230.

[2] Gupta, S. S. and Sobel, M. (1960). Selecting a subset containing the best of several binomial populations, *Chapter 20 in Contributions to Probability and Statistics (Hotelling Volume)*, Stanford University Press, Stanford.

[3] Khatri, C. G. (1962). Distributions of order statistics for discrete case, *Ann. Inst. Statist. Math.*, **14**, 167-171.

[4] Siotani, M. (1956). Order statistics for discrete case with a numerical application to the binomial distribution, *Ann. Inst. Statist. Math.*, **8**, 95-104.

[5] Siotani, M. and Ozawa, M. (1958). Tables for testing the homogeneity of k independent binomial experiments on a certain event based on the range, *Ann. Inst. Statist. Math.*, **10**, 47-63.

[6] Sobel, M. and Huyett, M. J. (1957). Selecting the best one of several binomial populations, *Bell System Tech. J.*, **36**, 537-576.