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AN ANALYSIS OF STATISTICAL RESPONSE OF BACKRASH*

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1. Introduction

Nowadays, analytical treatments of non-linear element without memory are well established but the effects of non-linearity with memory are quite diverse and an unified analytical treatment cannot be expected. In this paper we shall show some results of applications of the method of statistical estimation of frequency response function to the analysis of the response of backrash, to see the effects of non-linearity on the method.

2. Set-up of the experimental system

The effect of backrash is explained by Fig. 1. Experimental records are obtained by using the output of a low frequency random noise generator as input to a simulating circuit of backrash. Fig. 2 shows the block diagram of the experimental system. Observation was made on the effect of the width of backrash by changing the gain G in three levels for a fixed backrash with $b=0.5$ volt. The set-up of the simulating circuit and its block diagram is given in Fig. 3, where

$$G_B=\frac{1}{r_F C} \quad \left(\frac{1}{M\Omega\mu F}\right),$$

r_F=sum of r_b and the forward resistance of diode, $C=0.01\,\mu F$ and the real backrash corresponds to the case where $G_B=\infty$.

3. Numerical results

The following values of parameters were used for the computation:

$$\begin{aligned}&\Delta t=1/200 \text{ sec}\\&M=1500\\&H=199\\&h=80\\&K=0\\&S=0.5.\end{aligned}$$

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As to the definition of parameters, see the glossary in the paper by Akaike [pp. 5–6 of this supplement]. Normalized correlograms $C_{xx}(l)/C_{xx}(0)$, $C_{yy}(l)/C_{yy}(0)$ and $C_{yx}(l)/\sqrt{C_{xx}(0)C_{yy}(0)}$ with values $\sqrt{C_{xx}(0)}$ and $\sqrt{C_{yy}(0)}$ are given in Fig. 4 for three settings of G. It can be seen that the input $x(t)$ has a nearly exponential auto-correlation. In this example $\sqrt{C_{xx}(0)}$ and $\sqrt{C_{yy}(0)}$ are larger for minimum G than for medium G, but, as will be seen in Fig. 5, this is due to the fluctuation of very low frequency component in input and above 3 c.p.s. the powers of input and output are obviously lower for minimum G. We can also see that as G decreases the ordinate giving the maximum of $C_{yx}(l)$ shifts to the right, which will show that as the effect of backrash increases the lag of the response increases.

The results of spectral analysis are given in Fig. 5. As is intuitively expected the coherencies are high at low frequencies and decreases towards high frequency, but the response of the system is fairly high even for higher frequencies. Thus, if a detailed analysis of the response characteristics of the non-linearity is expected, it will be necessary to adopt a smaller value of Δt. Fig. 6 shows the estimates of impulsive response, in a statistical sense, of the system obtained by inverse Fourier transformation from the $\hat{A}(f)$s without compensation for S.

In these transformations any compensation was not made for the sampling variability of the estimate due to the low coherencies at higher frequencies. The results strongly suggest that for the analysis of this type, the present value of Δt is inappropriately large. On the other hand, the shapes of the power spectra $\hat{P}_{xx}(f)$, $\hat{P}_{yy}(f)$ of input and output suggest that the present Δt is sufficiently small for the analysis of ordinary frequency response characteristics of the system under the present inputs. It seems that this example is showing clearly the usually implicit difference between the standpoints of the two approaches, that from time-domain and that from frequency-domain.

Taking into account that $\sqrt{C_{xx}(0)}$ s are not so much greater than $b=0.5$ volt in the present examples, we may consider that the present values of $\hat{A}(f)$ suggest that so long as the backrash appears as an open loop element for the input treated here, the non-linearity is fairly insensitive to the variation of the power of input. This means that even in many practical applications of the statistical method of estimation of frequency response function to the evaluation of a system containing significant non-linearities, we may obtain, at least apparently, fairly stable results for a slight instability of input.

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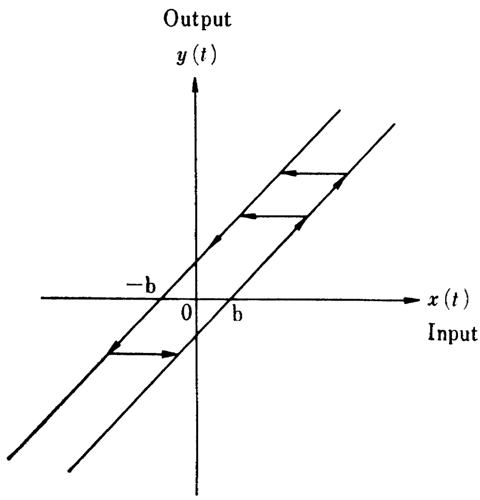

Fig. 1

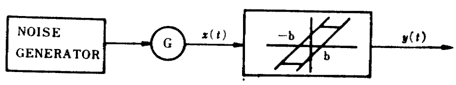

Fig. 2

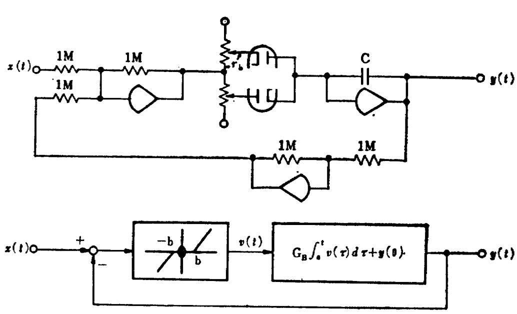

Fig. 3

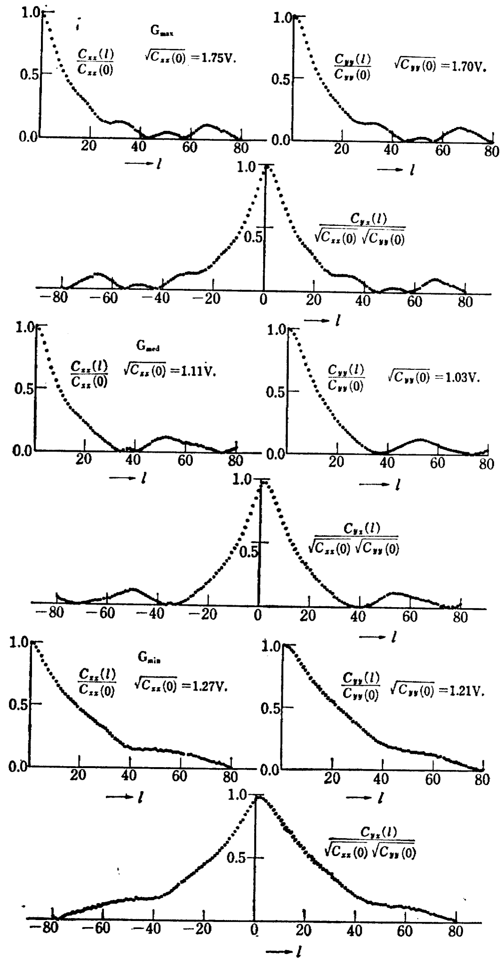

Fig. 4

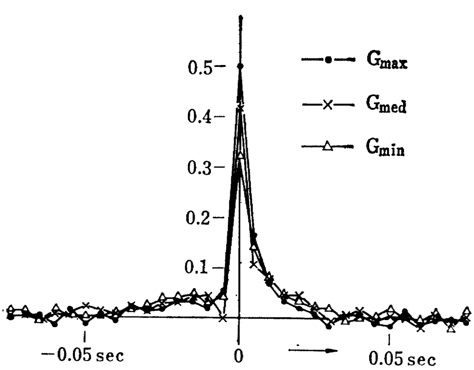

Fig. 6

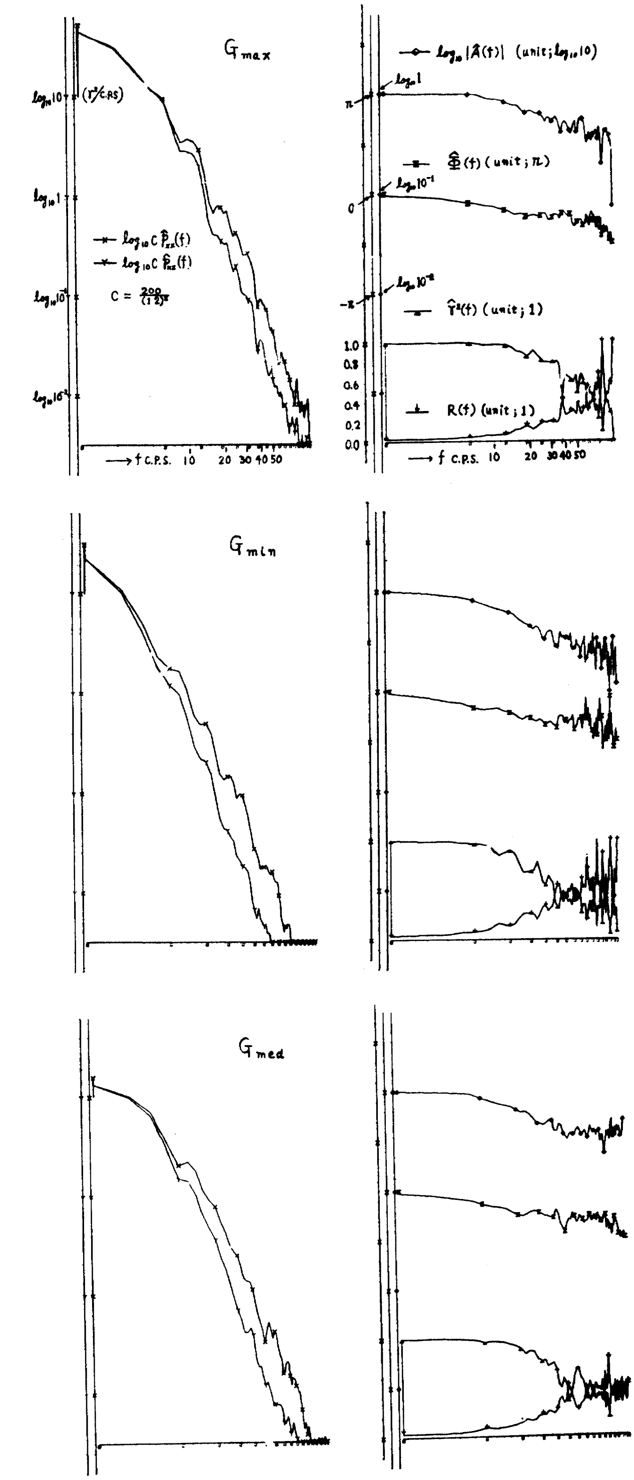

Fig. 5